THE ERINDALE ACADEMY
1576 Dundas Street West, Mississauga ON L5C 1E5
www.teacademy.ca Tel: 905-232-1576
Email: info@teacademy.ca

Teacher: Ms. Ella

Unit 6 – Exponential and Logarithmic functions

Chapter 8.7: Solve Problems with Logarithmic Functions

pH Scale (hydrogen ion concentration):

$\mathbf{pH} = -log_{10}H^+$,

where pH is the scaled measurement (0 to 14), and H+ is the concentration of hydrogen ions (mol/L)

Example1:

In chemistry, the pH (the measure of acidity or alkalinity of a substance) is based on a logarithmic scale. A logarithmic scale uses powers of 10 to compare numbers that vary greatly in size. For example, very small and very large concentrations of the hydrogen ion in a solution influence its classification as either a base or an acid.

Concentration of hydrogen ions compared to distilled water		Examples of solutions at this pH
10 000 000	pH = 0	battery acid, strong hydrofluoric acid
1 000 000	pH = 1	hydrochloric acid secreted by stomach lining
100 000	pH = 2	lemon juice, gastric acid, vinegar
10 000	pH = 3	grapefruit, orange juice, soda
1000	pH = 4	tomato juice, acid rain
100	pH = 5	soft drinking water, black coffee
10	pH = 6	urine, saliva
1	pH = 7	"pure" water
$\frac{1}{10}$	pH = 8	seawater
$\frac{1}{100}$	pH = 9	baking soda
$\frac{1}{1000}$	pH = 10	Great Salt Lake, milk of magnesia
$\frac{1}{10\,000}$	pH = 11	ammonia solution
$\frac{1}{100\,000}$	pH = 12	soapy water
$\frac{1}{1\,000\,000}$	pH = 13	bleaches, oven cleaner
$\frac{1}{10\,000\,000}$	pH = 14	liquid drain cleaner

A difference of one pH unit represents a tenfold (10 times) change in the concentration of hydrogen ions in the solution. For example, the acidity of a sample with a pH of 5 is 10 times greater than the acidity of a sample with a pH of 6. A difference of 2 units, from 6 to 4, would mean that the acidity is 100 times greater, and so on.

- A liquid with a pH less than 7 is considered *acidic*.
- A liquid with a pH greater than 7 is considered *alkaline*.
- A liquid with a pH of 7 is considered *neutral*. Pure distilled water has a pH value of 7.

Teacher: Ms. Ella

Example 1:

The relationship between pH and hydrogen ion concentration is given by the formula $pH = -\log[H^+]$, where $[H^+]$ is the concentration of hydrogen ions in moles per litre (mol/L).

a) Calculate the pH if the concentration of hydrogen ions is 0.0001 mol/L.

b) The pH of lemon juice is 2. Calculate the hydrogen ion concentration.

c) If the hydrogen ion concentration is a measure of the strength of an acid, how much stronger is an acid with pH 1.6 than an acid with pH 2.5?

Richter Scale (Earthquakes): $\mathbf{M} = \boldsymbol{log_{10}A}$, where M is the magnitude (approximately 0 to 10), and A is the amplitude on the seismograph.

Notes: This formula is useful on for comparing the relative intensity of earthquakes. The actual energy of the earthquake is more complex.

Example 2:

The Richter magnitude scale uses logarithms to compare intensity of earthquakes.

True Intensity	Richter Scale Magnitude
10^1	$\log_{10}10^1 = 1$
10^4	$\log_{10}10^4 = 4$
$10^{5.8}$	$\log_{10}10^{5.8} = 5.8$

An earthquake of magnitude 2 is actually 10 times more intense than an earthquake of magnitude 1. The difference between the magnitudes of two earthquakes can be used to determine the difference in intensity. If the average earthquake measures 4.5 on the Richter scale, how much more intense is an earthquake that measures 8?

Practice: If earthquake A has a magnitude of 6.4 and it is 25 time more intense then earthquake B, what is the magnitude of earthquake B?

THE ERINDALE ACADEMY
1576 Dundas Street West, Mississauga ON L5C 1E5
www.teacademy.ca Tel: 905-232-1576
Email: info@teacademy.ca

Teacher: Ms. Ella

Sound Loudness (decibel scale):

$$L = 10log_{10}\left(\frac{I}{I_o}\right),$$

where L is the loudness of the sound, I is the sound intensity (energy), and I_o is the threshold of human hearing.

Example 3:

The dynamic range of human hearing and sound intensity spans from 10^{-12} W/m^2 to about 10 W/m^2. The highest sound intensity that can be heard is 10 000 000 000 000 times as loud as the quietest! This span of sound intensity is impractical for normal use. A more convenient way to express loudness is a relative logarithmic scale, with the lowest sound that can be heard by the human ear, $I_0 = 10^{-12}$ W/m^2, given the measure of loudness of 0 dB.

Recall that the formula that is used to measure sound is $L = 10 \log\left(\frac{I}{I_0}\right)$, where L is the loudness measured in decibels, I is the intensity of the sound being measured, and I_0 is the intensity of sound at the threshold of hearing. The following table shows the loudness of a selection of sounds measured in decibels.

Sound	Loudness (dB)
soft whisper	30
normal conversation	60
shouting	80
subway	90
screaming	100
rock concert	120
jet engine	140
space-shuttle launch	180

How many times more intense is the sound of a rock concert than the sound of a subway?