



**Unit 5: One and Two Variable data analysis**

**Lesson 5.6: Measures of spread – Standard deviation & z – scores**

*Learning Goal: Analyze and describe data using statistical measure of spread*

**Standard deviation & variance:** shows how values in a distribution are centered about the mean

**Deviation:** is the difference between an individual value in a set of data and the mean for the data

- For a population, deviation =
- For a sample, deviation =
- The larger the size of the deviations, the greater the spread in the data
- Values less than the mean have negative deviations (if you add up all the deviations for a data set, they will cancel out)

**Standard deviation:** the square root of the mean of the squares of the deviations

- Gives greater weight to the larger deviations since it is based on the squares of the deviations

| Population standard deviation   | Sample standard deviation   |
|---|---|
| $\sigma$ = sigma is the symbol for the standard deviation of a population<br>N = the size of the population | s = standard deviation of a sample<br>n = is the size of the sample<br>n – 1 compensates for the fact that a sample taken from a population tends to underestimate the deviations in the population |

**Variance:** the mean of the squares of the deviations. This is the square of the standard deviation.

| Population variance | Sample variance |
|---------------------|-----------------|
|                     |                 |

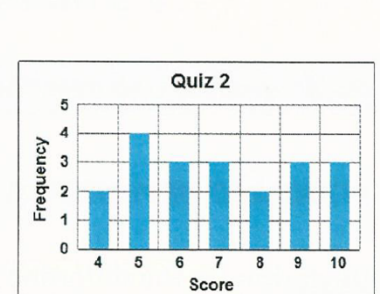
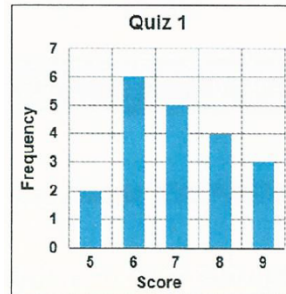


**Analyzing graphs:**

**Example 1:** These graphs show the scores on two quizzes.

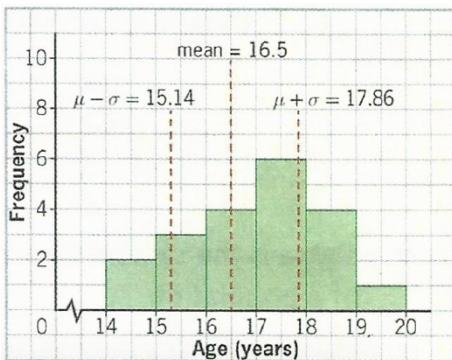
The mean score for each quiz is 7.0.

- a) Which quiz has a greater standard deviation? Why?
- b) The variance of Quiz 1 is 1.5. what is the standard deviation?
- c) What would the Quiz 1 graph look like if the standard deviation were 1.6?



**Example 2:** The ages of participants in a school's talent contest are listed below along with the mean, standard deviation, and a histogram of the data.

|    |    |    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|----|----|
| 16 | 17 | 18 | 16 | 15 | 16 | 17 | 15 | 18 | 14 |
| 17 | 19 | 18 | 16 | 17 | 17 | 17 | 14 | 16 | 18 |



- a) What would happen to the standard deviation if the first person's age were 18?
- b) What would happen to the standard deviation if the second person's age were 16 instead of 17?
- c) What would happen to the standard deviation if each person were one year older?
- d) Which ages are more than one standard deviation from the mean?



**Calculating standard deviation**

**Example 3:** For a game of basketball, a group of friends split into two randomly chosen teams. The heights of the players are shown in the table to the right.

| Falcons |             | Ravens |             |
|---------|-------------|--------|-------------|
| Player  | Height (cm) | Player | Height (cm) |
| Laura   | 183         | Sam    | 166         |
| Jamie   | 165         | Elle   | 163         |
| Deepa   | 148         | Tracy  | 168         |
| Colleen | 146         | Lia    | 161         |
| Ingrid  | 181         | Maria  | 165         |
| Justiss | 178         | Amy    | 166         |
| Sheila  | 154         | Selena | 166         |

Assumptions:

- We are estimating characteristics a population so we will use the population standard deviation
- The teams are very small samples so they could have significant random variations

Use means and standard deviations to compare the distributions to compare the distribution of heights for the two basketball teams.

| Falcons |             |                     |               | Ravens |             |                     |               |
|---------|-------------|---------------------|---------------|--------|-------------|---------------------|---------------|
| Player  | Height (cm) | Deviation $x - \mu$ | $(x - \mu)^2$ | Player | Height (cm) | Deviation $x - \mu$ | $(x - \mu)^2$ |
| Laura   |             |                     |               | Sam    |             |                     |               |
| Jamie   |             |                     |               | Elle   |             |                     |               |
| Deepa   |             |                     |               | Tracy  |             |                     |               |
| Colleen |             |                     |               | Lia    |             |                     |               |
| Ingrid  |             |                     |               | Maria  |             |                     |               |
| Justiss |             |                     |               | Amy    |             |                     |               |
| Sheila  |             |                     |               | Selena |             |                     |               |
|         | SUM         |                     |               |        | SUM         |                     |               |

Conclusions:



**Z-Score:** is a measure of how many standard deviations a particular data value is from the mean

- Divide the deviation of datum by the standard deviation
- Data with values below the mean have negative z-scores
- Data with values above the mean have positive z-scores
- Data with values equal to the mean have a zero z-score
- Later we will use z-scores to determine probabilities (next unit)

| Population z-score | Sample z-score | Population SD | Sample SD |
|--------------------|----------------|---------------|-----------|
|                    |                |               |           |

- Above, you can derive computational standard deviation formulas from the given formulas. These formulas simplify the calculations of standard deviation using a scientific calculator.

**Example 4:** A food manufacturer makes 2 – L jars of pasta sauce. Samples are tested for how close to 2 L the jars are filled. Fifteen samples were taken and their volumes, in liters, were as indicated.

|      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|
| 2.11 | 2.02 | 2.10 | 1.99 | 1.92 | 2.01 | 1.89 | 1.96 |
| 2.00 | 1.96 | 1.98 | 2.02 | 2.08 | 2.15 | 2.03 |      |

- Determine the sample mean and standard deviation.
- Calculate the z-score of the jar that was filled to a volume of 2.02 L. interpret its meaning.
- Calculate the z-score of the jar that was filled to a volume of 1.98L. interpret its meaning.
- The manufacturer rejects any jars that are filled to less than 1.5 standard deviations below the mean. Which jars would be rejected?