Unit 2 – Polynomials

Chapter 3.3: Characteristics of Polynomial functions in Factored form

Consider a polynomial in the form: $g(x) = a(x-p)(x-q)(x-r)$

The factors of the polynomial can be used to identify the zeros (or roots, or x-intercepts).

Set $g(x) = 0$, so $0 = a(x-p)(x-q)(x-r)$

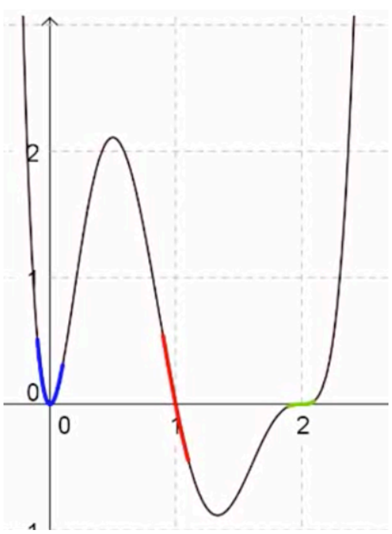
Because the leading coefficient cannot be equal to zero, so $(x-p) = 0, (x-q) = 0 \text{ } or \text{ } (x-r) = 0$

Hence, *x = p, x = q, x = r.*

Example 1:

The order or degree of the factors will determine the behavior of the graph near the x-axis.

Consider $f(x) = 5x^2(x-1)(x-2)^3$



To sketch the graph of a polynomial in factored form:

1) Use leading coefficient and order of polynomial to determine end behavior.
2) Plot x-intercepts (zeros) and y-intercepts,
3) Use order of factors to sketch behavior at x-axis.

In Summary

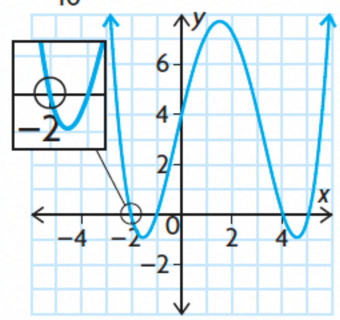
Key Idea

- The zeros of the polynomial function $y = f(x)$ are the same as the roots of the related polynomial equation, $f(x) = 0$.

Need to Know

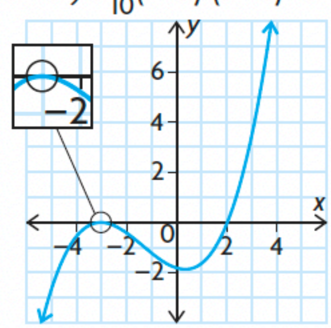
- To determine the equation of a polynomial function in factored form, follow these steps:
  - Substitute the zeros $(x_1, x_2, \ldots, x_n)$ into the general equation of the appropriate family of polynomial functions of the form $y = a(x - x_1)(x - x_2)\ldots(x - x_n)$.
  - Substitute the coordinates of an additional point for x and y, and solve for a to determine the equation.
- If any of the factors of a polynomial function are linear, then the corresponding x-intercept is a point where the curve passes through the x-axis. The graph has a linear shape near this x-intercept.

$y = \frac{1}{10}(x + 2)(x + 1)(x - 4)(x - 5)$



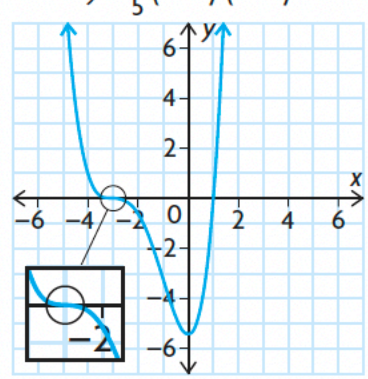
- If any of the factors of a polynomial function are squared, then the corresponding x-intercepts are turning points of the curve and the x-axis is tangent to the curve at these points. The graph has a parabolic shape near these x-intercepts.

$y = \frac{1}{10}(x + 3)^2(x - 2)$



- If any of the factors of a polynomial function are cubed, then the corresponding x-intercepts are points where the x-axis is tangent to the curve and also passes through the x-axis. The graph has a cubic shape near these x-intercepts.

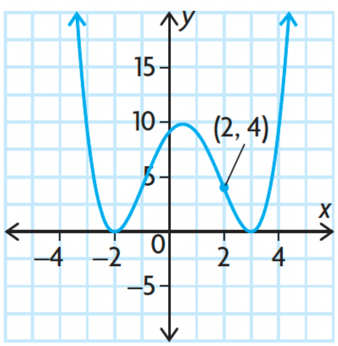
$y = \frac{1}{5}(x + 3)^3(x - 1)$



Example 2: Sketch a possible graph of the function $f(x) = -(x+2)(x-1)(x-3)^2$.

Example 3: Write the equation of a cubic function that has zeros at – 2, 3, and $\frac{2}{5}$. The function also has a y-intercept of 6.

Example 4: Write the equation of the function shown below. And state the domain and range of the function.



Example 5: Sketch the graph of $f(x) = x^4 + 2x^3$.