Chapter 1: linear Systems:

Vocabulary:

1. **Linear System:**
2. **Point of Intersection**
3. **Method of Substitution**
4. **Equivalent Linear Equations**
5. **Equivalent Linear Systems**
6. **Method of Elimination**

Basic equation of a line.

y = mx + b

other forms of an equation of a line is:
Ax + By = C
Ax + By + C = 0

m is the slope of the line

m = (y2 – y1)

(x2 - x1)

b is the y – intercept.

A simple equation of a line below:

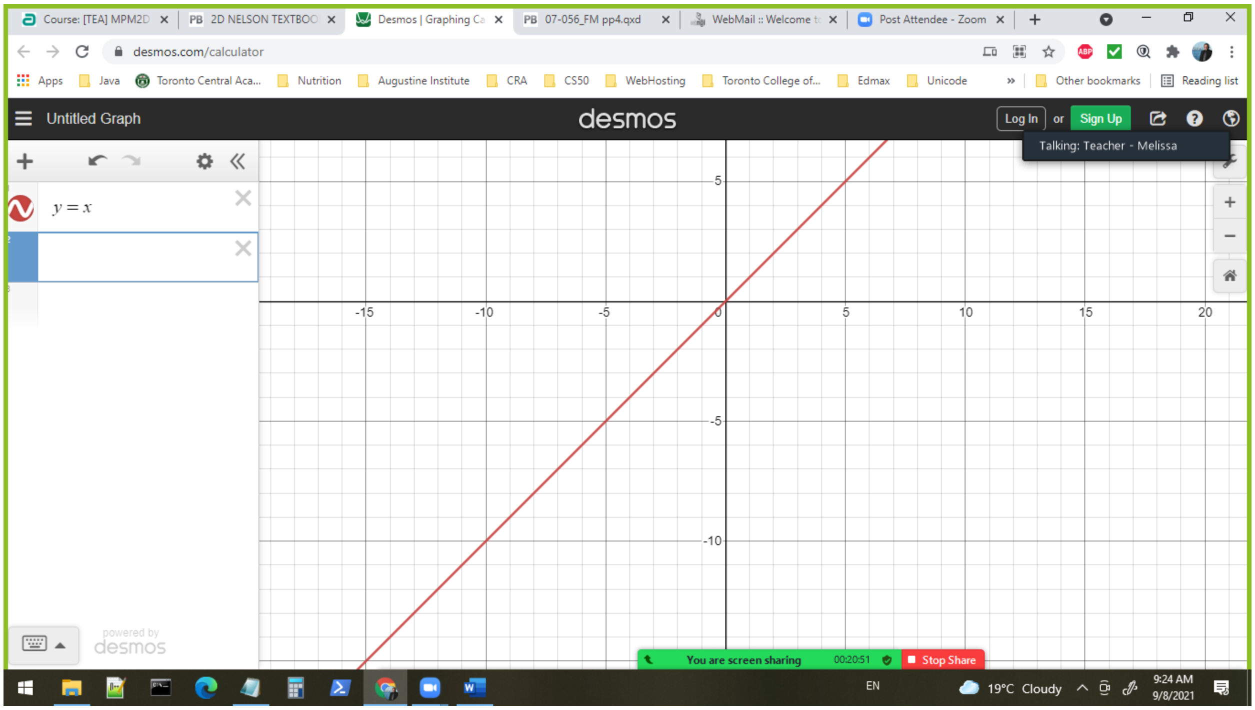
y = x

Independent variable x	Dependents variable y	Pairs (x, y)
Negative infinity …	Negative infinity…	
-5	-5	(-5.-5)
-4	-4	(-4, -4)
-3	-3	(-3, -3)
-2	-2	(-2, -2)
-1	-1	(-1, -1)
0	0	(0, 0)
1	1	(1, 1)
2	2	(2, 2)
3	3	(3, 3)
4	4	(4, 4)
5	5	(5, 5)
… positive infinity	… positive infinity	

When x = -5, what is y, y is -5.

When x = -4, what is y, y is -4…

Y = x



m = 1

point 1(x1, y1) = (0, 0)

point 2 (x2, y2) = (5, 5)

m = (y2 – y1) / (x2 – x1) = (5 -0) / (5 – 0) = 5/5 = 1

point 2 (x2, y2) = (5, 5)
point 1 (x1, y1) = (-5, -5)

m = (y2 – y1) / (x2 – x1) = (5 –(-5)) / (5 – (-5)) = (5 + 5) / (5 + 5) = 10 / 10 = 1

general equation is y = mx + b, where by is the y intercept.

y-intercept is when x = 0.

$y = 1x + b$

when $x = 0$, what is b.

$y = 1(0) + b$
$b = 0$.

Here is an example of when the slope of the line is negative.

y = -x

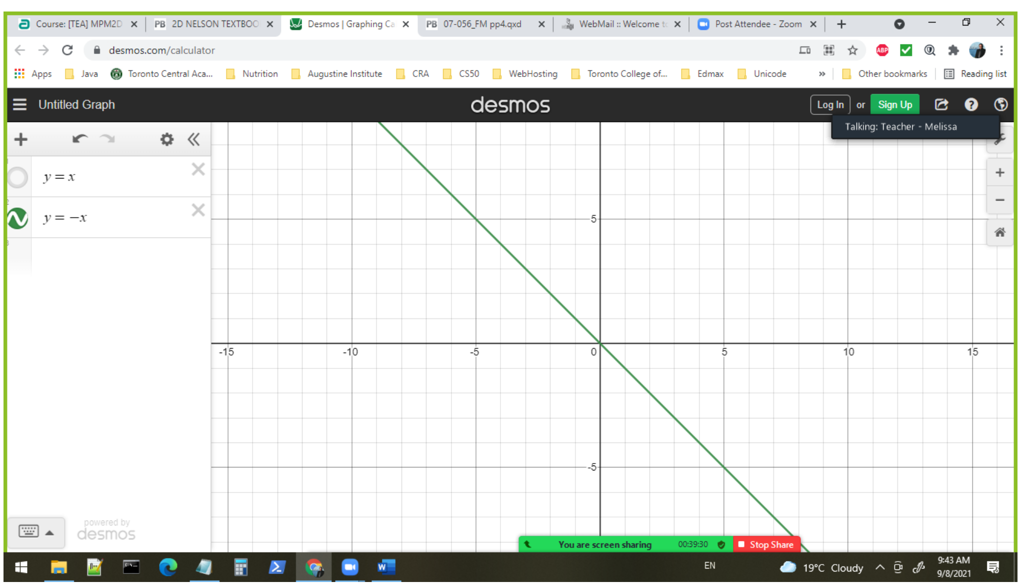
Independent variable x	**Dependents variable y**	**Pairs (x, y)**
Negative infinity …	**Negative infinity…**	
-5	**5**	**(-5.5)**
-4	**4**	**(-4, 4)**
-3	**-3**	**(-3, 3)**
-2	**-2**	**(-2, 2)**
-1	**-1**	**(-1, 1)**
0	**0**	**(0, 0)**
1	**1**	**(1, -1)**
2	**2**	**(2, -2)**
3	**3**	**(3, -3)**
4	**4**	**(4,-4)**
5	**5**	**(5, -5)**
… positive infinity	**… positive infinity**	

When x = -5, what is y, y = -(-5) = y = 5

When x = -4, what is y, y = -(-4) = y = 4

and so on….

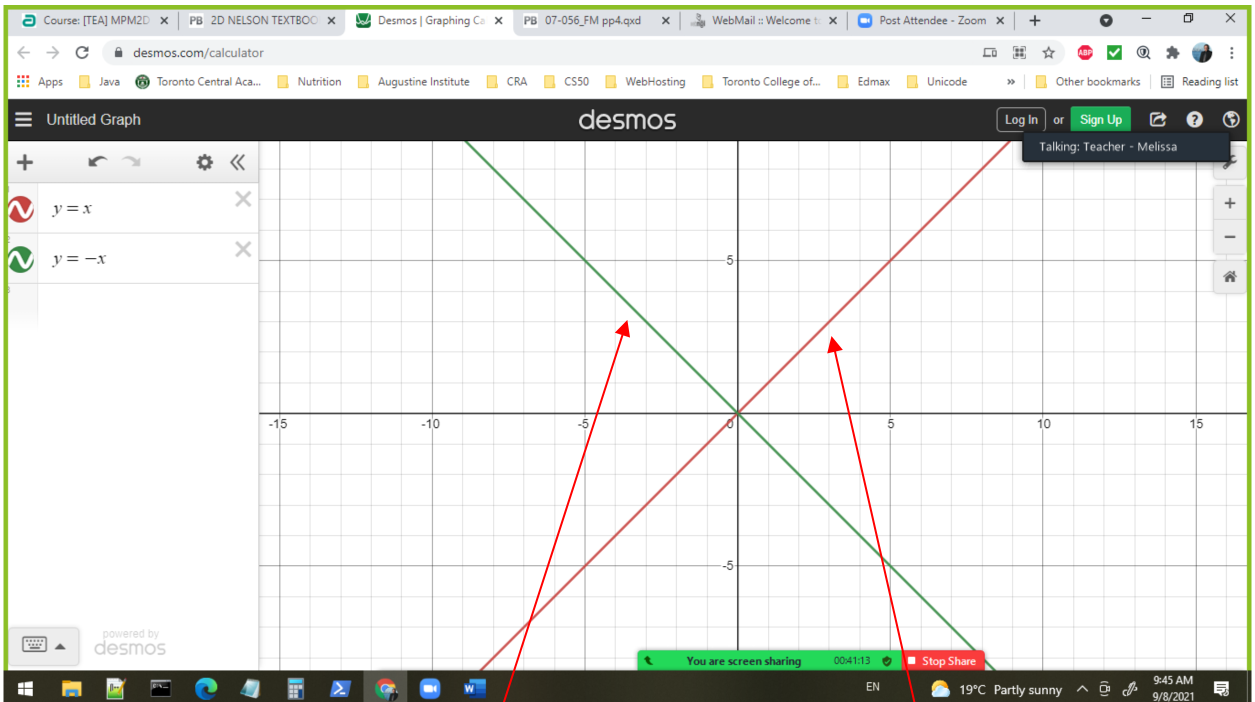
when x = 5, what is y, y = -(5) = -5



__**Combine both equations:**__

y = x (slope is positive) and

y = -x (slope is negative)



y = -x is an equation with a negative slope of m = -1.

y = x is an equation with a positive slope of m = 1.

Examples:

Questions:

Graph each line. Use a table of values or the slope, y-intercept method.

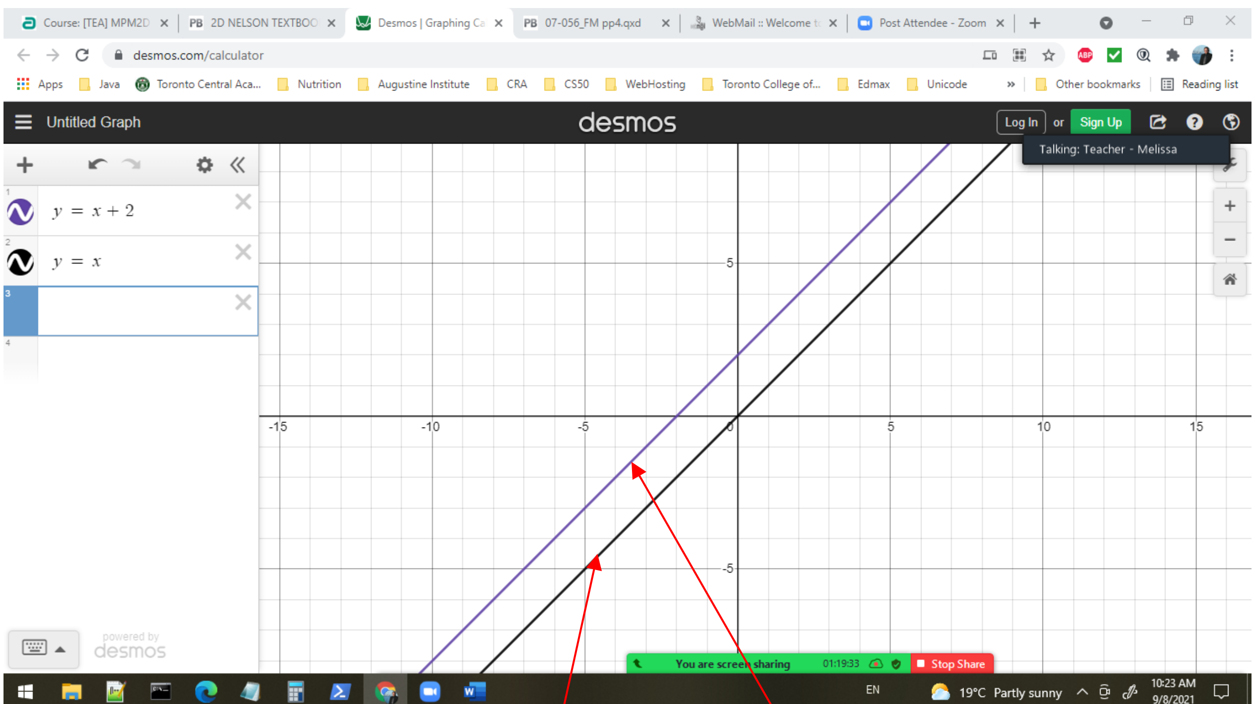
5a) y = x + 2

m = 1

when x = 0, y intercept is:

y = 0 + 2; y = 2.

Point of y intercept is (x, y) = (0, 2)



y = x, where slope (m = 1), y-intercept is 0.

y = x + 2, where slope (m = 1), y-intercept is 2.

<u>Based on the Nelson text book:</u>

<u>4a)</u> y = 5x - 1

Using the slope, y – intercept method, lets plot the graph.

y = mx + b

m = 5, slope is 5, rise / run.

$$m = \frac{rise}{run} = \frac{(y2 - y1)}{(x2 - x1)}$$

y-intercept, is what is the value of y, when x is 0.

y = 5x - 1, when x = 0
y = 5(0) - 1, y = -1

The point on the graph for the y-intercept is (x,y) = (0, -1)

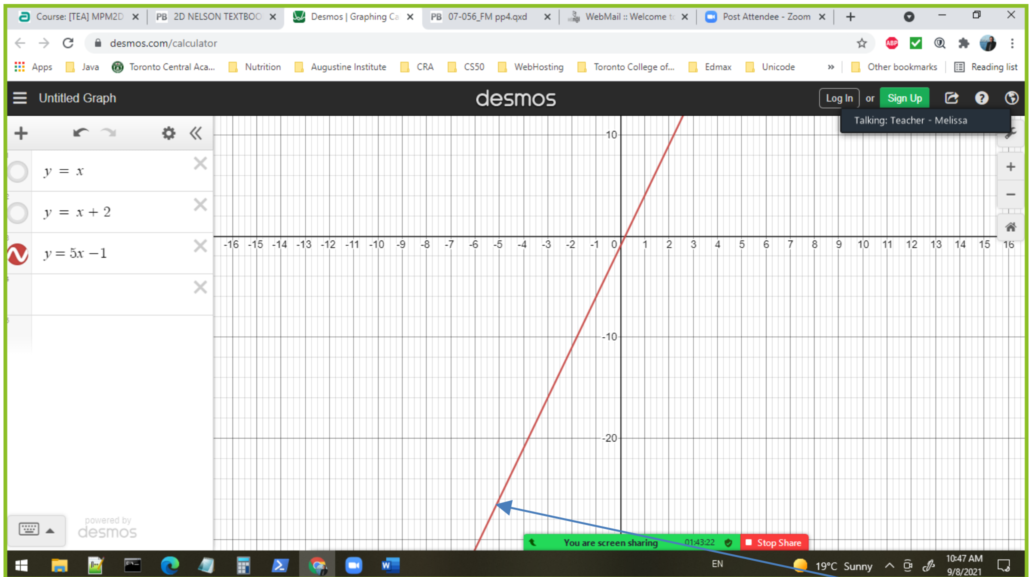
Here is an example of when the slope of the line is negative.

y = 5x - 1

Independent variable X Test domain (-5, … +5)	Calculate y based on formula y = 5x - 1	Dependents variable y	Pairs (x, y)
Negative infinity …		Negative infinity…	
-5	y = 5(-5) - 1	-26	(-5. -26)
-4	y = 5(-4) - 1	-21	(-4, -21)
-3	y = 5(-3) - 1	-16	(-3, -16)
-2	y = 5(-2) - 1	-11	(-2, -11)
-1	y = 5(-1) - 1	-6	(-1, -6)
0	y = 5(0) - 1	-1	(0, -1) ➔ Y intercept, when x = 0
1	y = 5(1) - 1	4	(1, 4)
2	y = 5(2) - 1	9	(2, 9)
3	y = 5(3) - 1	14	(3, 14)
4	y = 5(4) - 1	19	(4, 19)
5	y = 5(5) - 1	24	(5, 24)
… positive infinity		… positive infinity	

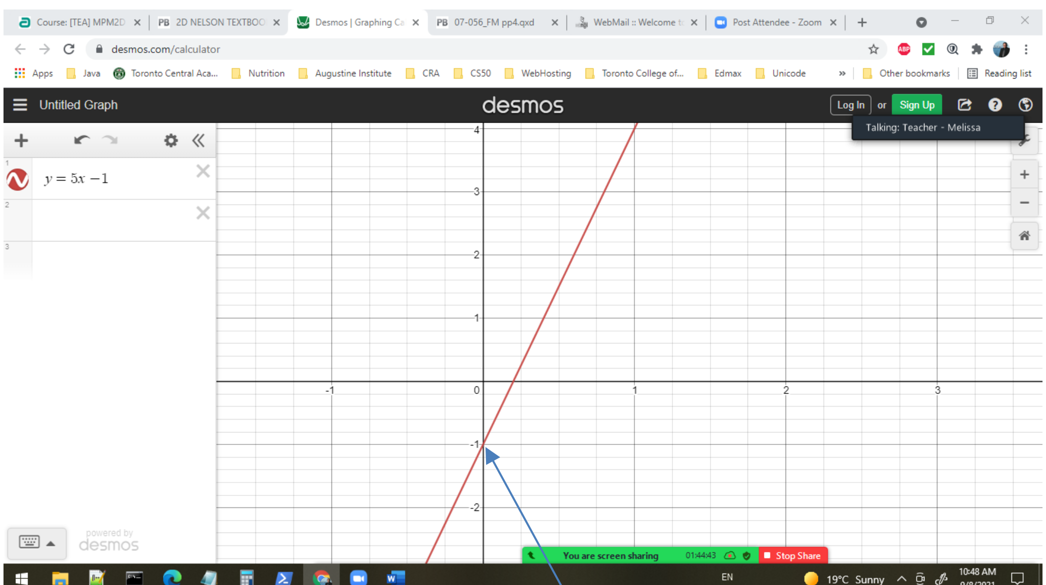
The graph below is for:

Y = 5x -1



Point (-5, -26)

zoomed in to show y-intercept more clearly.



Point of y-intercept (0, -1), when x is 0.

Assignment #1:
Exercise:

Use the graphing software:
https://www.desmos.com/calculator

plot the following graphs:

1. **y = x**
2. **y = -x**
3. **y = x + 2**
4. **y = 5x – 1**

Answer the following:

What is the slope of each equation, and what is the y – intercept, that is, when x = 0.

Note 1: remember, the general equation of a line is y = mx + b, where m is the slope of the line, and b is the y-intercept.

Note 2: remember, the y-intercept is when x is 0.

Date: Thursday, September 9th, 2021.

Example 1: Translate words into Algebra to Solve a Problem:

Ian owns a small airplane. He pays $50/h for flying time and $300/month to store the plane in a hangar at a local airport. A hangar is a garage for planes. If Ian rented the same type of plane at the local flying club, it would cost him $100/h only.

How many hours will Ian have to fly each month so that the cost of renting will be the same as the cost of flying his own plane?

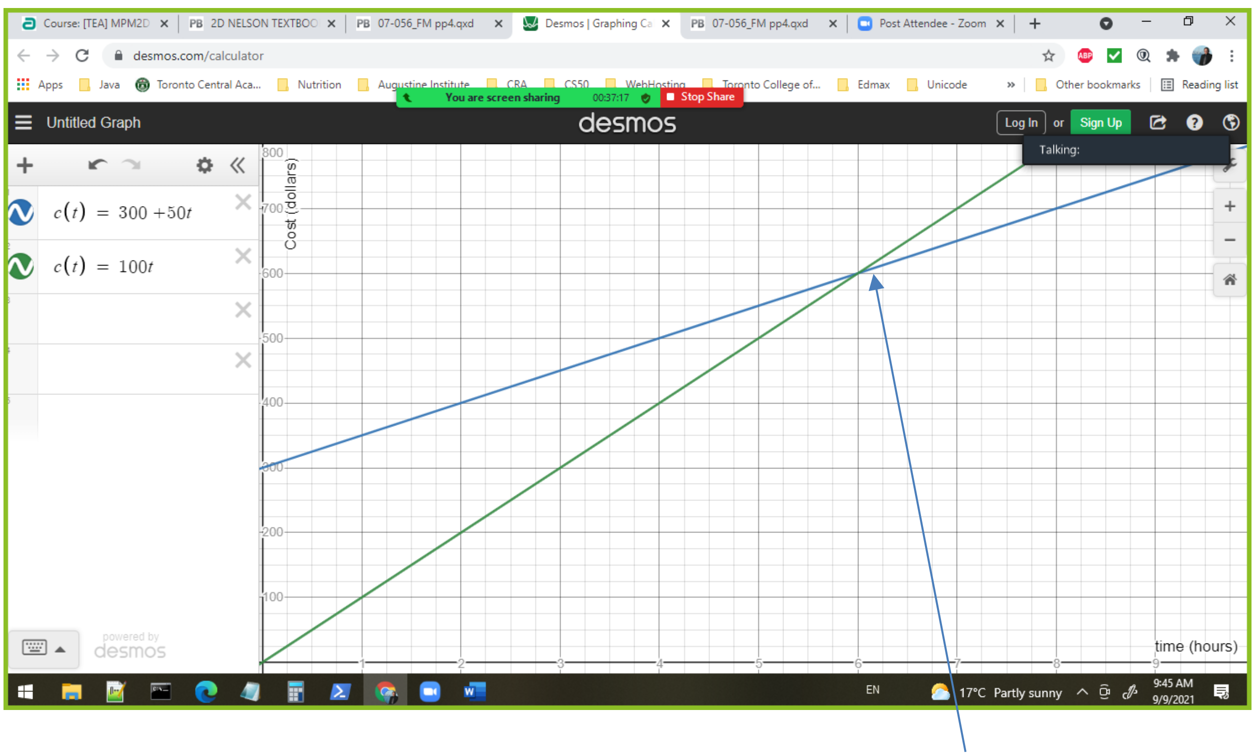
C(h) = $300/h + $50h

C(h) = $100/h

Formulas can also be written in the form of time.

C(t) = $300 + $50t

C(t) = $100t



After 6 hours, based on the graph, Ian will start saving money if he flies more than 6 hours.

Alternative to solve the above problem is using a method called substitution.

Solving Ian's answer using substitution for:
How many hours will Ian have to fly each month so that the cost of renting will be the same as the cost of flying his own plane?

C1(t) = $300 + $50t

C2(t) = $100t

C1(t) = C2(t)

$100t = $300 + 50t

$100t – $50t = $300
$50t = $300

t = $\frac{\$300}{\$50}$ **= 6 hours.**

Example 2: Find the point of Intersection via Substitution from the graph.

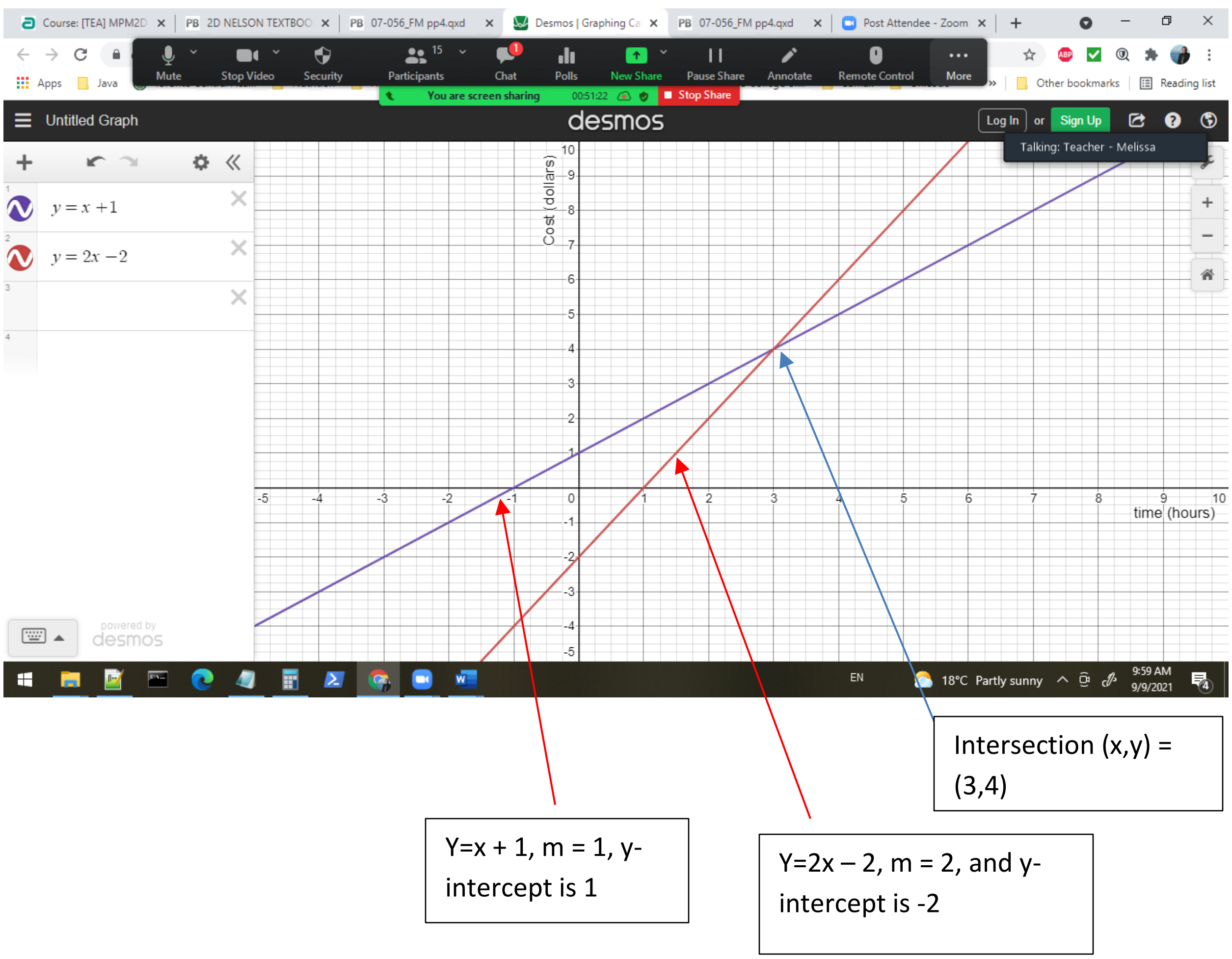
There are two linear equations:

1. **x – y = -1**
2. **2x – y = 2**

Re-arrange the equations in the form of: y = mx + b

1. **x – y = -1**
   -y = -x – 1 <= multiply by -1
   y = x + 1 m = 1, y-intercept is 1

2. **2x – y = 2**
   -y = -2x + 2 <= multiply by -1
   y = 2x -2 m = 2, y-intercept -2.



Through looking at the graph above, that is, visualization, we can see that the lines intersect at point, P(x,y) = (3, 4)

How do we know that P(x, y) = (3, 4) solves the equations. Here, we need to go back to the original equations and substitute (x,y) = (3, 4) and see if the left side (L.S.) = right side (R.S).

The original equations below:

1. **$x - y = -1$**
2. **$2x - y = 2$**

Substitute p(x,y) = (3,4) into equation 1.

L.S. = $x - y = 3 - 4 = -1$.

R.S. = -1

Substitute p(x,y) = (3,4) into equation 2.

L.S. = $2x - y$
L.S. = $2(3) - 4 = 6 - 4 = 2$.

R.S. = 2.

Therefore, L.S. = R.S. on both equations.

Therefore point P(x,y) = (3, 4) is the solution to the linear equations.

Example 2B: Find the point of Intersection via Substitution of one linear equation into another linear equation.

The original equations below:

1. **x – y = -1**
2. **2x – y = 2**

Equation 1:
x – y = -1 <= place into the form y = mx + b

-y = -x - 1 <= multiply both sides by -1.
y = x + 1

Substitute y = x + 1 into equation 2.

2. 2x – y = 2
2x – (x + 1) = 2
2x – x – 1 = 2
x – 1 = 2
x = 2 + 1 = 3

Substitute x = 3 into equation 1.

1. x – y = -1 <= x = 3
3 – y = -1
-y = -1 – 3 <= multiply both sides by -1
y = 1 + 3 = 4

Point P(x, y) = (3, 4)

The original equations below:

1. **x – y = -1**
2. **2x – y = 2**

Equation 1: x – 1 = -1 **L.S. = x – y** **L.S. = 3 – 4 = -1** **L.S. = -1** **R.S. = -1.** **L.S. = R.S.**	**Equation 2: 2x – y = 2** **L.S. = 2x -y** **L.S. = 2(3) -4 = 6 – 4 = 2** **L.S. = 2** **R.S. = 2** **L.S. = R.S.**

Assignment #2: Solve an Internet Cost Problem.

Brian and Catherine ant to get internet access for their home. There are two companies in the same area providing internet access.

The first company, IT Plus, charges a flat fee of $25 / month for unlimited use.

The second company, Techies Inc. charge $10/month plus $1 / hour for use.

If Brian and Catherine expect to use the internet for approximately 18 hours / month, which plan is the better option for them.

Find the equations or algebraic relations for both companies.

Find the point of interaction.

Draw the graphs of both lines.

Explain when is it cheaper or more expense of each plan.

In addition, on the Nelson text book, page 19, Q#7 a, b.

Date: Friday, September 10th, 2021

Goal: Method of Elimination.

Review:

One more problem for method of Substitution

Two equations are:

1. **x + y = 5**
2. **3x – y = 7**

Formula 1: place into the form of y = mx + b

y = -x + 5

substitute y = -x + 5 into equation 2.

3x – y = 7

3x – (-x + 5) = 7
3x + x -5 = 7
4x -5 + 5 = 7 + 5
4x = 12
x = 3

substitute x = 3 into the original equation 1: x + y = 5 to solve for y.
x + y = 5
(3) + y = 5
3 + y = 5
Solve for y;

3 -3 + y = 5 – 3
y = 5 – 3 = 2
y = 2.

Point (x,y) = (3,2)

Point (x,y) = (3,2)

Go back to the original equations: 1, 2 and determine if both the L.S. = R.S.

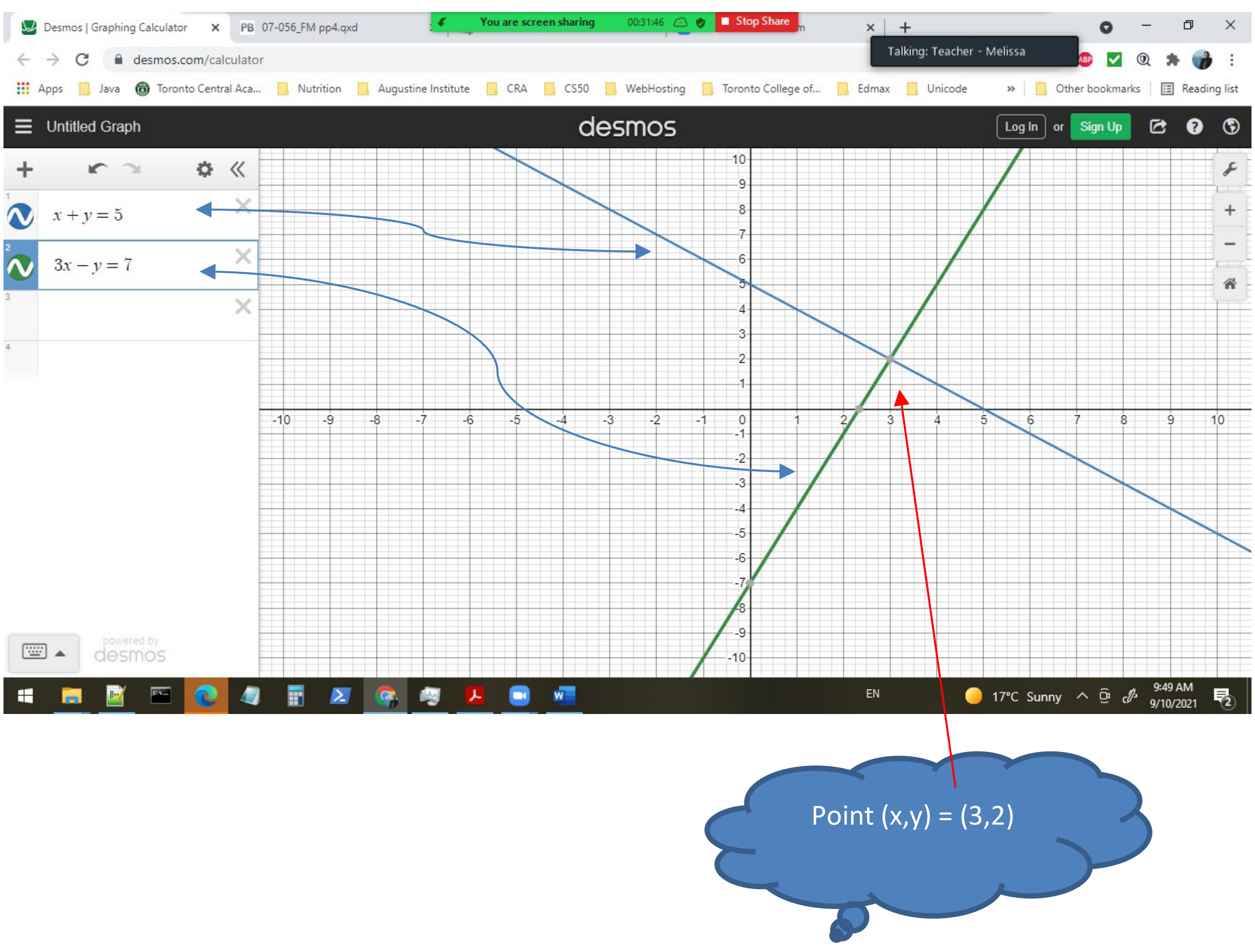
1. **x + y = 5**
2. **3x – y = 7**

In x + y = 2

In: x + y = 2	**In: 3x – y = 7**
L.S. = x + y **L.S. = 3 + 2** **L.S. = 5** **R.S. = 5** **L.S. = R.S.**	**L.S. = 3x – y** **L.S. = 3(3) – 2** **L.S. = 9 – 2 = 7** **L.S. = 7** **R.S. = 7** **L.S. = R.S**

Therefore, point (x,y) = (3,2) satisfies both equations. Therefore the solution for both linear equations is point(x,y) = (3,2)

Let's also place the formulas in our graphing calculator so visualize the solution for point of intersection.



Example 1: Method of Elimination:

Solve using method of elimination.

1. **3x + y = 19**
2. **4x – y = 2 <= Add both equations. (y will be eliminated)**

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3. **7x = 19 + 2**
   7x = 21
   x = $\frac{21}{7}$
   x = 3

Substitute x = 3, into the original equation.

1. **3x + y = 19**
   3(3) + y = 19
   9 + y = 19
   9 – 9 + y = 19 – 9
   y = 19 – 9
   y = 10

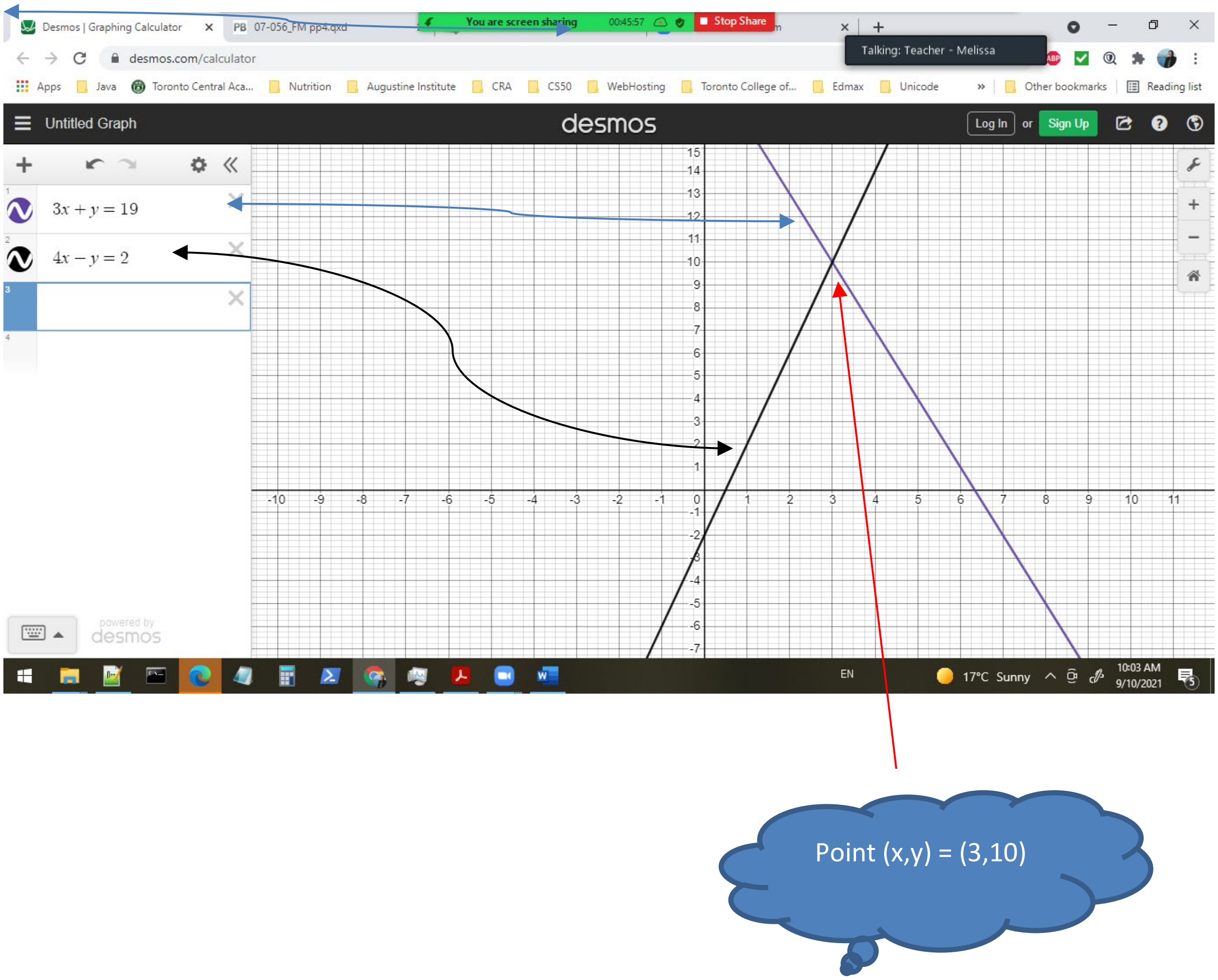
Point (x, y) = (3, 10)

Substitute point (x, y) = (3, 10) into both original equations to see if both the L.S. = R.S.

In: 3x + y = 19	In: 4x – y = 2
L.S. = 3x + y L.S. = 3(3) + 10 L.S. = 9 + 10 L.S. = 19 R.S. = 19 L.S. = R.S.	L.S. = 4x – y L.S. = 4(3) – 10 L.S. = 12 – 10 = 2 L.S. = 2 R.S = 2 L.S. = R.S

Therefore, point (x,y) = (3,10) satisfies both equations. Therefore the solution for both linear equations is point(x,y) = (3,10)

Let's also place the formulas in our graphing calculator so visualize the solution for point of intersection



Example 2: Method of Elimination:

1. $4x + 3y = 13$
2. $5x - 4y = -7$ **<= eliminate variable x, multiply equation 1 by 5, and equation 2 by 4.**

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3. $4x(5) + 3y(5) = 13(5)$
   $20x - 15y = 65$

4. $5x(4) + 4y(4) = -7(4)$
   $20x - 16y = -28$

$20x + 15y = 65$ **← = equation 3**
$20x - 16y = -28$ **← equation 4; subtraction 4 from 3. (Multiply by -1 on both sides)**

$20x + 15y = 65$
$-20x + 16y = 28$ **← add both of these equations to eliminate the variable x.**

$20x - 20x + 15y + 16y = 65 + 28$
$(20x - 20x) + (15y + 16y) = 65 + 28$
$31y = 65 + 28 = 93$
$31y = 93$
$y = \frac{93}{31} = 3$

Substitute y = 3, into the original equation 1.

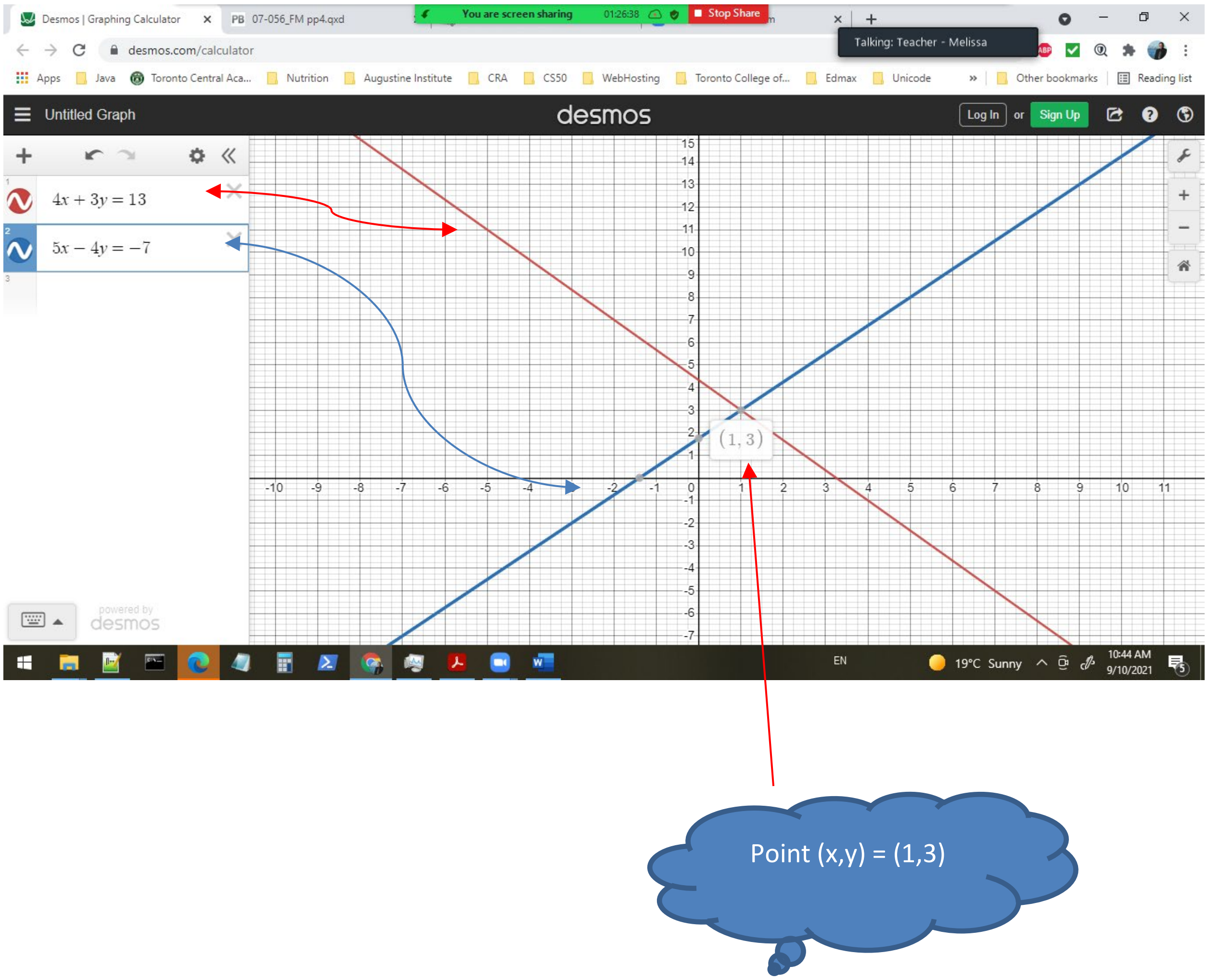
$4x + 3y = 13$
$4x + 3(3) = 13$
$4x + 9 = 13$
$4x + 9 - 9 = 13 - 9$
$4x = 13 - 9$
$4x = 4$
$x = 1$

Our point (x, y) = (1, 3).
Substitute (x,y) = (1, 3) into both original equations to see if L.S. = R.S in both equations.

In: $4x + 3y = 13$	In: $5x - 4y = -7$
L.S. = $4x + 3y$ L.S. = $4(1) + 3(3)$ L.S. = $4 + 9 = 13$ L.S = 13 R.S. = 13 L.S. = R.S.	L.S. = $5x - 4y$ L.S. = $5(1) - 4(3)$ L.S. = $5 - 12$ L.S. = -7 R.S = -7 L.S. = R.S.

Therefore, point (x,y) = (1,3) satisfies both equations. Therefore the solution for both linear equations is point(x,y) = (1,3)

Let’s also place the formulas in our graphing calculator so visualize the solution for point of intersection



Assignment #3:

Solve the linear equations:

1. **$10x + 4y = -1$**
2. **$8x - 2y = 7$**

Use the process of elimination to find the point of intersection point (x, y) = (x1, y1). Substitute point (x1, y1) into both original equations to find the point of intersection. Use L.S. = R.S. methodology to prove that the point (x1, y1) solves the linear equations.

Graph both linear equations using the graphing calculator.

Label both lines, and the point of intersection of the graph as well.

Language:

Linear Systems, to solve linear systems with two unknowns usually x, y, you need to equations.

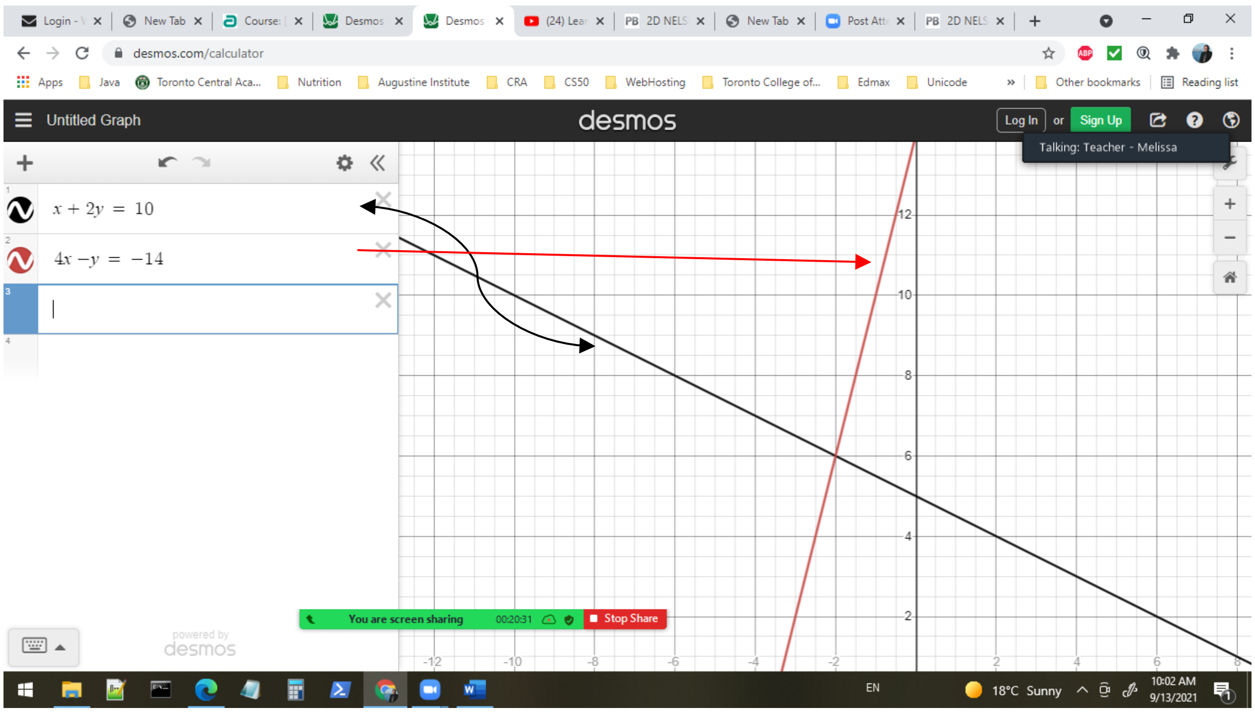
$x1 + y1 = c1$
$x2 + y2 = c2$

Linear systems can also be more than two variables, link 3 unknown variables, x, y, z. Then you would need 3 equations with x, y, z to the 3 unknown variables. And so on with n number of unknown variables, you would need n number of equations.

Section 1.5 – Equivalent Linear Systems

Equivalent Linear Systems are systems of equations that have the same solution.

1. **$x + 2y = 10$**
2. **$4x - y = -14$**



Point of intersection is p(x, y) = (-2, 6). This is the solution to the system of the two equations above.

Page 41, in Nelson text book.

If we have a set of linear equations, and the intersection point results in the same intersection point as the equations above, then we have equivalent linear systems.

How do we a another set of equations that have the same point of intersection. The text book creates the two new equations from the original equations above.

1. x + 2y = 10
2. 4x – y = -14 🡐 add the two equations.

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3. 5x + y = -4

1. x + 2y = 10
2. 4x – y = -14 🡐 subtract the two equations.

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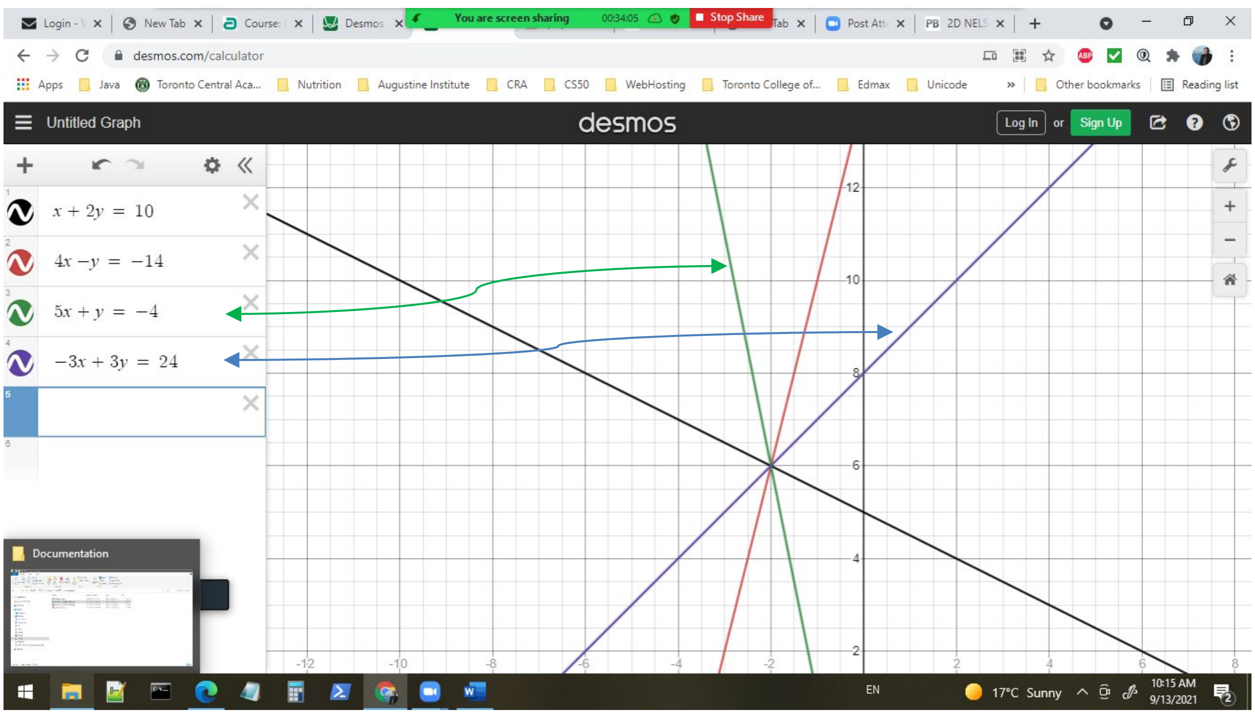
x – 4x +2y +y = 10 –(-14)

4. -3x + 3y = 24



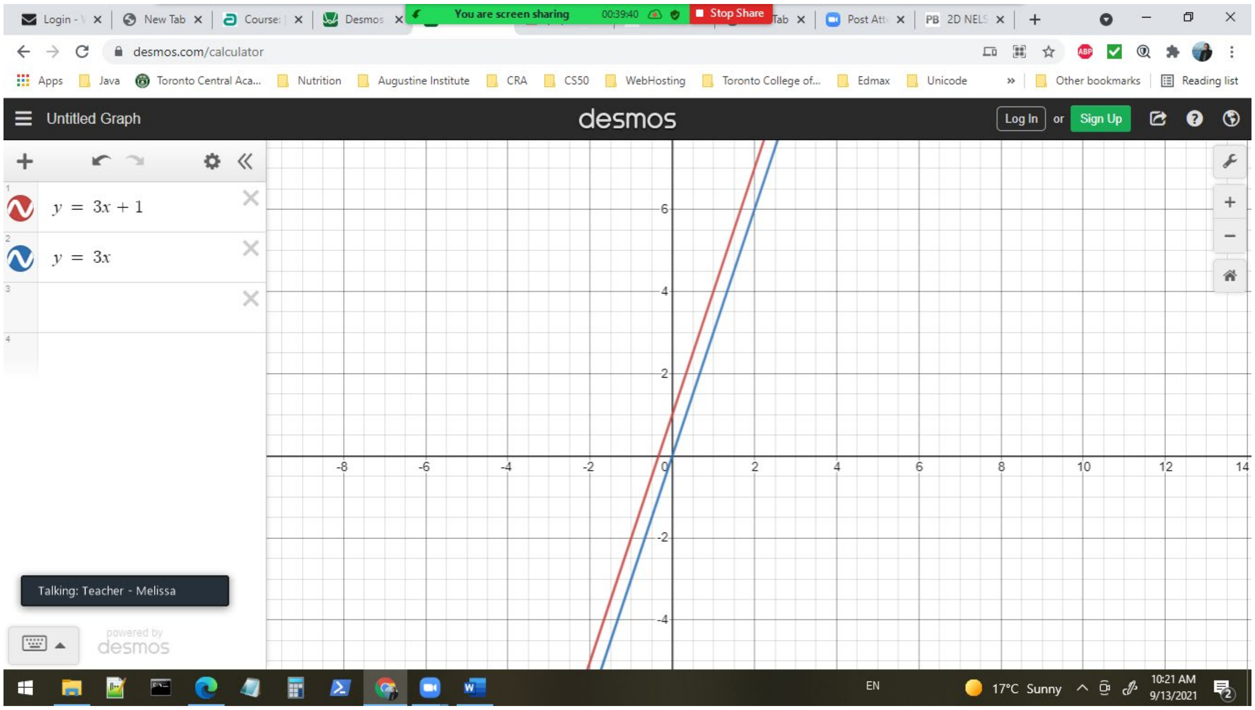
We add the last two lines, and they intersect at the same point (-2, 6). Therefore, the two linear systems are equivalent.

Section 1.7 – Exploring Linear Systems.

1. **Possibility #1: When the equations of the lines, have the same slope, there is no solution to the problem.**

y = 3x + 1
y = 3x

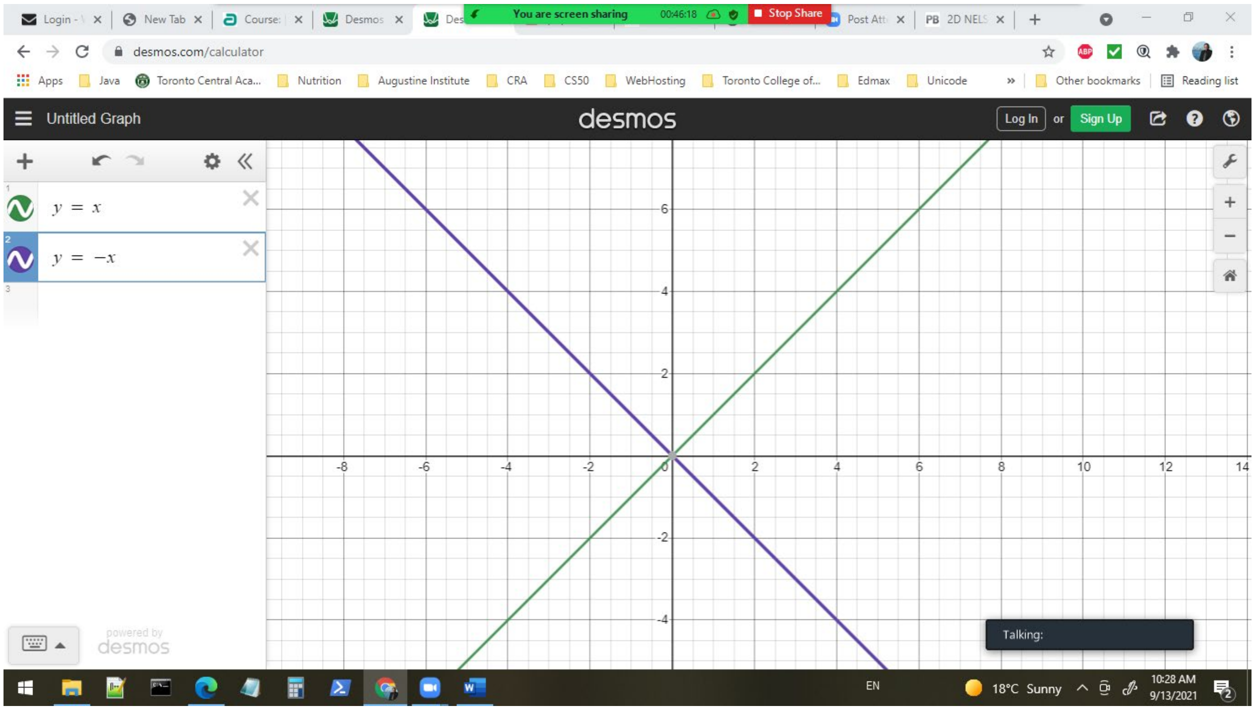
These lines are parallel, and the do not intersect, therefore they have no solution.



2. Possibility #2: One solution, the lines intersect at point (x, y) = (0, 0)

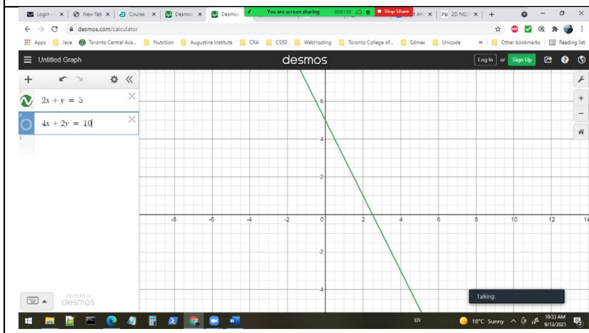
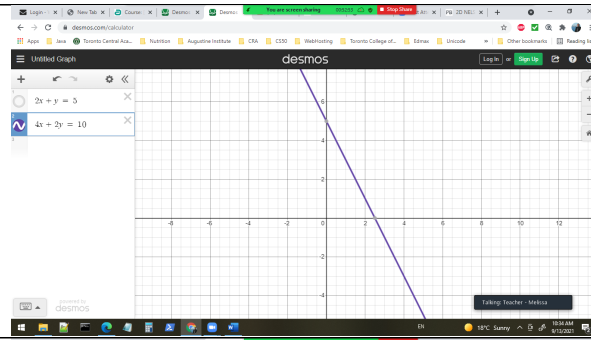
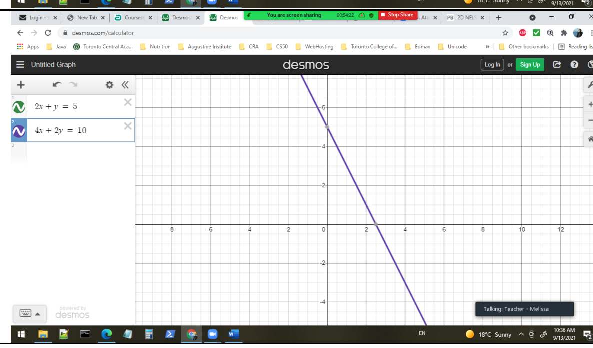
y = x

y = -x



These two lines intersect at point (x, y) = 0, 0. Therefore, the linear equations have a solution, with the point of intersection.

3. **Possibility #3: The lines are on top of each other, which means there are many (infinite number) of solutions.**

2x + y = 5	**4x + 2y = 10**
	
Overlapping the two lines ====➔ **Infinite number of solutions:**	

Page 57, Nelson text book.

1.7 Exploring Linear Systems

GOAL

Connect the number of solutions to a linear system with its equations and graphs.

YOU WILL NEED

- graphing calculator, or grid paper and ruler

EXPLORE the Math

Three different linear systems are given below.

A	B	C
$2x + 3y = -4$ $-4x - 3y = -1$	$2y = 6 - 3x$ $6x - 5 = -4y$	$x - y = 5$ $3x = 15 + 3y$

❓ How many solutions can a linear system have, and how can you predict the number of solutions without solving the system?

A. Solve each system of linear equations algebraically. Record the number of solutions you determine.

	A	B	C
Linear System	$2x + 3y = -4$ $-4x - 3y = -1$	$3x + 2y = 6$ $6x + 4y = 5$	$x - y = 5$ $3x - 3y = 15$
Number of Solutions			

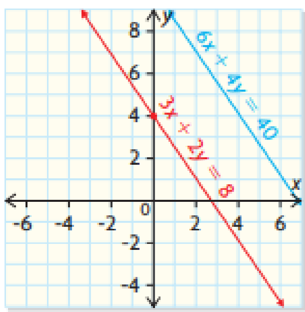
In Summary

Key Idea

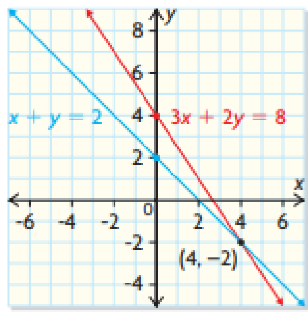
- A linear system can have no solution, one solution, or an infinite number of solutions.

Need to Know

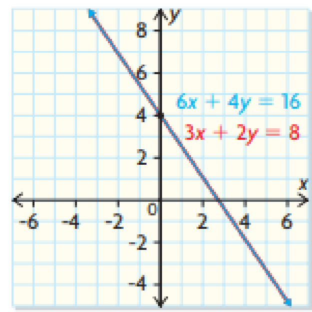
- When a linear system has no solution, the graphs of both lines are parallel and never intersect. For example, the system $3x + 2y = 8$ and $6x + 4y = 40$ does not have a solution. The coefficients in the equations are multiplied by the same amount, but the constants are not.



- When a linear system has one solution, the graphs of the two lines intersect at a single point. For example, the system $3x + 2y = 8$ and $x + y = 2$ has one solution. The coefficients and constants in the equations are not multiplied by the same amount.



- When a linear system has an infinite number of solutions, the graphs of both equations are identical and intersect at every point. For example, the system $3x + 2y = 8$ and $6x + 4y = 16$ has an infinite number of solutions. The coefficients and constants in the equations are multiplied by the same amount.



Nelson Text book, page 63.

b) Use addition and subtraction to create another linear system that is equivalent to the system in the graph.

c) Use multiplication to create another linear system that is equivalent to the system in the graph.

11. **a)** Create two linear systems that are equivalent to the following system.

$-2x - 3y = 5$

$3x - y = 9$

b) Verify that all three systems have the same solution.

Lesson 1.6

12. Use elimination to solve each linear system.

a) $2x - 3y = 13$
$5x - y = 13$

b) $x - 3y = 0$
$3x - 2y = -7$

c) $3x + 21 = 5y$
$4y + 6 = -9x$

d) $x - \frac{1}{3}y = -1$
$\frac{2}{3}x - \frac{1}{4}y = -1$

15. Solve the linear system.

$2(2x - 1) - (y - 4) = 11$

$3(1 - x) - 2(y - 3) = -7$

16. Juan is a cashier at a variety store. He has a total of $580 in bills. He has 76 bills, consisting of $5 bills and $10 bills. How many of each type does he have?

17. **a)** Sketch a linear system that has no solution.

b) Determine two possible equations that could represent both lines in your sketch.

c) Explain how the slopes of these lines are related.

18. The linear system $6x + 5y = 10$ and $ax + 2y = b$ has an infinite number of solutions. Determine a and b.

Assignment #4.

Page 63, Q# 11 a, b. Q#17 a, b, Q#18.