ANSWER

3. a) $\frac{3}{8}$ b) $\frac{1}{8}$

4. a) $\frac{1}{2}$ b) $\frac{2}{7}$

5. a) $\frac{8}{12}$ or $\frac{2}{3}$ b) $\frac{7}{12}$ c) 0

6. a) independent b) dependent
   c) independent d) dependent

7. a) Tree diagram outcomes: (H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)
   b) independent; the outcome of flipping a coin does not affect the outcome of flipping another coin.

8. a) $\frac{1}{156}$ b) $\frac{1}{78}$ c) $\frac{15}{26}$

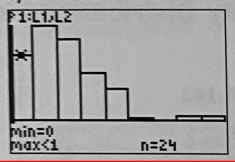
9. a) 26 400 b) 31 119

10. a) 108 b) 252 c) 894

11. a) $\frac{15!}{3!3!3!3!3!}$ or ${}_{15}C_3 \times {}_{12}C_3 \times {}_9C_3 \times {}_6C_3 \times {}_3C_3$ or 168 168 000
    b) ${}_{15}C_5 \times {}_{10}C_5 \times {}_5C_5$ or 756 756

12. a) 44 352 b) 0.0707… c) 0.3426 …
    d) 0.2304 e) 0.0823 …

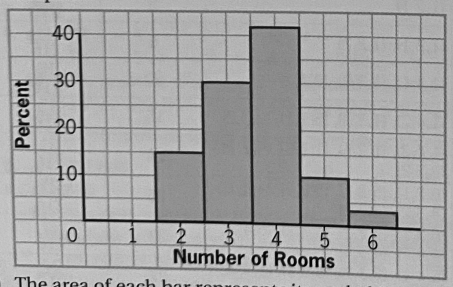
13. a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
    b) $1024x^5 + 3840x^4y + 5760x^3y^2 + 4320x^2y^3 + 1620xy^4 + 243y^5$
    c) 1 d) 1

14. 

4.1 Probability Distributions, pages 144–153

Example 1 Your Turn

a) the number of rooms in apartments in a particular complex

b) 

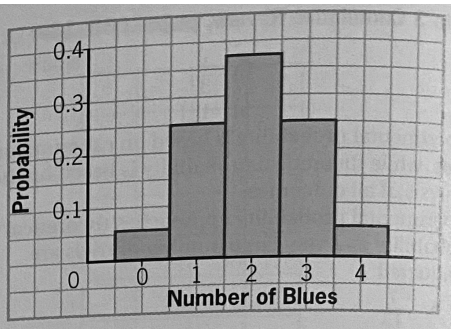
c) The area of each bar represents its probability, as a percent. The width of each bar is 1, so the probability, as a percent is shown on the vertical axis.

d) Four-room apartments occur most frequently, and the probability decreases as the room value increases or decreases from four.

e) Sum of the probabilities is 1. Yes.

Example 2 Your Turn

a) Tree diagram outcomes:
(R, R, R, R), (R, R, R, B), (R, R, B, R), (R, R, B, B), (R, B, R, R), (R, R, R, B), (R, B, B, R), (R, B, B, B), (B, R, R, R), (B, R, R, B), (B, R, B, R), (B, R, B, B), (B, B, R, R), (B, B, R, B), (B, B, B, R), (B, B, B, B)

b) 

c) 2

d) On average, the spinner would land on blue 2 out of the 4 spins.

Example 3 Your Turn

a) −$2.25

b) Answers may vary. No; You lose money if you buy a ticket.

c) Answers may vary. The price could be reduced or more prizes could be given away.

Reflect

R1. While it is impossible to have 1.8 children, expected values are predicted average values and should not be rounded.

R2. Answers may vary. The number of email messages you receive each day of the week and the number of students in each mathematics class at your school. These examples are discrete because each must be a whole number.

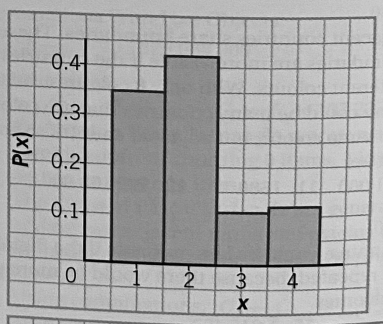
R3. Answers may vary. Create a table showing all possible sums of the two 12-sided dice. Determine the frequency of each sum and its probability. Then, construct a histogram to illustrate the probability distribution.

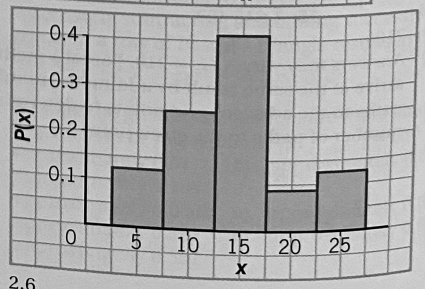
Practise

1. a) discrete b) continuous c) discrete
   d) discrete e) continuous

2. A

3. C

4. a) 

   b) 

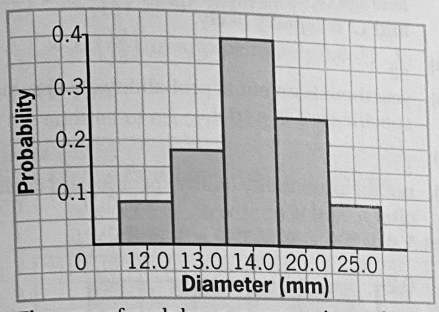
5. a) 2.6 b) 3.8

Apply

6. a) the diameter of the marbles in a bag

b) discrete; the number of marbles in a bag is a whole number

c)



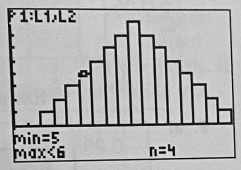
d) The area of each bar represents its probability. The width of each bar is 1, so the probability is shown on the vertical axis.

e) about 16.067; the weighted mean of the outcomes equals the expectation

7. a)

Sum, x	Frequency
2	1
3	2
4	3
5	4
6	5
7	6
8	7
9	8
10	7
11	6
12	5
13	4
14	3
15	2
16	1

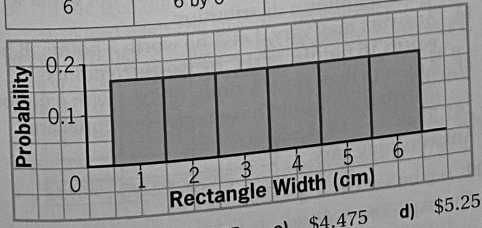
b)



c) 9; On average, the expected sum of two dice is 9.

8. Answers may vary. Perimeter = 24 cm.

Rectangle Width (cm), x	Distribution of Dimensions	Frequency	Probability, P(x)
1	1 by 11	1	$\frac{1}{6}$
2	2 by 10	1	$\frac{1}{6}$
3	3 by 9	1	$\frac{1}{6}$
4	4 by 8	1	$\frac{1}{6}$
5	5 by 7	1	$\frac{1}{6}$
6	6 by 6	1	$\frac{1}{6}$



9. a) $\frac{9}{2000}$ **b)** \$0.525 **c)** \$4.475 **d)** \$5.25

11. a) Let D represent rolling doubles and ND represent not rolling doubles. Tree diagram outcomes: (D, D, D), (D, D, ND), (D, ND, D), (D, ND, ND), (ND, D, D), (ND, D, ND), (ND, ND, D), (ND, ND, ND)

b)

Number of Doubles	Distribution of Doubles	Probability
0	(ND,ND,ND)	$\frac{125}{216}$
1	(D,ND,ND) (ND,D,ND) (ND,ND,D)	$\frac{75}{216}$
2	(D,D,ND) (D,ND,D) (ND,D,D)	$\frac{15}{216}$
3	(D,D,D)	$\frac{1}{216}$

c) 0.5

12.

Sum	Possible Groupings	Number of Outcomes	Probability
3	(1,1,1)	1	$\frac{1}{216}$
4	(1,2,1)	3	$\frac{3}{216}$
5	(1,3,1), (1,2,2)	6	$\frac{6}{216}$
6	(1,4,1), (1,3,2), (2,2,2)	10	$\frac{10}{216}$
7	(1,4,2), (1,3,3), (5,1,1), (3,2,2)	15	$\frac{15}{216}$
8	(1,4,3), (1,2,5), (1,1,6), (4,2,2), (3,3,2)	21	$\frac{21}{216}$
9	(6,2,1), (5,3,1), (5,2,2), (4,4,1), (4,3,2), (3,3,3)	25	$\frac{25}{216}$
10	(6,3,1), (6,2,2), (5,3,2), 5,4,1), (4,4,2), (4,3,3)	27	$\frac{27}{216}$
11	(6,4,1), (6,3,2), (5,5,1), (5,4,2), (5,3,3), (4,4,3)	27	$\frac{27}{216}$
12	(6,5,1), (6,4,2), (6,3,3), (5,5,2), (5,4,3), (4,4,4)	25	$\frac{25}{216}$
13	(6,6,1), (6,5,2), (6,4,3), (5,5,3), (5,4,4)	21	$\frac{21}{216}$
14	(6,4,4), (6,5,3), (5,5,4), (6,6,2)	15	$\frac{15}{216}$
15	(6,6,3), (6,4,5), (5,5,5)	10	$\frac{10}{216}$
16	(6,6,4), (6,5,5)	6	$\frac{6}{216}$
17	(6,6,5)	3	$\frac{3}{216}$
18	(6,6,6)	1	$\frac{1}{216}$

13. Answers may vary.

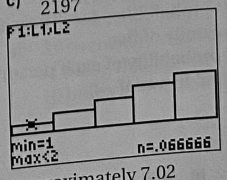
14. a) Answers may vary. There are many more values in these data sets that start with one compared to other digits.

b) 3.441

15. a) $\frac{4}{52}$ or $\frac{1}{13}$ **b)** $\frac{12}{169}$

c) $\frac{144}{2197}$ **d)** $\left(\frac{48}{52}\right)^{n-1} \times \frac{4}{52}$

16.



17. approximately 7.02

4.2 Uniform Distributions, pages 154–159

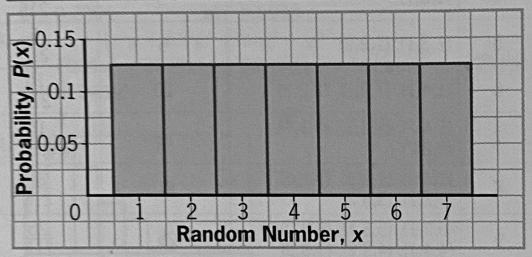
Example 1 Your Turn

a) Yes. Each randomly generated radius is equally likely and there is a single trial.

b)

Random Number, x	$P(x)$	$x \cdot P(x)$
1	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{8}$	$\frac{2}{8}$
3	$\frac{1}{8}$	$\frac{3}{8}$
4	$\frac{1}{8}$	$\frac{4}{8}$
5	$\frac{1}{8}$	$\frac{5}{8}$
6	$\frac{1}{8}$	$\frac{6}{8}$
7	$\frac{1}{8}$	$\frac{7}{8}$
8	$\frac{1}{8}$	$\frac{8}{8}$

c)



d) 4.5; On average, the expected radius length will be 4.5.

Example 2 Your Turn

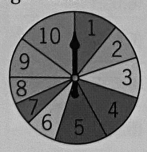
No. A fair game will have an expectation equal to 0. This is not a fair game because the player will win 0.75 point on each turn, on average.

Reflect

R1. Yes, randomly selecting students by their student number is uniform. Each randomly generated student number is equally likely in a single trial.

R2. a)

b)



R3. The expected profit per ticket is $\$2 - \$0.75 = \$1.25$. This gives an advantage to the school.

Practise

1. a) no; there are different probabilities for different sums in a trial

b) yes; there is an equal probability of selecting each card in a trial

c) yes; there is an equal probability of each song being randomly selected in a trial

d) no; there are different probabilities for different numbers of boys in a family of five

e) yes; there is an equal probability of each person being randomly selected in a trial

2. C

3. D

4. 6 green balls

5. a) 15 **b)** 3.5

Apply

6. a) 6.5

b) No. The expectation is simply the predicted average of all outcomes. Each number between 1 and 12 is equally likely.

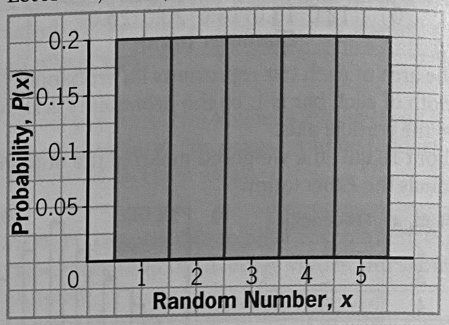
7. a) $\frac{1}{52}$

b) yes; there is an equal probability of selecting any specific card in a trial

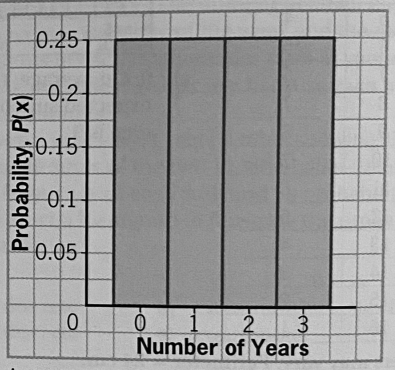
c) $\frac{1}{51}$

d) no; the probability of any specific card changes after a card is removed

8. Let A = 1, B = 2, C = 3, D = 4, and E = 5.



9. a)



b) Answers may vary. The only chance of going free is if prisoner P confesses. Prisoner P should confess. If Q confesses, then P should also confess. If Q denies, then P should confess because he will be set free.

10. a) Answers may vary. 2.5, 3.5, 4.5, 6.5, and 10.5 (the average of the face values for each platonic solid).

b) four faces: 2.5; six faces: 3.5; eight faces: 4.5; twelve faces: 6.5

c) Answers may vary. My findings in part b) confirm my prediction for the icosahedron.

11. $700.

12. a) The areas of the three regions are not equal so this is not a uniform distribution.

b) Answers may vary. Let A be worth 5 points, B be worth 16 points, and C be worth 20 points.

c) Answers may vary. A similar target with a uniform distribution would have regions of equal area.

13. −$1.3726 **14.** $5.55

16. Answers may vary.

Extend

17. The expected outcome is $\frac{n+1}{2}$.

18. Choosing 1 gives the greatest expected outcome at $70.50.