THE ERINDALE ACADEMY

# Course Outline

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| **Course Title:** | Calculus and Vectors, Grade 12, University Preparation |
| **Department:** | Mathematics |
| **Grade Level:** | Grade 12 |
| **Course Code:** | MCV4U |
| **Developed from:** | The Ontario Curriculum, Grades 11 and 12: Mathematics, 2007 (Revised) |
| **Prerequisite**: | The new Advanced Functions course (MHF4U) must be taken prior to or concurrently with Calculus and Vectors (MCV4U). |
| **Credits**: | 1.0 |
| **Developed by:** | George Vanderkuur |
| **Development Date:** | November 2016 |
| **Revised By:** | Nash Vadsaria |
| **Revised Date:** | May 2017 |

**COURSE DESCRIPTION**

This course builds on students' previous experience with functions and their developing understanding of rates of change. Students will solve problems involving geometric and algebraic representations of vectors and representations of lines and planes in three-dimensional space; broaden their understanding of rates of change to include the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions; and apply these concepts and skills to the modelling of real-world relationships. Students will also refine their use of the mathematical processes necessary for success in senior mathematics. This course is intended for students who choose to pursue careers in fields such as science, engineering, economics, and some areas of business, including those students who will be required to take a university-level calculus, linear algebra, or physics course.

## OVERALL EXPECTATIONS:

### Strand A – Rate of Change

- demonstrate an understanding of rate of change by making connections between average rate of change over an interval and instantaneous rate of change at a point, using the slopes of secants and tangents and the concept of the limit;
- graph the derivatives of polynomial, sinusoidal, and exponential functions, and make connections between the numeric, graphical, and algebraic representations of a function and its derivative;
- verify graphically and algebraically the rules for determining derivatives; apply these rules to determine the derivatives of polynomial, sinusoidal, exponential, rational, and radical functions, and simple combinations of functions; and solve related problems.

### Strand B – Derivatives and Their Applications

- make connections, graphically and algebraically, between the key features of a function and its first and second derivatives, and use the connections in curve sketching;
- solve problems, including optimization problems, that require the use of the concepts and procedures associated with the derivative, including problems arising from real-world applications and involving the development of mathematical models.

### Strand C – Geometry and Algebra of Vectors

- demonstrate an understanding of vectors in two-space and three-space by representing them algebraically and geometrically and by recognizing their applications;
- perform operations on vectors in two-space and three-space, and use the properties of these operations to solve problems, including those arising from real-world applications;
- distinguish between the geometric representations of a single linear equation or a system of two linear equations in two-space and three-space, and determine different geometric configurations of lines and planes in three-space;
- represent lines and planes using scalar, vector, and parametric equations, and solve problems involving distances and intersections.

**OUTLINE OF COURSE CONTENT**

| Unit | Topic | Time |
|---|---|---|
| 1 | Introduction to Calculus | 20 hours |
| 2 | Derivatives | 15 hours |
| 3 | Optimization | 12 hours |
| 4 | Curve Sketching | 12 hours |
| 5 | Introduction to Vectors | 12 hours |
| 6 | Applications of Vectors | 18 hours |
| 7 | Lines and Planes | 15 hours |
| Final Evaluation | Exam Review and Final Exam | 6 hours |
| | Total | 110 hours |

**TEACHING AND LEARNING STRATEGIES:**

There are seven mathematical processes that support effective learning in mathematics. Attention to the mathematical processes is considered to be essential to a balanced mathematics program. The processes are to be applied in all strands of the mathematics course and are part of the evaluation of student achievement.

***The seven mathematical processes are:***

*Communicating:* To improve student success there will be several opportunities for students to share their understanding both in oral as well as written form.

*Problem solving:* Scaffolding of knowledge, detecting patterns, making and justifying conjectures, guiding students as they apply their chosen strategy, directing students to use multiple strategies to solve the same problem, when appropriate, recognizing, encouraging, and applauding perseverance, discussing the relative merits of different strategies for specific types of problems.

*Reasoning and proving:* Asking questions that get students to hypothesize, providing students with one or more numerical examples that parallel these with the generalization and describing their thinking in more detail.

*Reflecting:* Modeling the reflective process, asking students how they know.

*Selecting Tools and Computational Strategies:* Modeling the use of tools and having students use technology to help solve problems.

*Connecting:* Activating prior knowledge when introducing a new concept in order to make a smooth connection between previous learning and new concepts, and introducing skills in context to make connections between particular manipulations and problems that require them.

*Representing:* Modeling various ways to demonstrate understanding, posing questions that require students to use different representations as they are working at each level of conceptual development - concrete, visual or symbolic, allowing individual students the time they need to solidify their understanding at each conceptual stage.

These seven learning strategies are used throughout the course to support strategies including:

*Guided Exploration* *Problem Solving* *Direct Instruction* *Ideal Problem Solving*
*Graphing* *Graphing Applications* *Model Analysis* *Visuals*

## ASSESSMENT AND EVALUATION STRATEGIES

**Diagnostic Assessment (For)**

is the process of gathering evidence of student learning prior to commencing instruction. This information is useful for planning instruction, and in particular for individualizing program delivery. It is not used to determine student achievement levels.

***Implementation***

- Pretest for each section of the Course
- Interview with teacher
- Mathematics Language competency test

**Formative assessment (As)**

is the process of gathering information during the learning process. It involves constructive and specific feedback to students aimed to improve learning. This evidence may be used for determining a grade/level when there is insufficient evidence from summative assessments.

***Implementation***

- Learning expectations and criteria for assessment are communicated to students in advance.
- Frequent use of Quizzes with feedback to student
- Class room observation with teacher feedback to student
- Assignments with rubrics
- Discussion of achievement chart relation to a specific expectations
- Teachers provide students with ongoing and descriptive feedback on their learning to help them establish goals for improvement
- Peer assessment is used for formative feedback may take the form of marking each other's quizzes and making suggestions for improvement.
- Students periodically assess their own work and set goals for improvement.
- Student portfolios to demonstrate growth over time
- Students are provided with examples and models to assist them in understanding how to achieve excellence.
- Exemplars that illustrate achievement levels inform students about their own achievement level.

**Summative assessment (Of)**

may occur throughout a course. Summative assessment is designed to allow students to demonstrate achievement toward the expectations of a course. It forms the primary basis for establishing the report card levels of achievement and the final mark. Determination of grading levels for formal reporting purposes should primarily reflect student performance on summative tasks. Students' level grades will reflect their most consistent level of achievement with an eye to their most recent levels of achievement at the time of reporting.

***Implementation***

- Summative tests throughout term
- Final exam that covers the overall expectations of the course.

## STUDENT ACHIEVEMENT OF LEARNING EXPECTATIONS

Evaluation will be based on the provincial curriculum expectations and the achievement levels outlined in the curriculum document and will be evaluated according to the following breakdown.

| Categories of the Achievement Chart | Description | Weight |
|---|---|---|
| ***Knowledge/Understanding*** | · understanding concepts,<br>· knowledge of facts and terms<br>· transfer of concepts to new contexts<br>· understanding of relationships between concepts<br>· performing algorithms | 25% |
| ***Thinking /Inquiry / Problem Solving*** | · reasoning<br>· applying the steps of an inquiry/ problem-solving process (e.g., formulating questions; selecting strategies, resources, technology, and tools; representing in mathematical form; interpreting information and forming conclusions; reflecting on the reasonableness of results) | 25% |
| ***Communication*** | · communicating reasoning orally, in writing, and graphically<br>· using mathematical language, symbols, visuals, and conventions | 25% |
| ***Application*** | · applying concepts and procedures relating to familiar and unfamiliar settings | 25% |
| | | 100% |

## FINAL MARK

The percentage grade represents the quality of the student's overall achievement of the expectations for the course and reflects the corresponding level of achievement as described in the achievement chart for mathematics.

70% of the grade will be based upon evaluations conducted throughout the course. This portion of the grade will reflect the student's most consistent level of achievement throughout the course, although special consideration will be given to more recent evidence of achievement.

30% of the grade will be based on a final evaluation. The final evaluation will consist of an exam.

## SOME CONSIDERATIONS FOR PROGRAM PLANNING IN MATHEMATICS

## INSTRUCTIONAL APPROACHES

To make new learning more accessible to students, teachers build new learning upon the knowledge and skills students have acquired in previous years – in other words, they help activate prior knowledge. It is important to assess where students are in their mathematical growth and to bring them forward in their learning.

In order to apply their knowledge effectively and to continue to learn, students must have a solid conceptual foundation in mathematics. Successful classroom practices engage students in activities that require higher-order thinking, with an emphasis on problem solving. Learning experienced in the primary, junior, and intermediate divisions should have provided students with a good grounding in the investigative approach to learning new mathematical concepts, including inquiry models of problem solving, and this approach continues to be important in the senior mathematics program.

Students in a mathematics class typically demonstrate diversity in the ways they learn best. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through investigation involving hands-on experience, and through examples followed by practice. In mathematics, students are required to learn concepts, acquire procedures and skills, and apply processes with the aid of the instructional and learning strategies best suited to the particular type of learning.

The approaches and strategies used in the classroom to help students meet the expectations of this curriculum will vary according to the object of the learning and the needs of the students. For example, even at the secondary level, manipulatives can be important tools for supporting the effective learning of mathematics. These concrete learning tools, such as connecting cubes, measurement tools, algebra tiles, and number cubes, invite students to explore and represent abstract mathematical ideas in varied, concrete, tactile, and visually rich ways. Other representations, including graphical and algebraic representations, are also a valuable aid to teachers. By analysing students' representations of mathematical concepts and listening carefully to their reasoning, teachers can gain useful insights into students' thinking and provide supports to help enhance their thinking.

All learning, especially new learning, should be embedded in well-chosen contexts for learning – that is, contexts that are broad enough to allow students to investigate initial understandings, identify and develop relevant supporting skills, and gain experience with varied and interesting applications of the new knowledge. Such rich contexts for learning open the door for students to see the "big ideas" of mathematics – that is, the major underlying principles or relationships that will enable and encourage students to reason mathematically throughout their lives.

**Promoting Positive Attitudes Towards Learning Mathematics**

Students' attitudes have a significant effect on how students approach problem solving and how well they succeed in mathematics. Students who enjoy mathematics tend to perform well in their mathematics course work and are more likely to enrol in the more advanced mathematics courses.

Students develop positive attitudes when they are engaged in making mathematical conjectures, when they experience breakthroughs as they solve problems, when they see connections between important ideas, and when they observe an enthusiasm for mathematics on the part of their teachers. With a positive attitude towards mathematics, students are able to make more sense of the mathematics they are working on, and to view themselves as effective learners of mathematics.

They are also more likely to perceive mathematics as both useful and worthwhile, and to develop the belief that steady effort in learning mathematics pays off.

It is common for people to feel inadequate or anxious when they cannot solve problems quickly and easily, or in the right way. To gain confidence, students need to recognize that, for some mathematics problems, there may be several ways to arrive at a solution. They also need to understand that problem solving of almost any kind often requires a considerable expenditure of time and energy and a good deal of perseverance. To counteract the frustration they may feel when they are not making progress towards solving a problem, they need to believe that they are capable of finding solutions. Teachers can encourage students to develop a willingness to persist, to investigate, to reason, to explore alternative solutions, to view challenges as opportunities to extend their learning, and to take the risks necessary to become successful problem solvers. They can help students develop confidence and reduce anxiety and frustration by providing them with problems that are challenging but not beyond their ability to solve. Problems at a developmentally appropriate level help students to learn while establishing a norm of perseverance for successful problem solving.

Collaborative learning enhances students' understanding of mathematics. Working cooperatively in groups reduces isolation and provides students with opportunities to share ideas and communicate their thinking in a supportive environment as they work together towards a common goal. Communication and the connections among ideas that emerge as students interact with one another enhance the quality of student learning.

## PLANNING MATHEMATICS PROGRAMS FOR STUDENTS WITH SPECIAL EDUCATION NEEDS

This is not applicable since The Erindale Academy does not have students with special needs.

## PROGRAM CONSIDERATIONS FOR ENGLISH LANGUAGE LEARNERS

Young people whose first language is not English enter Ontario secondary schools with diverse linguistic and cultural backgrounds. Some English language learners may have experience of highly sophisticated educational systems, while others may have come from regions where access to formal schooling was limited. All of these students bring a rich array of background knowledge and experience to the classroom, and all teachers must share in the responsibility for their English-language development. Teachers of mathematics must incorporate appropriate adaptations and strategies for instruction and assessment to facilitate the success of the English language learners in their classrooms. These adaptations and strategies include:

- modification of some or all of the course expectations so that they are challenging but attainable for the learner at his or her present level of English proficiency, given the necessary support from the teacher;
- use of a variety of instructional strategies (e.g., extensive use of visual cues, scaffolding, manipulatives, pictures, diagrams, graphic organizers; attention to clarity of instructions); modelling of preferred ways of working in mathematics; previewing of textbooks; pre-teaching of key vocabulary; peer tutoring; strategic use of students' first languages);

- use of a variety of learning resources (e.g., visual material, simplified text, bilingual dictionaries, materials that reflect cultural diversity);
- use of assessment accommodations (e.g., granting of extra time; simplification of language used in problems and instructions; use of oral interviews, learning logs, portfolios, demonstrations, visual representations, and tasks requiring completion of graphic organizers or cloze sentences instead of tasks that depend heavily on proficiency in English).

Many ESL students are mathematically competent in their native language. Teachers should take advantage of this competency to place emphasis on developing the students' ability to comprehend and do math in English through strategies like:

- *Adjusted speech:* teacher changes speech patterns to increase student comprehension. Includes facing the students, paraphrasing often, clearly indicating most important ideas, limiting asides, etc.
- *Explicit vocabulary building through random recurrent assessments:* Using brief assessments to help students build basic subject-specific vocabulary and also gauge student retention of subject-specific vocabulary.
- *Native language support:* providing auditory or written content input to students in their native language.
- *Peer to Peer explaining:* students build language and mathematical skills as they explain problem solving steps and strategies to each other.

**ANTIDISCRIMINATION EDUCATION IN MATHEMATICS**

To ensure that all students in the province have an equal opportunity to achieve their full potential, the curriculum must be free from bias, and all students must be provided with a safe and secure environment, characterized by respect for others, that allows them to participate fully and responsibly in the educational experience.

Learning activities and resources used to implement the curriculum should be inclusive in nature, reflecting the range of experiences of students with varying backgrounds, abilities, interests, and learning styles. They should enable students to become more sensitive to the diverse cultures and perceptions of others, including Aboriginal peoples. By discussing aspects of the history of mathematics, teachers can help make students aware of the various cultural groups that have contributed to the evolution of mathematics over the centuries. Finally, students need to recognize that ordinary people use mathematics in a variety of everyday contexts, both at work and in their daily lives.

Connecting mathematical ideas to real-world situations through learning activities can enhance students' appreciation of the role of mathematics in human affairs, in areas including health, science, and the environment. Students can be made aware of the use of mathematics in contexts

such as sampling and surveying and the use of statistics to analyse trends. Recognizing the importance of mathematics in such areas helps motivate students to learn and also provides a foundation for informed, responsible citizenship.

Teachers should have high expectations for all students. To achieve their mathematical potential, however, different students may need different kinds of support. Some boys, for example, may need additional support in developing their literacy skills in order to complete mathematical tasks effectively. For some girls, additional encouragement to envision themselves in careers involving mathematics may be beneficial. For example, teachers might consider providing strong role models in the form of female guest speakers who are mathematicians or who use mathematics in their careers.

## LITERACY AND INQUIRY/RESEARCH SKILLS

Literacy skills can play an important role in student success in mathematics courses. Many of the activities and tasks students undertake in mathematics courses involve the use of written, oral, and visual communication skills. For example, students use language to record their observations, to explain their reasoning when solving problems, to describe their inquiries in both informal and formal contexts, and to justify their results in small-group conversations, oral presentations, and written reports. The language of mathematics includes special terminology. The study of mathematics consequently encourages students to use language with greater care and precision and enhances their ability to communicate effectively.

The Ministry of Education has facilitated the development of materials to support literacy instruction across the curriculum. Helpful advice for integrating literacy instruction in mathematics courses may be found in the following resource documents:

- *Think Literacy: Cross-Curricular Approaches, Grades 7–12, 2003*
- *Think Literacy: Cross-Curricular Approaches, Grades 7–12 – Mathematics: Subject-Specific Examples, Grades 10–12, 2005*

In all courses in mathematics, students will develop their ability to ask questions and to plan investigations to answer those questions and to solve related problems. Students need to learn a variety of research methods and inquiry approaches in order to carry out these investigations and to solve problems, and they need to be able to select the methods that are most appropriate for a particular inquiry. Students learn how to locate relevant information from a variety of sources, such as statistical databases, newspapers, and reports. As they advance through the grades, students will be expected to use such sources with increasing sophistication. They will also be expected to distinguish between primary and secondary sources, to determine their validity and relevance, and to use them in appropriate ways.

## THE ROLE OF INFORMATION AND COMMUNICATION TECHNOLOGY IN MATHEMATICS

Information and communication technologies (ICT) provide a range of tools that can significantly extend and enrich teachers' instructional strategies and support students' learning in mathematics. Teachers can use ICT tools and resources both for whole-class instruction and to design programs that meet diverse student needs. Technology can help to reduce the time spent on routine mathematical tasks, allowing students to devote more of their efforts to thinking and concept development. Useful ICT tools include simulations, multimedia resources, databases, sites that give access to large amounts of statistical data, and computer-assisted learning modules.

Applications such as databases, spreadsheets, dynamic geometry software, dynamic statistical software, graphing software, computer algebra systems (CAS), word-processing software, and presentation software can be used to support various methods of inquiry in mathematics. Technology also makes possible simulations of complex systems that can be useful for problem-solving purposes or when field studies on a particular topic are not feasible.

Information and communications technologies can be used in the classroom to connect students to other schools, at home and abroad, and to bring the global community into the local classroom.

Although the Internet is a powerful electronic learning tool, there are potential risks attached to its use. All students must be made aware of issues of Internet privacy, safety, and responsible use, as well as of the ways in which this technology is being abused – for example, when it is used to promote hatred. Teachers, too, will find the various ICT tools useful in their teaching practice, both for whole class instruction and for the design of curriculum units that contain varied approaches to learning to meet diverse student needs.

## CAREER EDUCATION IN MATHEMATICS

Teachers can promote students' awareness of careers involving mathematics by exploring applications of concepts and providing opportunities for career-related project work. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Students should be made aware that mathematical literacy and problem solving are valuable assets in an ever-widening range of jobs and careers in today's society. The knowledge and skills students acquire in mathematics courses are useful in fields such as science, business, engineering, and computer studies; in the hospitality, recreation, and tourism industries; and in the technical trades.

## THE ONTARIO SKILLS PASSPORT AND ESSENTIAL SKILLS

Teachers planning programs in mathematics need to be aware of the purpose and benefits of the Ontario Skills Passport (OSP).The OSP is a bilingual web-based resource that enhances the relevancy of classroom learning for students and strengthens school-work connections. The OSP provides clear descriptions of Essential Skills such as Reading Text, Writing, Computer Use, Measurement and Calculation, and Problem Solving and includes an extensive database of occupation-specific workplace tasks that illustrate how workers use these skills on the job. The Essential Skills are transferable, in that they are used in virtually all occupations. The OSP also includes descriptions of important work habits, such as working safely, being reliable, and providing excellent customer service. The OSP is designed to help employers assess and record students' demonstration of these skills and work habits during their cooperative education placements. Students can use the OSP to identify the skills and work habits they already have, plan further skill development, and show employers what they can do.

The skills described in the OSP are the Essential Skills that the Government of Canada and other national and international agencies have identified and validated, through extensive research, as the skills needed for work, learning, and life. These Essential Skills provide the foundation for learning all other skills and enable people to evolve with their jobs and adapt to workplace change. For further information on the OSP and the Essential Skills, visit: http://skills.edu.gov.on.ca.

## COOPERATIVE EDUCATION AND OTHER FORMS OF EXPERIENTIAL LEARNING

This is not applicable since The Erindale Academy does not offer cooperative education and other forms of experiential learning.

## PLANNING PROGRAM PATHWAYS AND PROGRAMS LEADING TO A SPECIALIST HIGH-SKILLS MAJOR

This is not applicable since The Erindale Academy does not offer programs leading to a specialist high-skills major.

## HEALTH AND SAFETY IN MATHEMATICS

Although health and safety issues are not normally associated with mathematics, they may be important when learning involves fieldwork or investigations based on experimentation. Out-of-school fieldwork can provide an exciting and authentic dimension to students' learning experiences. It also takes the teacher and students out of the predictable classroom environment and into unfamiliar settings. Teachers must preview and plan activities and expeditions carefully to protect students' health and safety.

## RESOURCES

- Calculus and Vectors, Nelson
- OMCA/OAME lesson materials (created summer 2007)
- Online Calculator https://www.desmos.com/calculator

**ACHIEVEMENT CHART – GRADES 11 AND 12, MATHEMATICS**

| Category | 50-59% (Level 1) | 60-69% (Level 2) | 70-79% (Level 3) | 80-100% (Level 4) |
|---|---|---|---|---|
| **Knowledge/Understanding** | **The student:** | | | |
| understanding concepts | demonstrates limited understanding of concepts | demonstrates some understanding of concepts | demonstrates considerable understanding of concepts | demonstrates thorough understanding of concepts |
| performing algorithms | performs only simple algorithms accurately by hand and by using technology | performs algorithms with inconsistent accuracy by hand, mentally, and by using technology | performs algorithms accurately by hand, mentally, and by using technology | selects the most efficient algorithm and performs it accurately by hand, mentally, and by using technology |
| **Thinking /Inquiry / Problem Solving** | **The student:** | | | |
| reasoning | follows simple mathematical arguments | follows arguments of moderate complexity and makes simple arguments | follows arguments of considerable complexity, judges the validity of arguments, and makes arguments of some complexity | follows complex arguments, judges the validity of arguments, and makes complex arguments |
| applying the steps of an inquiry/ problem-solving process (e.g., formulating questions; selecting strategies, resources, technology, and tools; representing in mathematical form; interpreting information and forming conclusions; reflecting on the reasonableness of results) | applies the steps of an inquiry/ problem-solving process with limited effectiveness | applies the steps of an inquiry/ problem-solving process with moderate effectiveness | applies the steps of an inquiry/ problem- solving process with considerable effectiveness | applies the steps of an inquiry/problem- solving process with a high degree of effectiveness and poses extending questions |
| **Communication** | **The student:** | | | |
| communicating reasoning orally, in writing, and graphically using mathematical language, symbols, visuals, and conventions | communicates with limited clarity and limited justification of reasoning | communicates with some clarity and some justification of reasoning | communicates with considerable clarity and considerable justification of reasoning | communicates concisely with a high degree of clarity and full justification of reasoning |
| | infrequently uses mathematical language, symbols, visuals, and conventions correctly | uses mathematical language, symbols, visuals, and conventions correctly some of the time | uses mathematical language, symbols, visuals, and conventions correctly most of the time | routinely uses mathematical language, symbols, visuals, and conventions correctly and efficiently |
| **Application** | **The student:** | | | |
| applying concepts and procedures relating to familiar and unfamiliar settings | applies concepts and procedures to solve simple problems relating to familiar settings | applies concepts and procedures to solve problems of some complexity relating to familiar settings | applies concepts and procedures to solve complex problems relating to familiar settings; recognizes major mathematical concepts and procedures relating to applications in unfamiliar settings | applies concepts and procedures to solve complex problems relating to familiar and unfamiliar settings |