Exploring Operations with Functions

Suppose we had two functions, $f(x)$ and $g(x)$.

Now, let's think about what $f(x)+g(x)$ means.

Well, for a moment, let's consider the case where $x = 2$. If we want to find $f(2)+g(2)$, we could simply evaluate both functions at $x = 2$ and then add the results.

A few things to think about with $f(2)+g(2)$…

- **Notice that it is the two output values (y-values) that we're actually adding here!**
- **Notice that we are adding the output values that correspond to a single input value. That is, if we substitute $x = 2$ into $f(x)$, we must substitute $x = 2$ into $g(x)$.**
- **Notice that we can only perform this addition if 2 is in the domain of both $f(x)$ and $g(x)$.**

Let's get to the point…

- **When we add two functions, we add the two output values (y-values) for every valid input value.**
- **We can add the two functions only where their domains overlap.**
- **The same ideas apply to the subtraction and multiplication of two functions.**

Some examples…

1) If $f = \{(1, 2), (2, -3), (3, 4), (5, 8), (6, -7)\}$ and $g = \{(1, 2), (2, 5), (3, -6), (4, -8), (5, 3)\}$, find the following:

a) $f + g = \{(1, 4), (2, 2), (3, -2), (5, 11)\}$

b) $f - g = \{(1, 0), (2, -8), (3, 10), (5, 5)\}$

c) $g - f = \{(1, 0), (2, 8), (3, -10), (5, -5)\}$

d) $fg = \{(1, 4), (2, -15), (3, -24), (5, 24)\}$

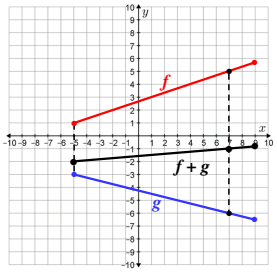
Question for Discussion: How does the domain of each of the results above compare to the domains of the original two functions?

2) If $f(x) = 7x + 5$ and $g(x) = 9x - 12$, find an equation for the following:

a) $$\begin{aligned} f(x)-g(x) &= (7x+5)-(9x-12) \\ &= 7x+5-9x+12 \\ &= -2x+17 \end{aligned}$$

b) $$\begin{aligned} f(x)\times g(x) &= (7x+5)(9x-12) \\ &= 63x^2-84x+45x-60 \\ &= 63x^2-39x-60 \end{aligned}$$

3) Use the graphs of functions f and g to sketch the graph of $f + g$.



Questions for Discussion:

- Would the graph of $f - g$ look the same as the graph of $g - f$?
- What result would you expect to obtain if you were to sketch the graph of fg?