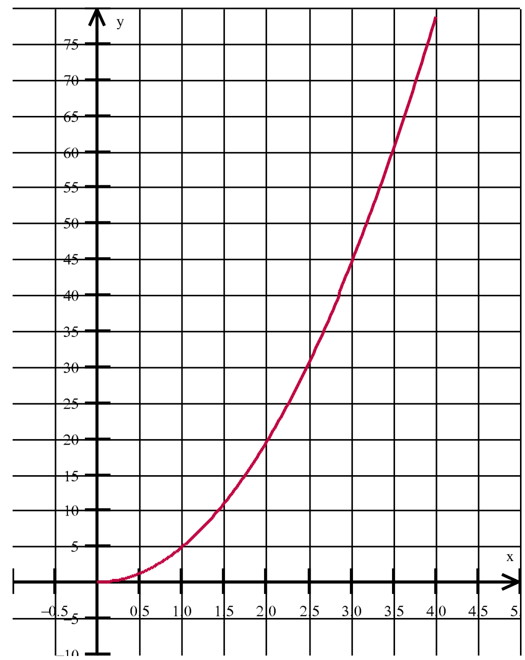
Lesson #1: Average and Instantaneous Rates of Change

L.G.: I can calculate the average and instantaneous rate of change of a function and interpret these values as the slope of the secant and tangent respectively.

Minds On

When parachuting from a plane, the jumper accelerates at a constant rate for a while, until the chute cord is pulled and the parachute opens. If air resistance is ignored the distance in metres the jumper travels before the chute opens is given by the equation:

$d(t) = 4.9t^2$ where t is the time in seconds.



What is the average rate of change of the distance traveled by the jumper between $t = 1$s and $t = 4$s? i.e. find ____________________

Find the instantaneous rate of change of the distance travelled by the jumper at 2 seconds.

Let's consider the average rate of change over successively smaller time periods.

Time Interval	Average Rate of Change
[2, 3]	
[2, 2.5]	
[2, 2.1]	
[2, 2.01]	
[2, 2.001]	
[2, 2.0001]	
[1.9999, 2]	

To find the **instantaneous** velocity we must find the value of the average velocity as Δt approaches 0. Since the average velocity does not exist when $\Delta t = 0$, this is called a ***limiting*** value.

Checking from the other side of 3

Homework: p29 #2, **4**, **7**,10,**11-13, 15** (Note: IRC calculations are to be done using a **table of values** from both sides)

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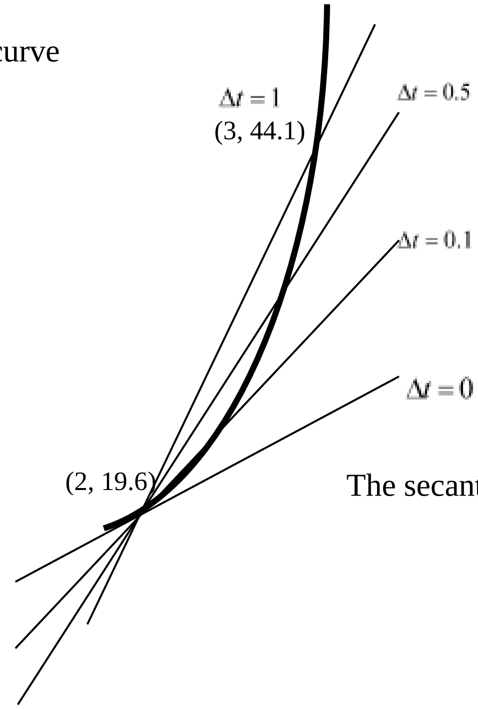
Graphically: Looking at a section of the curve

$d = 4.9t^2 \quad t \geq 0\,\text{s}$

NOTE:
The slope of the secant represents

The slope of the tangent represents



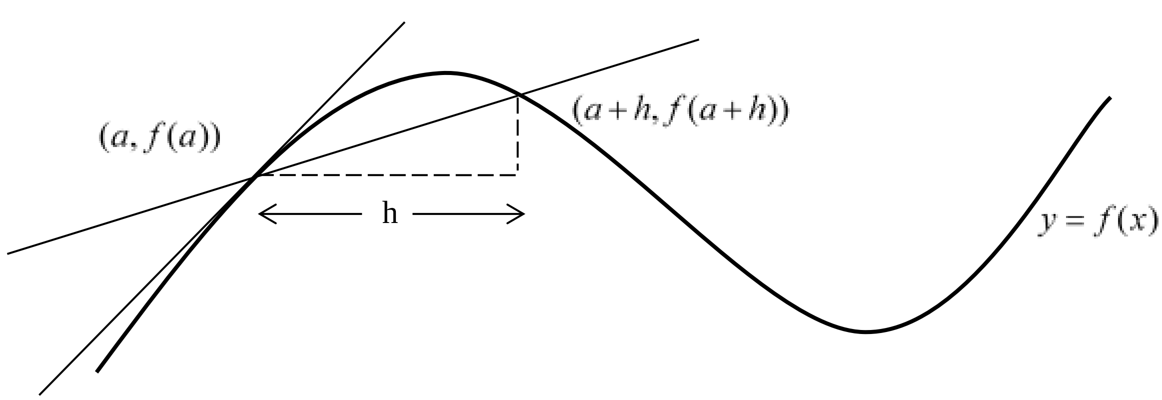
The secant becomes tangent as $\Delta t \to 0$

Definition of a Limit

<u>Limit</u>: For a function $f(x)$, $\lim_{x \to a} f(x) = L$ means that as the value of *x* gets closer and closer to *a*, the value of $f(x)$ (i.e. y) gets closer and closer to *L*.

$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$ is read " the limit of the change in *y* with respect to the change in *x* as the change in *x* approaches 0"

Average Rate of Change of a Function Vs. Instantaneous Rate of Change



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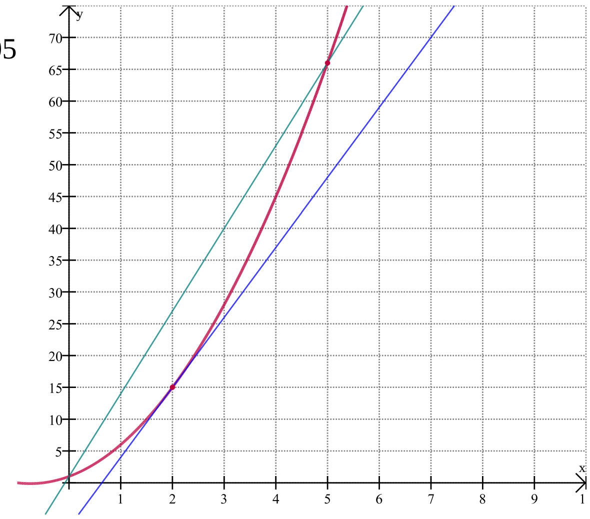
Problem1: The population of fish in a pond is given by $P(t) = 2t^2 + 3t + 1$ where P is the population in thousands and t is the time in years since January 2000.

a) Find the **average** rate of change of the population from 2000 to 2005



b) Estimate the **instantaneous** rate of change in population i) January 2002 ii) January 2005

Method 1: Cave Man Approach (number crunching)	**Method 2:** Industrial Revolution Approach (First Principles)

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