

Chapter 2: Analytic Geometry:

Wikipedia – Definition:

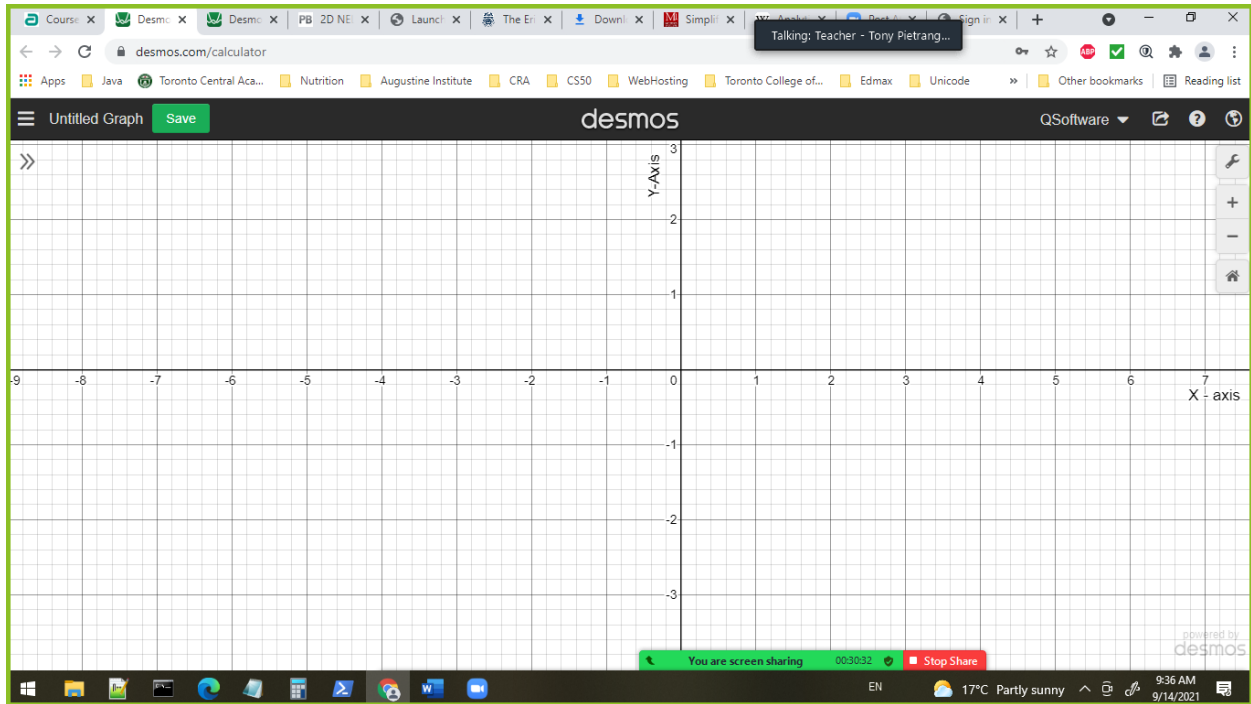
In classical mathematics, **analytic geometry**, also known as **coordinate geometry** or **Cartesian geometry**, is the study of **geometry** using a **coordinate system**. This contrasts with **synthetic geometry**.

Analytic geometry is used in **physics** and **engineering**, and also in **aviation**, **rocketry**, **space science**, and **spaceflight**. It is the foundation of most modern fields of geometry, including **algebraic**, **differential**, **discrete** and **computational geometry**.

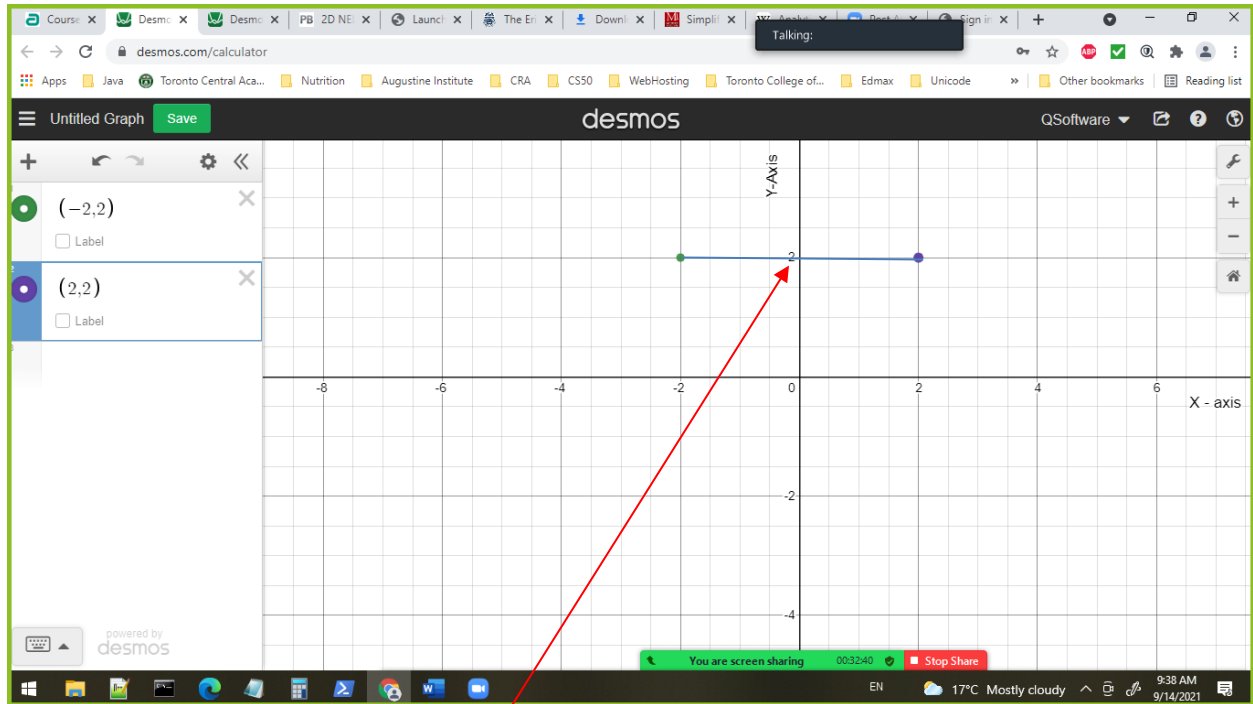
Vocabulary:

1. **Fractal**
2. **Cartesian grid**
3. **Midpoint**
4. **Equidistant**
5. **Right bisector**
6. **Altitude**
7. **Diagonal**
8. **Scalene triangle**
9. **Perpendicular bisector**
10. **Median of a triangle**
11. **Midsegment of a triangle**
12. **Cartesian coordinate system**
13. **Isosceles triangle**
14. **Equilateral triangle**

Cartesian Grid: — is a grid of x, and y coordinates in a two-plane system that have perpendicular axis.



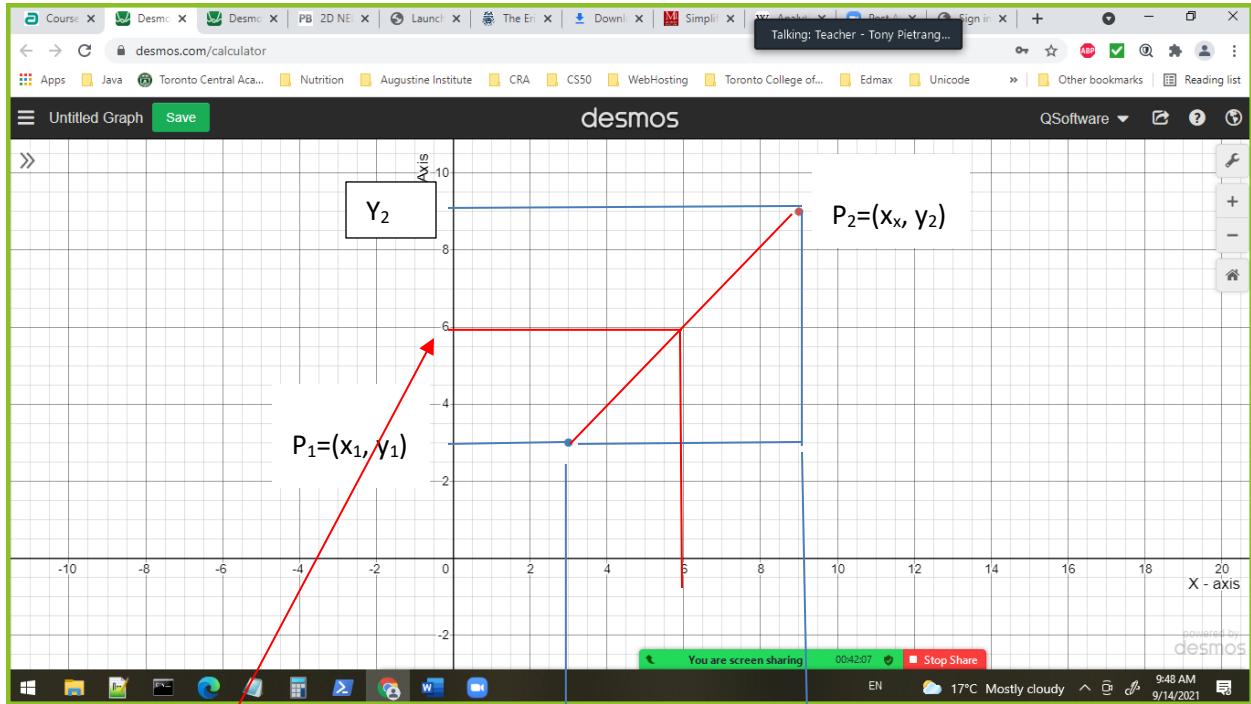
Midpoint: – is a point that divides a line segment into two equal line segments.



Length Midpoint of line segment = $(x_2 - x_1) = (2 - (-2)) = 4 / 2 = 2$
Start from $-2 + 2 = 0$. The midpoint is at $x = 0, y = 2$.

Generic Midpoint of a line Segment:

Plot any two generic Points $P_2(x_2, y_2)$, $P_1(x_1, y_1)$

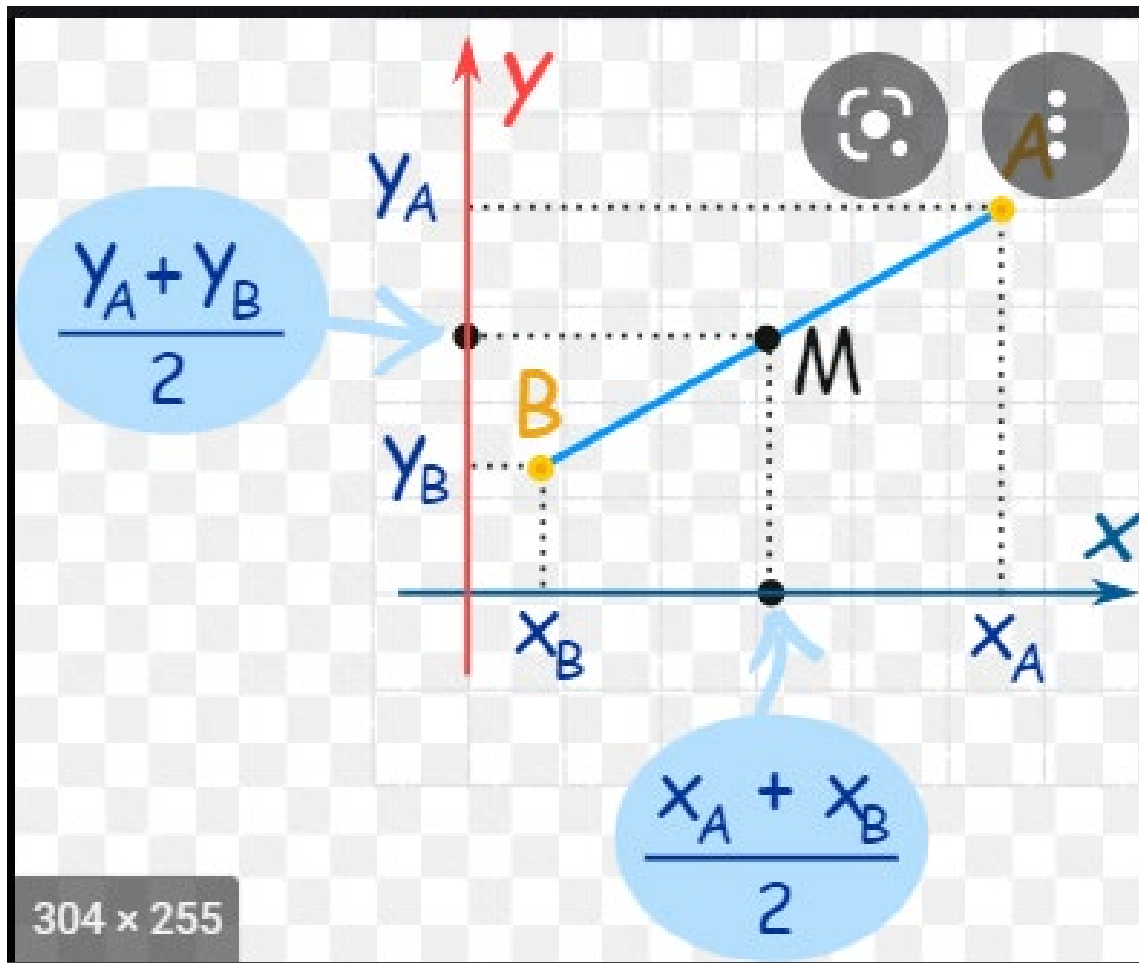


$$\frac{Y_1 + Y_2}{2}$$

$$\frac{X_1 + X_2}{2}$$

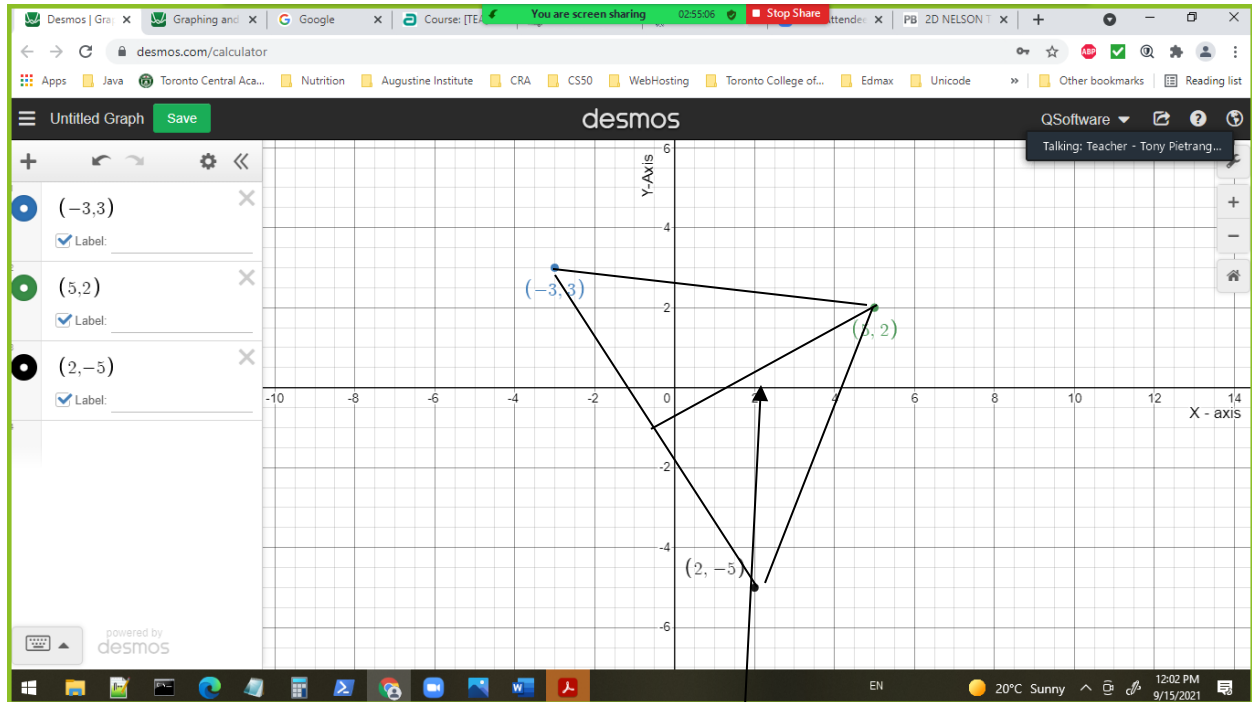
Midpoint $M(x, y) = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$

From the internet:



$$\text{Midpoint } M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Median of a triangle: - is the line segment joining a vertex of a triangle to the midpoint of the opposite side.



The median is the line segment that divides the triangle above.

Example 1: Find the Midpoint of two points:

Find the Midpoint - A(3,5) , B(11,14)

$$\text{Midpoint } M(x, y) = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$$

$$= ((3 + 11) / 2, (5 + 14) / 2)$$

$$= ((14) / 2, (19 / 2))$$

$$\text{Midpoint } (x, y) = (7, 9.5)$$

Three examples: Find the Midpoint.

a) $P_1 = (-4, 4), P_2 = (-4, -2)$

b) $P_1 = (-1, 3), P_2 = (-1, -1)$

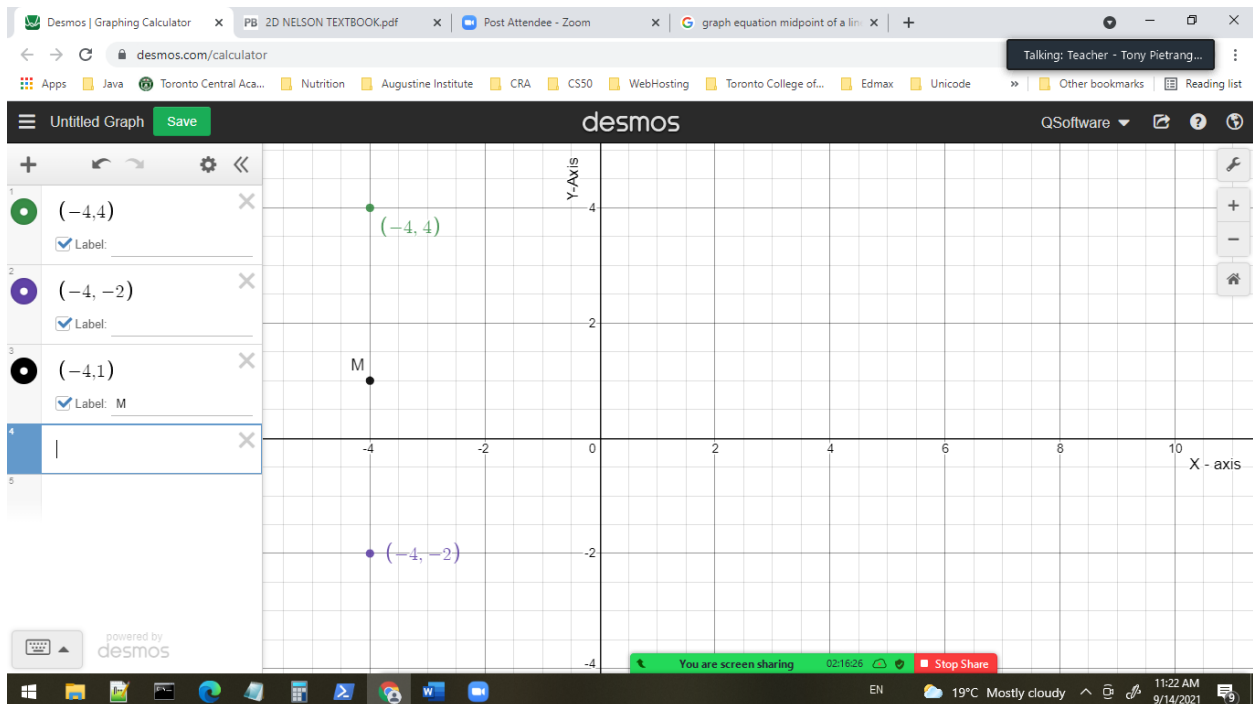
c) $P_1 = (5, -2), P_2 = (5, 3)$

Plot these points on their own graph, and determine the midpoint using the equation for the midpoint.

a) $P_1 = (-4, 4), P_2 = (-4, -2)$

Answer a)

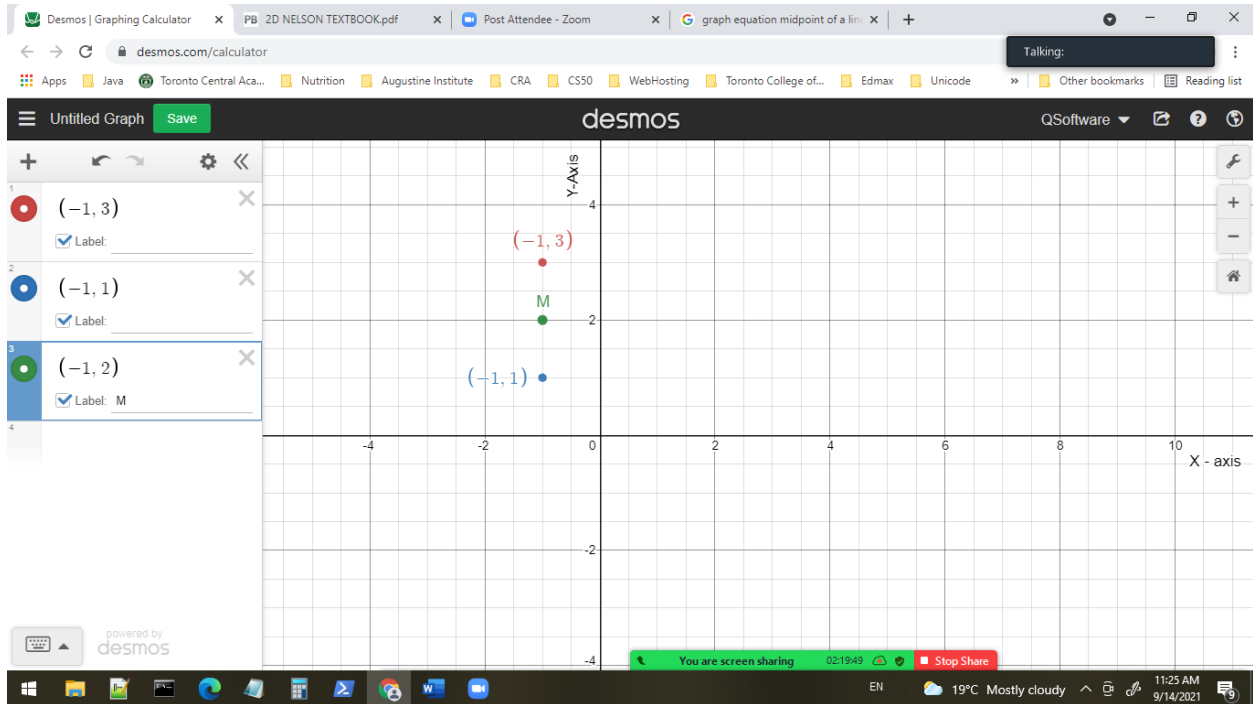
$$M(x, y) = ((-4 + -4) / 2, (4 + -2) / 2) = (-8 / 2, 2 / 2) = (-4, 1)$$



b) $P_1 = (-1, 3)$, $P_2 = (-1, -1)$

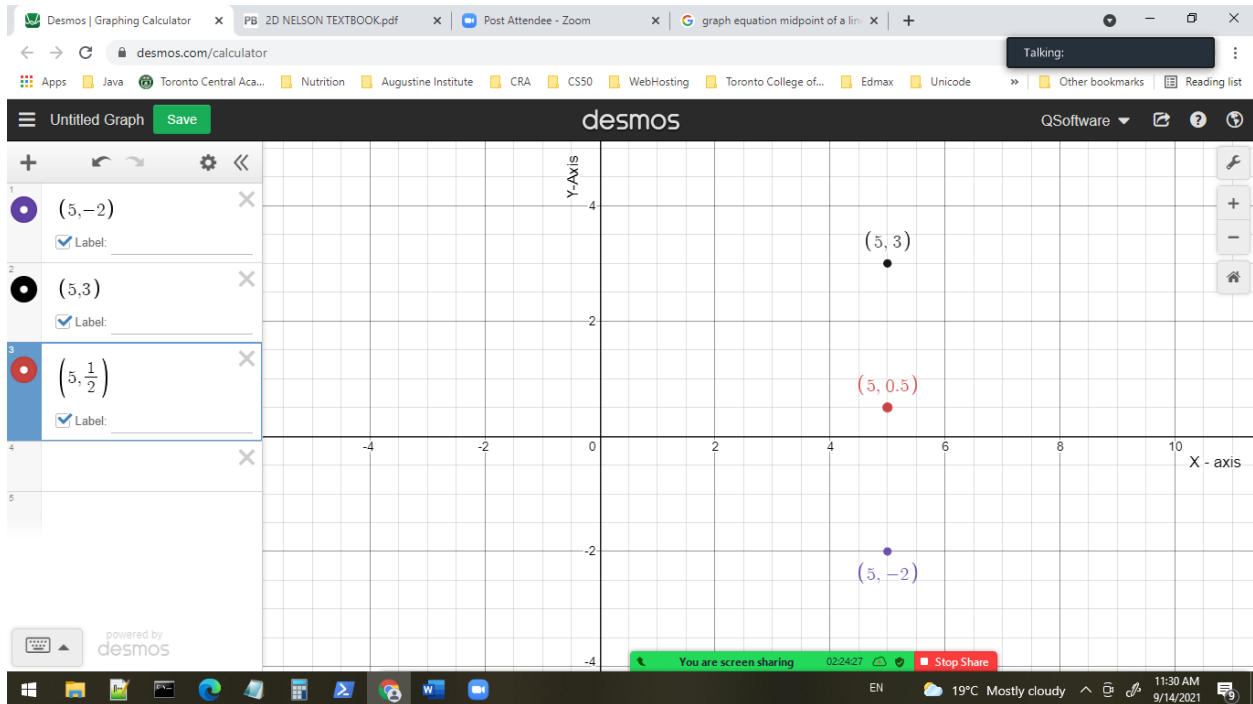
Answer b)

$$M(x, y) = \left(\frac{-1 + -1}{2}, \frac{3 + (-1)}{2} \right) = \left(\frac{-2}{2}, \frac{2}{2} \right) = (-1, 1)$$



C) $P_1 = (5, -2), P_2 = (5, 3)$

$M(x, y) = ((5 + 5) / 2, (-2 + 3) / 2) = (10 / 2) = (5, 1/2)$



Section 2.1 – Finding the Length of a Line Segment

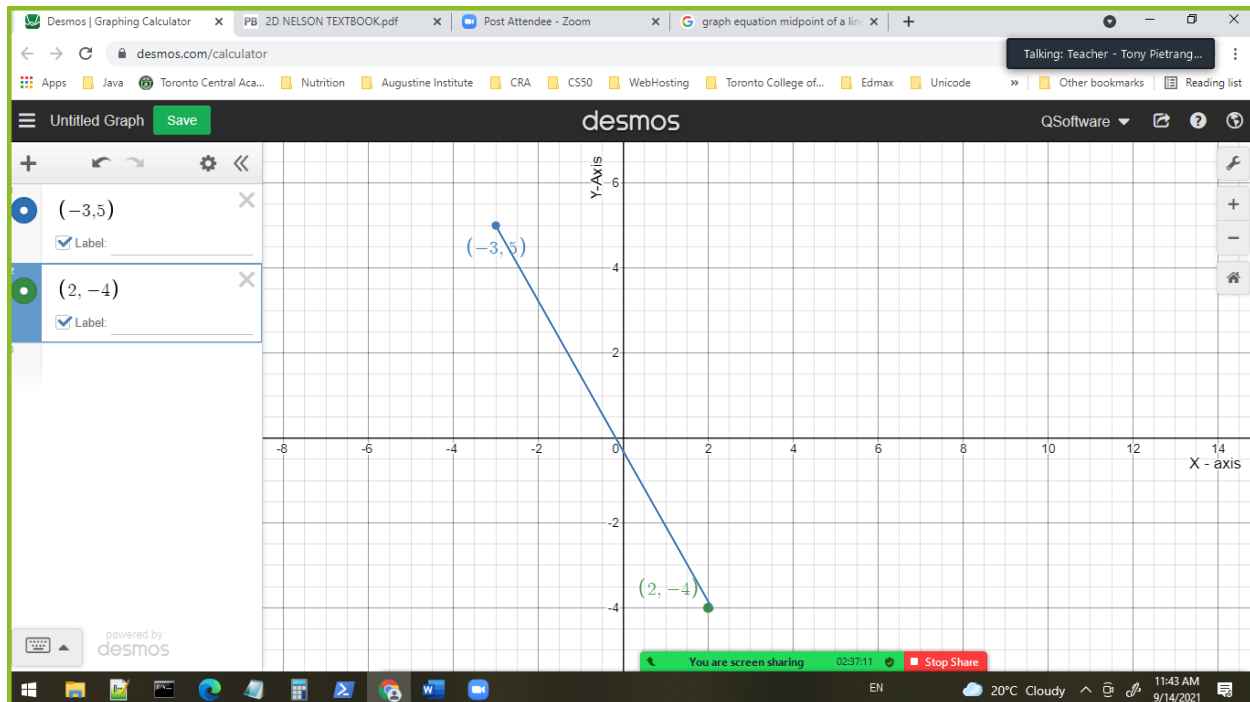
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \leftarrow \text{Pythagorean Theorem}$$

Point A(2, -4), Point B(-3, 5)

$$AB = \sqrt{(-3 - 2)^2 + (5 - (-4))^2}$$

$$AB = \sqrt{(-3 - 2)^2 + (5 - (-4))^2}$$

$$AB = \sqrt{(-5)^2 + (9)^2} = \sqrt{25 + 81} = \sqrt{106} = 10.3$$



Date Created: Wednesday, September 15th, 2021

Goal:

1. Find the Median of a Triangle and connect this to an equation of a line.
2. Find the Right Bisector of a Right-Angle Triangle.

Moodle Whiteboard:

Chapter_2_AnalyticGeometry.docx [Compatibility Mode] - Word

Whiteboard - Zoom

Select Text Draw

Participants can now see your whiteboard

Who can see what you share here? Recording On

Clear Save

Date: Wednesday, September 15th, 2021
Course: MPM2D - Principles of Mathematics.

Previously (Yesterday)

1. Midpoint of a line segment.
2. Length of a line segment (Pathagorean Theorem).

Goal:

1. Median (Midpoint) of a Triangle - Definition of Median.
2. Determine Right Bisector of a Right Angle Triangle. (Right Bisector)

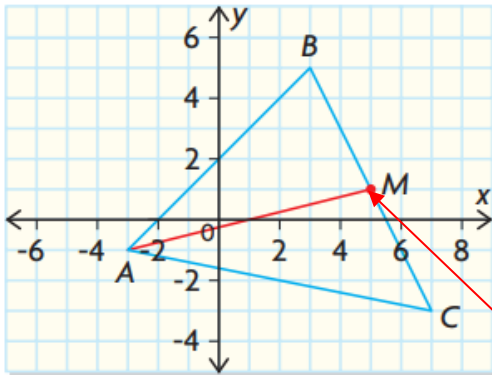
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Nelson text book: page 75.

EXAMPLE 3**Connecting the midpoint to an equation of a line**

A triangle has vertices at $A(-3, -1)$, $B(3, 5)$, and $C(7, -3)$. Determine an equation for the **median** from vertex A .

Graeme's Solution

I plotted A , B , and C and joined them to create a triangle.

I saw that the side opposite vertex A is BC , so I estimated the location of the midpoint of BC . I called this point M . Then I drew the median from vertex A by drawing a straight line from point A to M .

$A(-3, -1)$, $B(3, 5)$, $C(7, -3)$

$$\underline{M}_{BC}(x, y) = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$$

$$\underline{B}(x_1, y_1) = (3, 5)$$

$$\underline{C}(x_2, y_2) = (7, -3)$$

$$\underline{M}_{BC}(x, y) = ((3 + 7) / 2, (5 - 3) / 2)$$

$$\underline{M}_{BC}(x, y) = (10 / 2, 2 / 2)$$

$$\underline{M}_{BC}(x, y) = (5, 1)$$

Objective:

1. Find $M(x, y)$, which is the midpoint of line segment BC
2. Find the equation of line segment AM

Find the equation of line segment AM, A(-3,-1), M(5, 1).

General form of an equation of a line is: $y = mx + b$.

Calculate slope, which $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

$$P_2(x_2, y_2) = M(5, 1)$$

$$P_1(x_1, y_1) = A(-3, -1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-1)}{5 - (-3)} = \frac{1+1}{5+3} = \frac{2}{8} = \frac{1}{4}$$

$$m = \frac{1}{4}$$

$$y = \frac{1}{4}x + b.$$

b is the y-intercept, when $x = 0$.

Choose the coordinate or point A(-3, -1) to determine b.

$$-1 = \frac{1}{4}(-3) + b.$$

$$-1 = -\left(\frac{3}{4}\right) + b \quad \leftarrow \text{Add } \left(\frac{3}{4}\right) \text{ on both sides.}$$

$$-1 + \left(\frac{3}{4}\right) = -\left(\frac{3}{4}\right) + \left(\frac{3}{4}\right) + b$$

$$-\left(\frac{1}{4}\right) = b$$

$$b = -\left(\frac{1}{4}\right)$$

Equation of the line segment AM is equal to: $y = \frac{1}{4}x - \frac{1}{4}$

Let's see if we use the Midpoint (5, 1) to set the same equation for the line segment AM.

$$y = \frac{1}{4}x + b. \quad \leftarrow \text{Substitute } M(5, 1) \text{ into equation}$$

$$1 = \frac{1}{4}(5) + b$$

$$1 = \frac{1}{4}(5) + b$$

$$1 = \frac{5}{4} + b \quad \leftarrow \text{Substitute } 1 = \frac{4}{4}$$

$$\frac{4}{4} = \frac{5}{4} + b$$

$$\frac{4}{4} = \frac{5}{4} + b \quad \leftarrow \text{Subtract } \frac{5}{4} \text{ from both sides.}$$

$$\frac{4}{4} - \frac{5}{4} = \frac{5}{4} - \frac{5}{4} + b$$

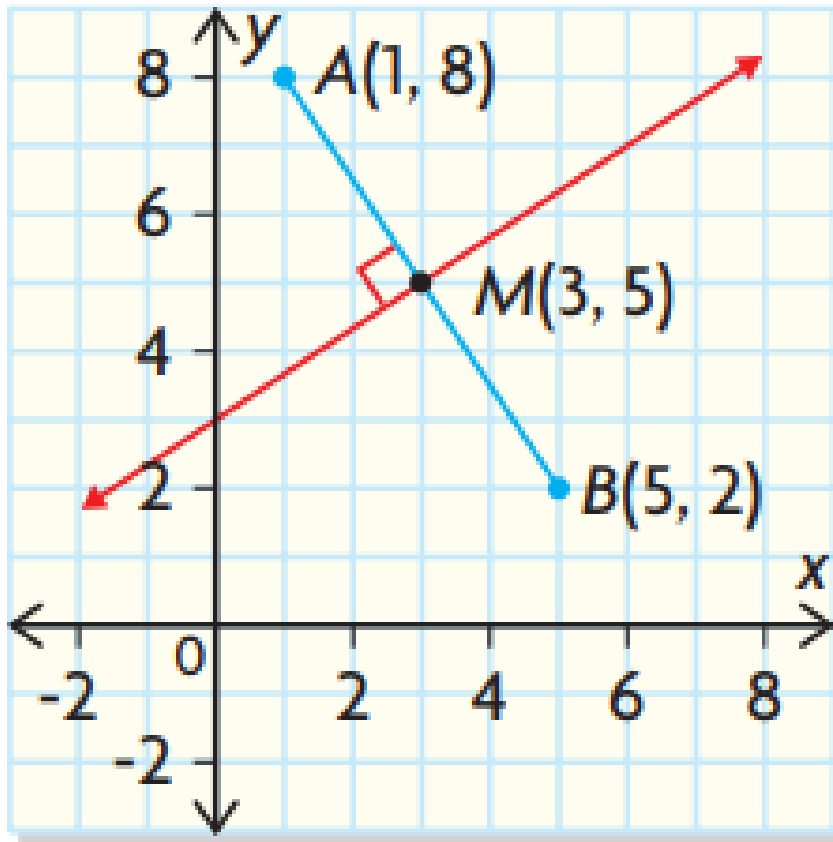
$$-\frac{1}{4} = b.$$

$$b = -\frac{1}{4}$$

Therefore, $b = -\frac{1}{4}$, so *it does not* matter if we use point A or M as the point to calculate b, y – intercept.

Example 4: See Nelson text book for details.

Objective: To find a line perpendicular to line segment AB.



Two lines that are perpendicular, that is, 90° , or right angles to each other, have slopes that are the product of:

$$M_1 \times m_2 = -1 \quad \text{or}$$

$$M_2 = -\frac{1}{M_1}$$

Based on example 4, in the Nelson text book, the slope of AB = $m_1 = -\frac{3}{2}$

$$M_2 = -\frac{1}{M_1} = -\frac{1}{-\frac{3}{2}} =$$

$$M_2 = -\frac{1}{-\frac{3}{2}} \times \frac{-\frac{2}{3}}{-\frac{2}{3}} =$$

Note: $\frac{-\frac{2}{3}}{-\frac{2}{3}} = 1$

$$M2 = -\frac{1}{-\frac{3}{2}} \times \frac{-\frac{2}{3}}{-\frac{2}{3}} = \frac{2}{\frac{3}{1}} = \frac{2}{3}$$

Find b based on text book, which is: $b = -\frac{1}{4}$

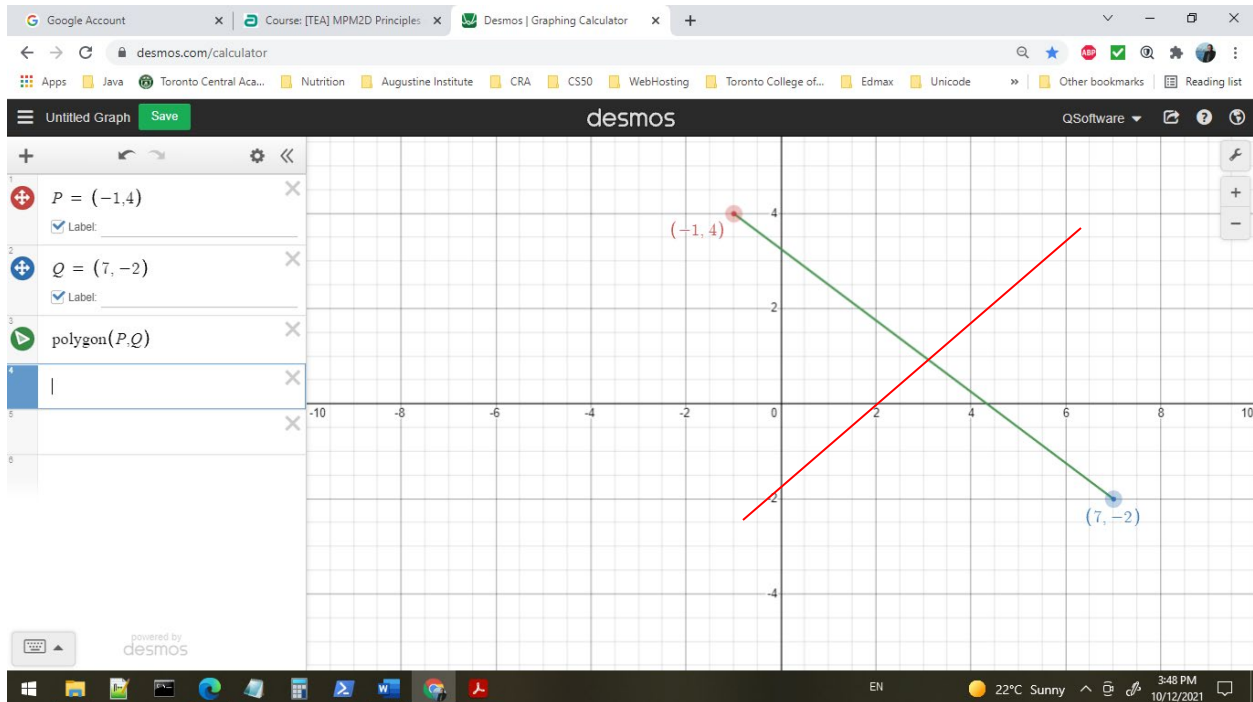
Equation of line for land fill would be any point on this line: $y = \frac{2}{3}x - \frac{1}{4}$

Assignment #5:

Text: McGraw-Hill Ryerson

Example: Equation to Right Bisector of a Triangle.

Two schools are located at the points $P(-1, 4)$ and $Q(7, -2)$ on a town map. The school board is planning a new sport complex equidistant from the two schools. Use an equation to represent the possible locations of the sports complex.



Steps to logic:

to find the equation above of the red line above, which is a line perpendicular to lines segment PQ.

1. Find midpoint to PQ.
2. Find the slope of PQ.
3. Find the slope of a line perpendicular to PQ.
4. Use the Midpoint coordinate to find the b intercept for the equation of line in red above.
5. Find equation to a line that is perpendicular to line segment PQ.

Nelson text book:

Section 2.1 - Midpoint of a line segment (page 78)

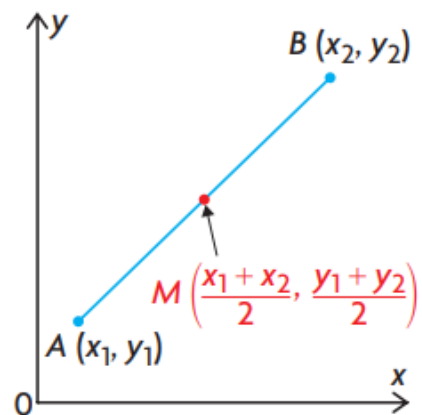
In Summary

Key Idea

- The coordinates of the midpoint of a line segment are the means of the coordinates of the endpoints.

Need to Know

- The formula $(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ can be used to calculate the coordinates of a midpoint.
- The coordinates of a midpoint can be used to determine an equation for a median in a triangle or the perpendicular bisector of a line segment.



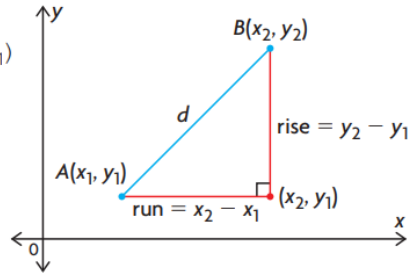
Section 2.2 – Length of a Line Segment (page 85, Nelson text book)

In Summary

Key Idea

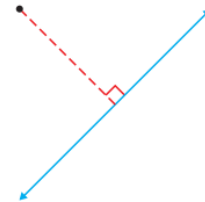
- The distance, d , between the endpoints of a line segment, $A(x_1, y_1)$ and $B(x_2, y_2)$, can be calculated using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



Need to Know

- The Pythagorean theorem is used to develop the distance formula, by calculating the straight-line distance between two points.
- The distance between a point and a line is the shortest distance between them. It is measured on a perpendicular line from the point to the line.

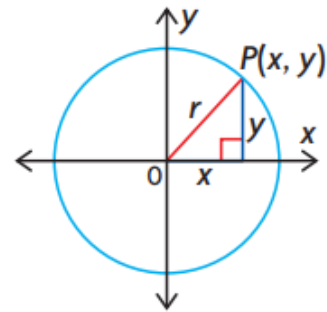


$$x^2 + y^2 = r^2$$

In Summary

Key Idea

- Using the distance formula, you can show that the equation of a circle with centre $(0, 0)$ and radius r is $x^2 + y^2 = r^2$.



Need to Know

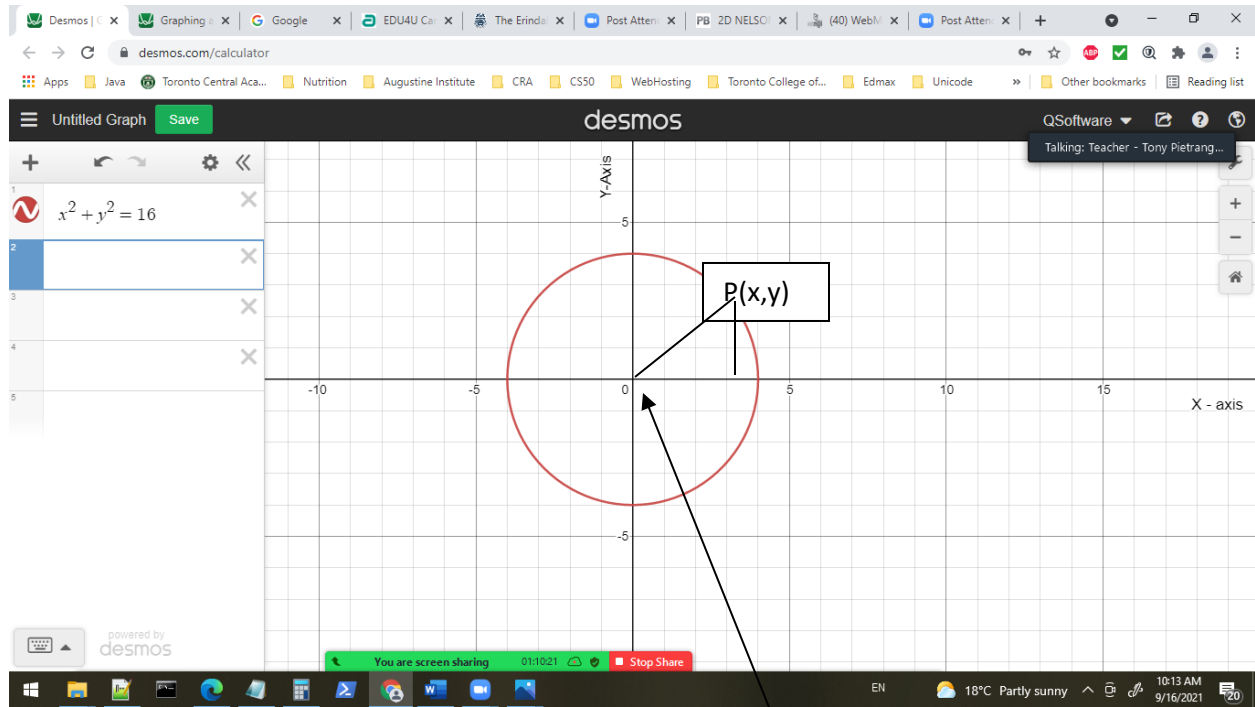
- Every point on the circumference of a circle is the same distance from the centre of the circle.
- Once you know one point on a circle with centre $(0, 0)$, you can determine other points on the circle using symmetry. If (x, y) is on a circle with centre $(0, 0)$, then so are $(-x, y)$, $(-x, -y)$, and $(x, -y)$.

Example 1: Equation of a Circle (McGraw-Hill Ryerson):

Find an equation for circle with center (0, 0) and radius 4.

$$x^2 + y^2 = 4^2$$

$$x^2 + y^2 = 16$$



P(x,y) is a point on the circle from the origin (0,0)

$$O(x,y) = O(0,0)$$

There are two point on the graph above:

$$O(x,y) = (0,0)$$

P(x,y) = (x,y) ← is a random point the circumference of the circle.

Length of a Line Segment: (Pythagorean Theorem)

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$P(x,y) = P(x_x, y_2)$$

$$O(x,y) = P(x_1, y_1) = (0, 0)$$

$$OP = r = 4$$

Using the distance formula:

$$OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$O(x,y) = P(x_1, y_1) = (0, 0)$ ← based on the diagram above substitute into distance formula.

$$OP = \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$$

$$OP = \sqrt{(x_2)^2 + (y_2)^2}$$

Therefore,

$$r = \sqrt{x^2 + y^2}$$

C

Square both sides.

$$r^2 = x^2 + y^2$$

Logic used behind the problem above, and connecting the use of:

1. The distance formula of two points.
2. Radius of a circle, plus using the point of origin $(x, y) = (0,0)$

Whiteboard - Zoom

Any two points, Length of Line segment between any two points is: $\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$P(x_1, y_1) = O(0,0)$

$A(2, 4) = (x_1, y_1)$
 $B(-3, 5) = (x_2, y_2)$

$B(x_2, y_2) = B$

$y = -0.8x + 1.5$

$\Delta y = \text{rise} = y_2 - y_1$

$\Delta x = x_2 - x_1 = \text{run}$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = 10/3$

$(0,0) = (x_1, y_1)$

$r^2 = b^2 + c^2$
 $r^2 = x^2 + y^2$
 $r = \sqrt{x^2 + y^2} = r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(-3-0)^2 + (5-0)^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$

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Goal or outcome

Whiteboard - Zoom

$r = \sqrt{x^2 + y^2} \Rightarrow r^2 = x^2 + y^2 \Rightarrow (4)^2 = 16 = x^2 + y^2 \Rightarrow x^2 + y^2 = 16$ = Desmos graphing calc

$r = 4$ (Square both sides)
 $r^2 = x^2 + y^2 = 4^2 = 16$

$A(x_1, y_1) = O(x_1, y_1) = (0,0)$

$\vec{AB} = \vec{OP}$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 $= \sqrt{(x_2 - 0)^2 + (y_2 - 0)^2}$
 $= \sqrt{x_2^2 + y_2^2}$

$B(x_2, y_2) = B$

$y = -0.8x + 1.5$

$\Delta y = \text{rise} = y_2 - y_1$

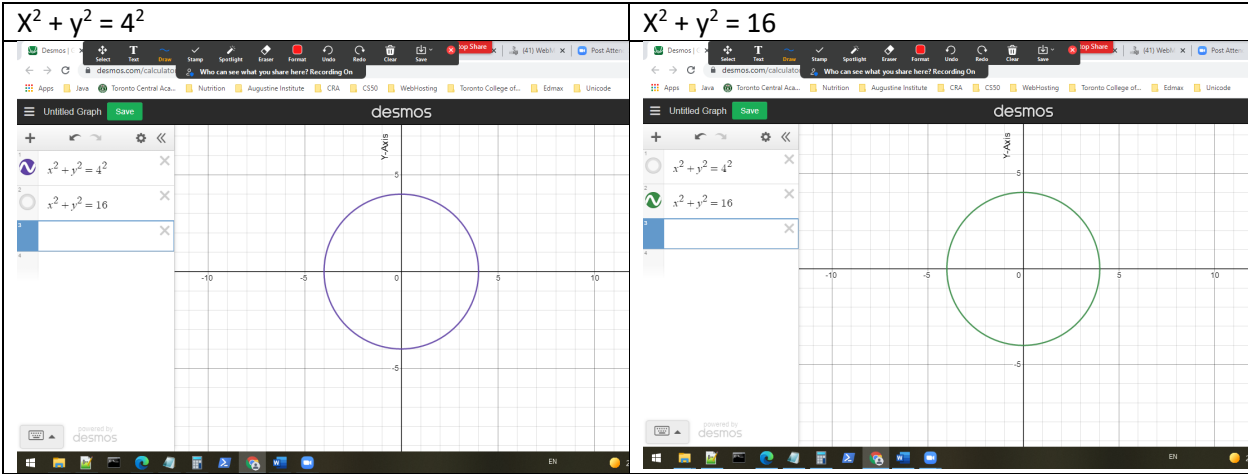
$\Delta x = x_2 - x_1 = \text{run}$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = 10/3$

$(0,0) = (x_1, y_1)$

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Inside our desmos graphing Calculator:



HomeWork (See Moodle)

page 91, Nelson Text book, Q#1 a, d, Q#2

For question #2, use Desmos graphing calculator to draw the circles in Q#2.

Date Created: Friday, September 17th, 2021

Topic: Geometric Shapes:

The screenshot shows a whiteboard interface with the following content:

- Handwritten word: *hypotenuse*
- Text: *Date: Friday, September 17th, 2021*
Course: MPM2D - Principles of Mathematics.
- Text: *Topic: 1. Studying Geometric Shapes in a Grid System.*
- Text: *Previous Topics:*
 - 1. Midpoint of a line segment.*
 $M(x,y) = ((x_1 + x_2) / 2, (y_1 + y_2) / 2)$
 - 2. Length of a Line Segment*
see arrow
 - 3. Formula for a Circle:*
 - 4. Proved that formula using P1(0,0), P2(x, y), with radius of 4.*
- Handwritten formulas:
 - $v = L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 - $x^2 + y^2 = r^2$
 - $x^2 + y^2 = 4^2$
 - $x^2 + y^2 = 16$
- Handwritten equations:
 - $v = y - y_1$
 - $u = x_2 - x_1$

Home Work:

Take up homework.

The screenshot shows a textbook page with the following content:

2D NELSON TEXTBOOK.pdf 95 / 698 150% +

a circle with centre (0, 0), then so are $(-x, y)$, $(-x, -y)$, and $(x, -y)$.

CHECK Your Understanding

- The graph at the right shows a circle with its centre at (0, 0).
 - State the x -intercepts of the circle.
 - State the y -intercepts.
 - State the radius.
 - Write the equation of the circle.
- Write the equation of a circle with centre (0, 0) and radius r .
 - $r = 3$
 - $r = 50$
 - $r = 2\frac{1}{3}$
 - $r = 400$
 - $r = 0.25$

The graph shows a coordinate plane with a circle centered at the origin (0,0). The circle passes through the points (4,0), (-4,0), (0,4), and (0,-4). The x and y axes are labeled from -8 to 8.

NEL Chapter 2 91

1a) What are the x-intercepts of the circle above?

Note:

x-intercepts are when $y = 0$:

Answer:

Circle intercepts at the following points: x-intercepts $(-7, 0)$, $(7, 0)$

1b) What are the y-intercepts of the circle above?

Note:

y-intercepts are when $x = 0$;

Circle intercepts at the following points: y-intercepts $(0, 7)$, $(0, -7)$

1c) State the radius of the circle:

Radius of the circle is 7 units.

1d) The general equation of a circle is: $x^2 + y^2 = r^2$

$r = 7$.

$$x^2 + y^2 = 7^2 = 49$$

$$x^2 + y^2 = 7^2 = 49$$

Question #2:

Note: General Equation of a circle is below:

$$x^2 + y^2 = r^2$$

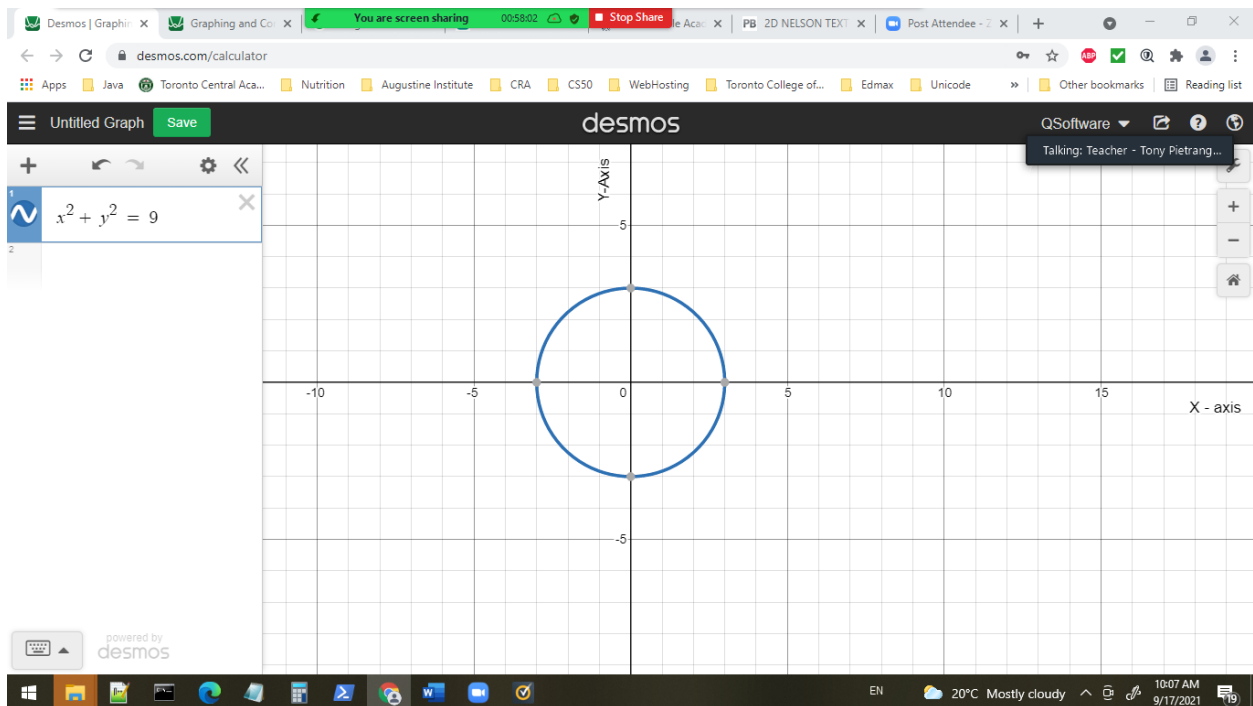
Write the equation of the circles for the following:

radius:

a) $r = 3$

$$x^2 + y^2 = 3^2 = 9$$

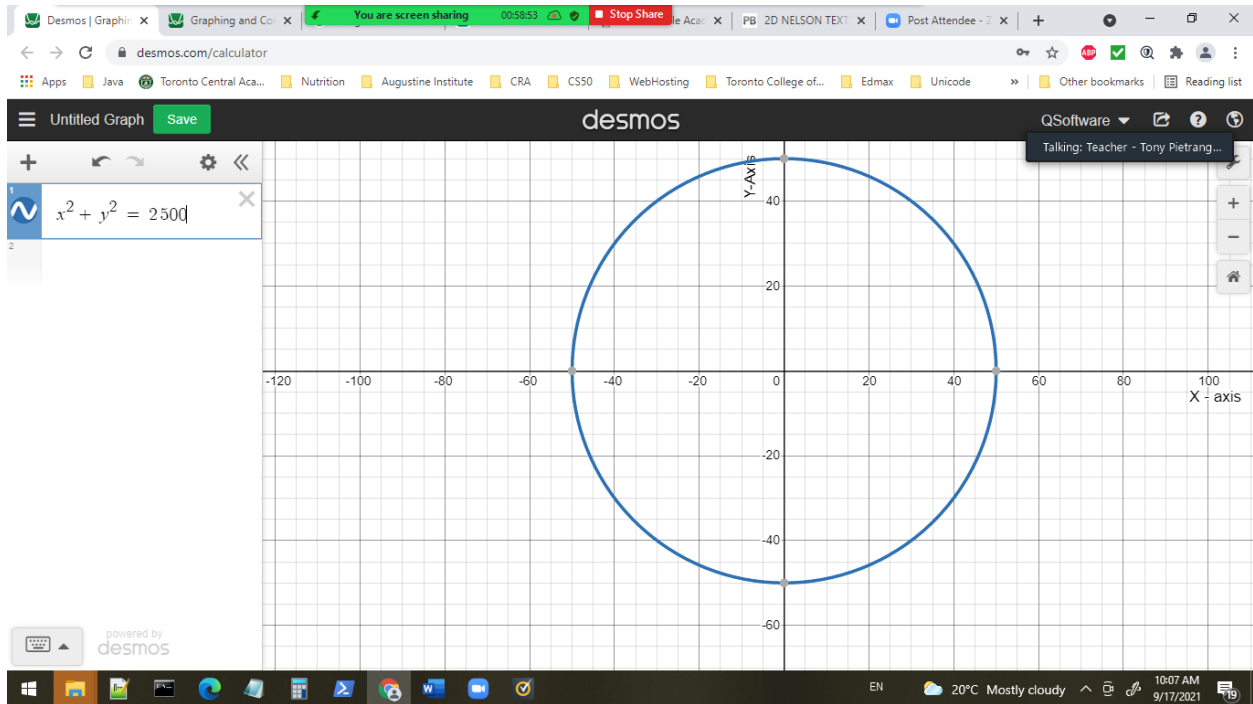
$$x^2 + y^2 = 9$$



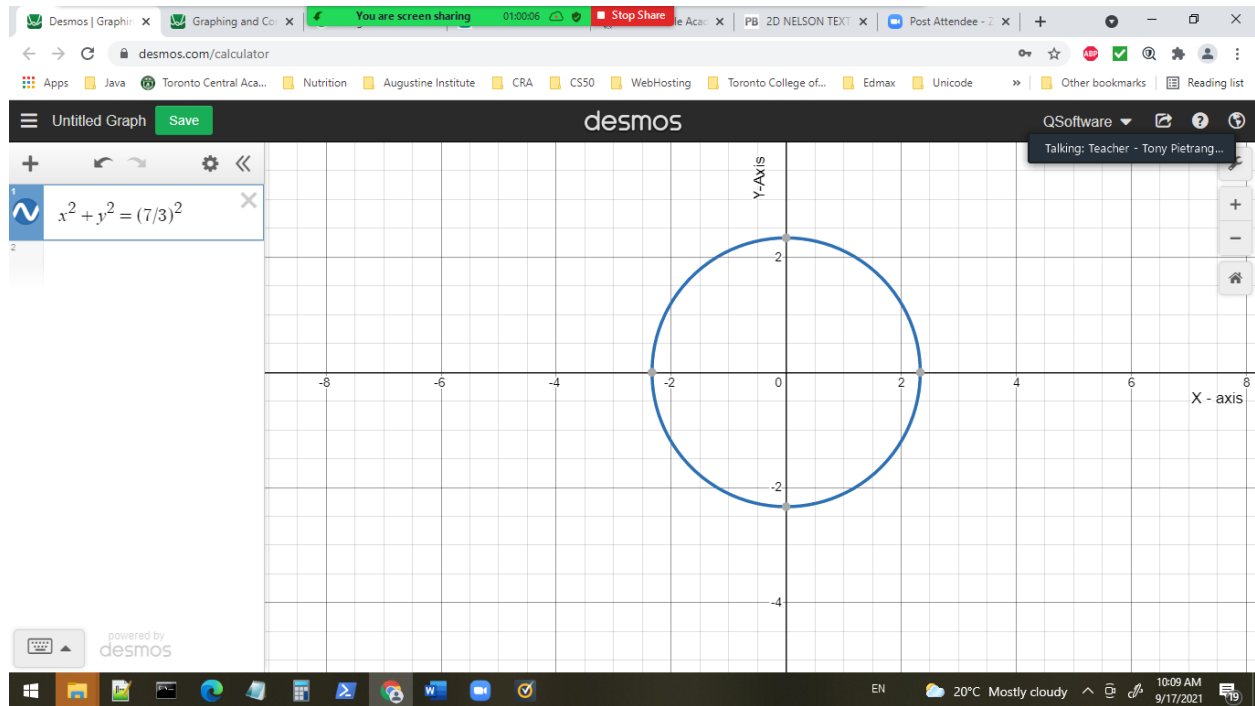
b) $r = 50$

$$x^2 + y^2 = 50^2 = 2500$$

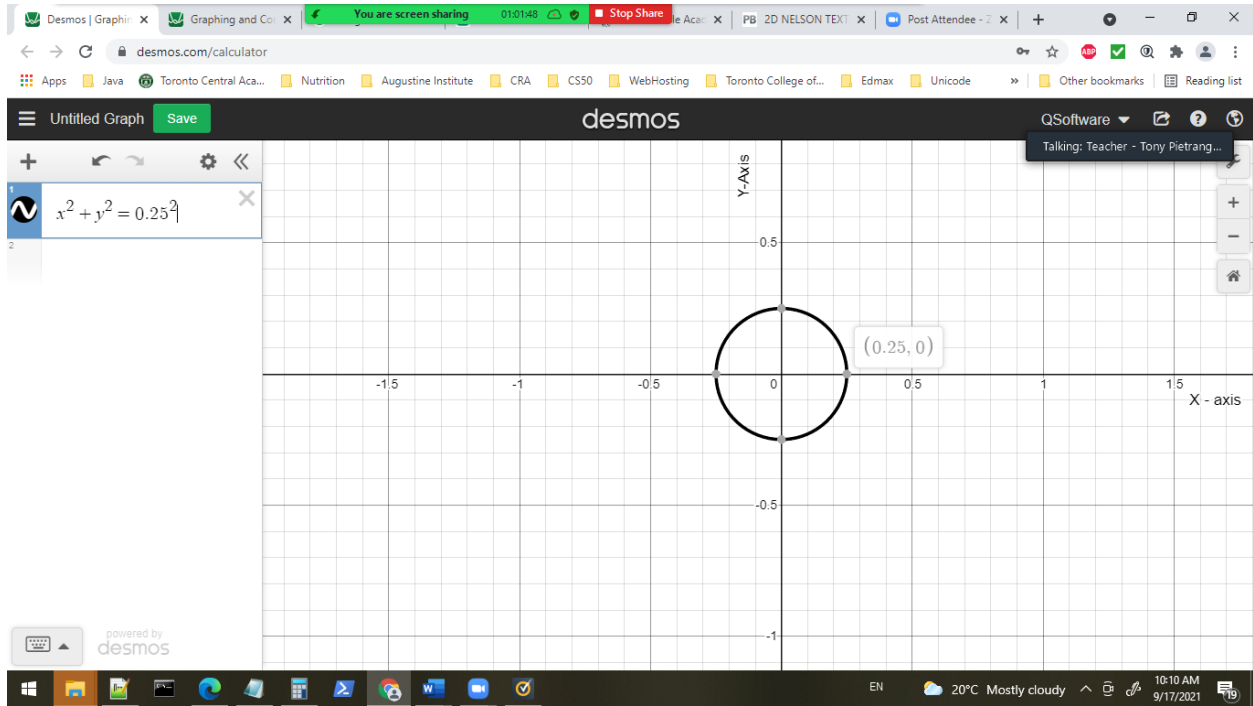
$$x^2 + y^2 = 2500$$



$$c) r = 2\frac{1}{3} = \frac{7}{3}$$
$$x^2 + y^2 = \left(\frac{7}{3}\right)^2$$

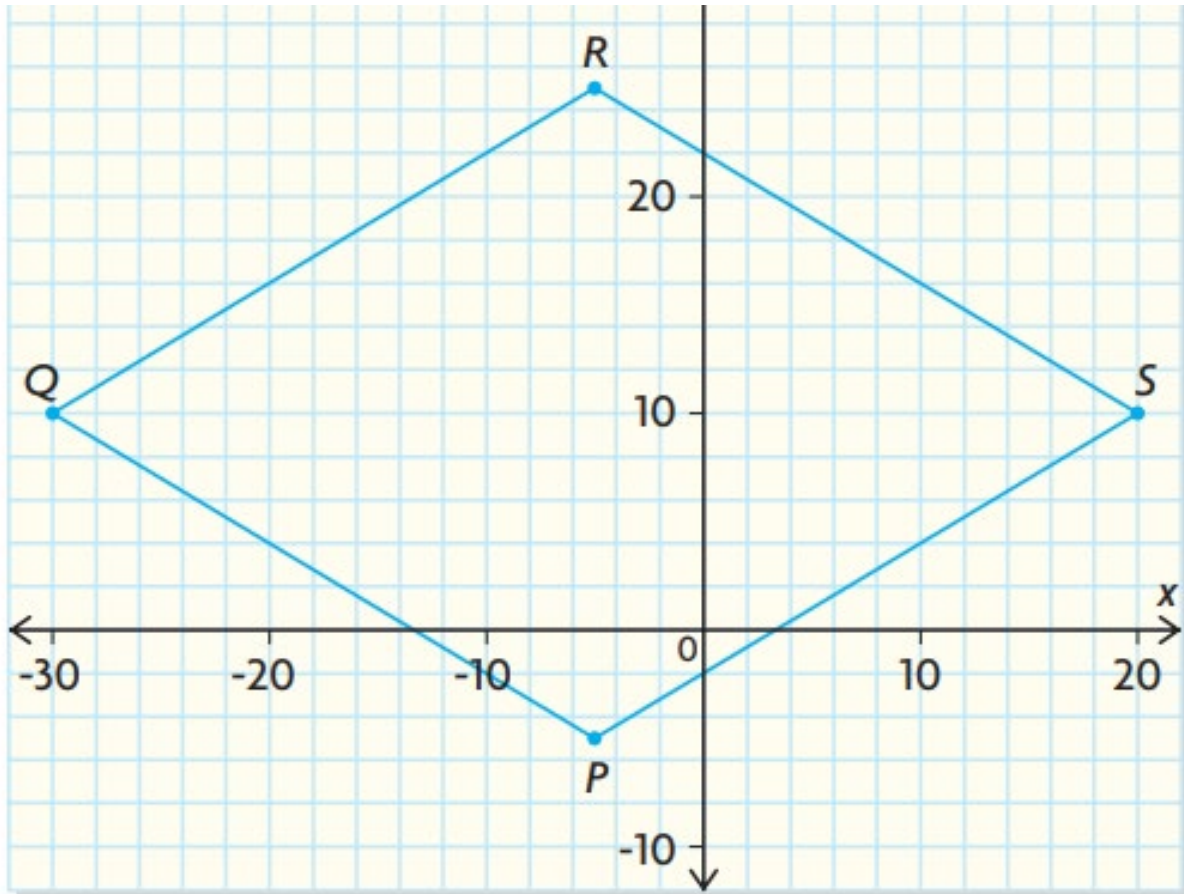


c) $r=0.25$
 $x^2 + y^2 = 0.0625$



Section 2.4 - Classifying Figures on a Coordinate Grid

A surveyor has marked the corners of a lot where a building is going to be constructed. The corners have coordinates $P(-5, -5)$, $Q(-30, 10)$, $R(-5, 25)$ and $S(20, 10)$. Each unit represents 1 m. The builder wants to know the perimeter and shape of this building lot.



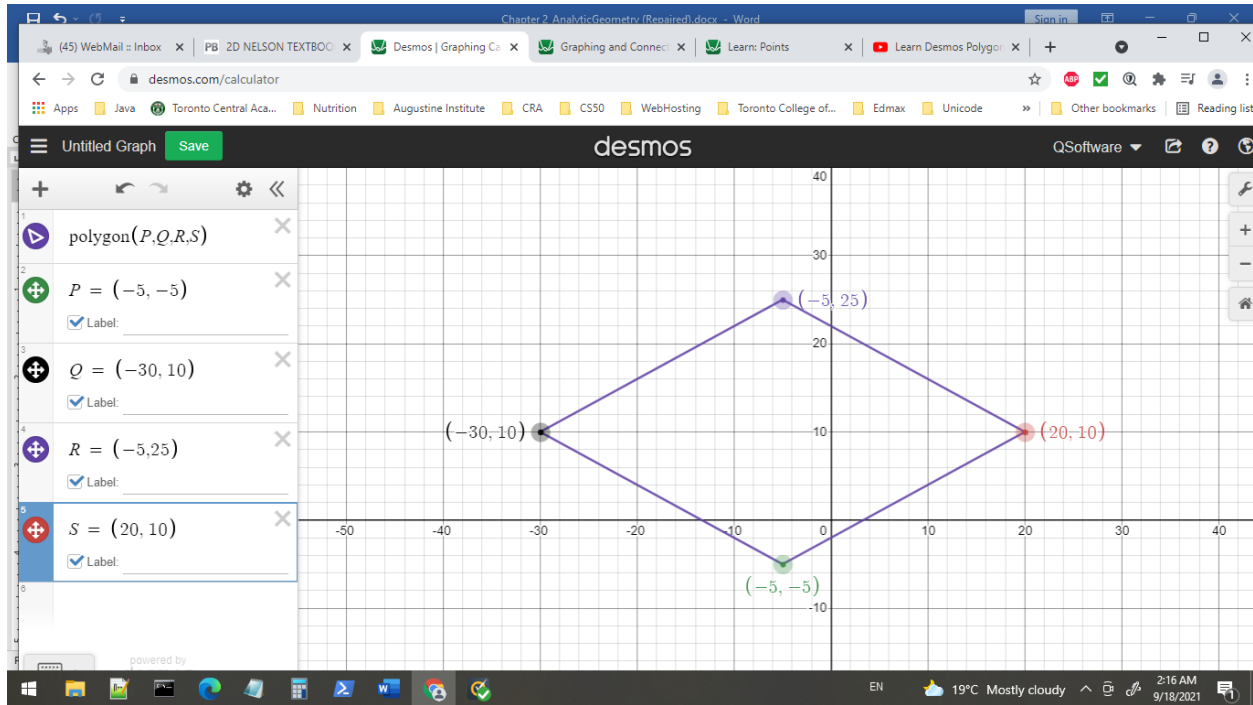
Analyze all the points of the shape above.

P(-5,-5)

Q(-30, 10),

R(-5,25),

S(20,10),



Objective:

1. Find the perimeter of the shape above. We need to have the lengths of all the line segments.
2. Find what type of shape is above. Questions we ask ourselves? Is it a square? Is it a parallelogram? Etc.

1. Find the perimeter of the shape above, we would need to find the lengths of each line segment and add them together.

Formulas:

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

$$L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Points on the shape (Quadrilateral)	Line Segment Name of Line segment.	Length of Line Segment $L = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Slope of Line segment $M = \frac{y_2 - y_1}{x_2 - x_1}$
<u>P(-5,-5), Q(-30, 10),</u>	PQ	$L = \sqrt{(-30 - (-5))^2 + (10 - (-5))^2}$ $L = \sqrt{(-25)^2 + (15)^2}$ $L = \sqrt{625 + 225} = \sqrt{850} \cong 29.15$	$M_{pq} = \frac{10 - (-5)}{-30 - (-5)} = = \frac{15}{-25}$ $m_{pq} = -\frac{3}{5}$
<u>Q(-30, 10), R(-5,25),</u>	QR	$L = \sqrt{(-5 - (-30))^2 + (25 - (10))^2}$ $L = \sqrt{(25)^2 + (15)^2}$ $L = \sqrt{625 + 225} = \sqrt{850} \cong 29.15$	$M_{qr} = \frac{25 - 10}{-5 - (-30)} = = \frac{15}{25}$ $M_{qr} = \frac{3}{5}$
<u>R(-5,25), S(20,10),</u>	RS	$L = \sqrt{(20 - (-5))^2 + (10 - 25)^2}$ $L = \sqrt{(25)^2 + (-15)^2}$ $L = \sqrt{625 + 225} = \sqrt{850} \cong 29.15$	$M_{rs} = \frac{10 - 25}{20 - (-5)} = = \frac{-15}{25}$ $M_{rs} = -\frac{3}{5}$
<u>S(20,10), P(-5,-5),</u>	SP	$L = \sqrt{(20 - (-5))^2 + (10 - (-5))^2}$ $L = \sqrt{(25)^2 + (15)^2}$ $L = \sqrt{625 + 225} = \sqrt{850} \cong 29.15$	$M_{sp} = \frac{-5 - 10}{-5 - (-20)} = = \frac{-15}{-25}$ $M_{sp} = \frac{3}{5}$

We have determined the lengths of all line segments are the same, which is about 29.15 units.
 $PQ = QR = RS = SP = 29.15$ Units.

Therefore, the perimeter of the 4-sided polygon is $4 \times (29.15) = 116.6$ units or 116.6 meters.

To find the type of 4-sided polygon, we need to analyze the slopes of the line segments as well.
 The following lines segments are parallel: $PQ \parallel RS$, and $QR \parallel SP$.

The slopes of the line segments are as follows:

$$M_{pq} = M_{rs} = -\frac{3}{5}$$

$$M_{sp} = M_{qr} = \frac{3}{5}$$

The slopes of the lines segments are not the negative reciprocals of each other. Therefore, the line segments (PQ, QR) and (RS, SP) are not perpendicular to each other. Thus, this parallelogram is not a square, but rather a rhombus.

Rhombus – is a parallelogram, that has 4 equals sides in length, but the angles are not 90° to each other.

In Summary

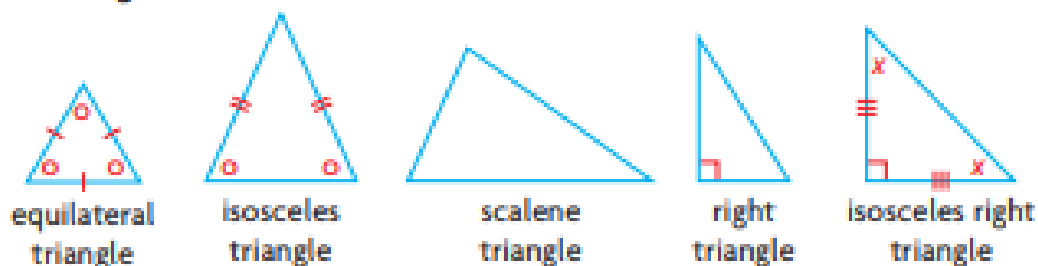
Key Idea

- When a geometric figure is drawn on a coordinate grid, the coordinates of its vertices can be used to calculate the slopes and lengths of the line segments, as well as the coordinates of the midpoints.

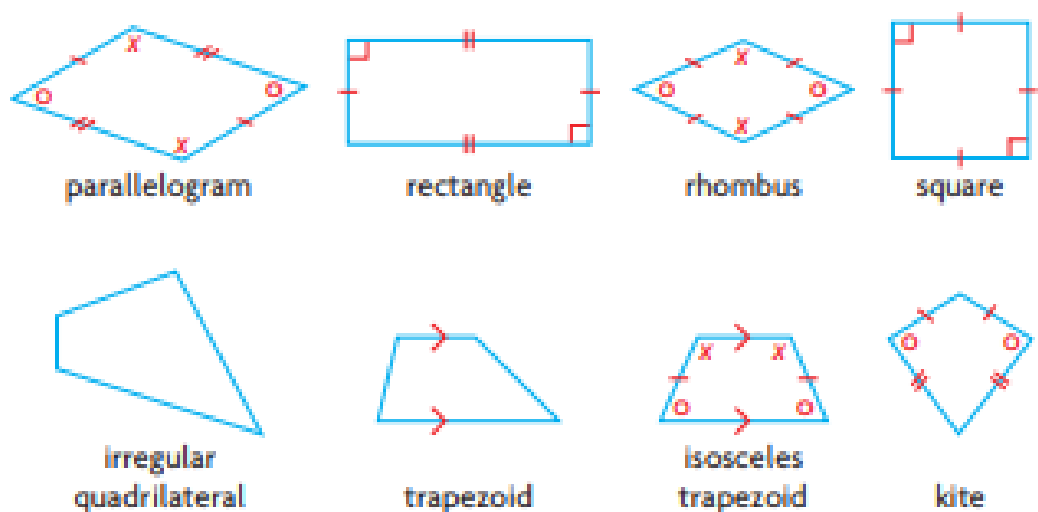
Need to Know

- Triangles and quadrilaterals can be classified by the relationships between their sides and their interior angles.

Triangles



Quadrilaterals



- To solve a problem that involves a geometric figure, it is a good idea to start by drawing a diagram of the situation on a coordinate grid.
- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals.

Homework:

Nelson text book, pages 101, Q1 and Q2.

We will do more on geometric shapes in the next class as well.

Date: Monday, September 20th, 2021

The image shows a Zoom whiteboard interface. At the top, there is a toolbar with various drawing and editing tools. A notification bar at the top center reads "Who can see what you share here? Recording" and "You are screen sharing 02:36:39". A "Saved as PNG" notification is also visible. The whiteboard content includes:

- Date:** Monday, September 20th, 2021
- Course:** MPM2D - Principles of Mathematics
- Previously covered in last lesson:**
 - 1. A rhombus, a parallelogram with all sides equal.
 - slope of all line segments to be parallel, opposite sides.
 - lengths of all sides to be the same (length)
- Topic/goal:**
 - 1. Properties of midpoints of a 4-sided polygon (Quadrilateral) forms a parallelogram (Varignon Parallelogram).
 - 2. Reviewed some properties of
 - a. Triangles.
 - b. Quadrilaterals.

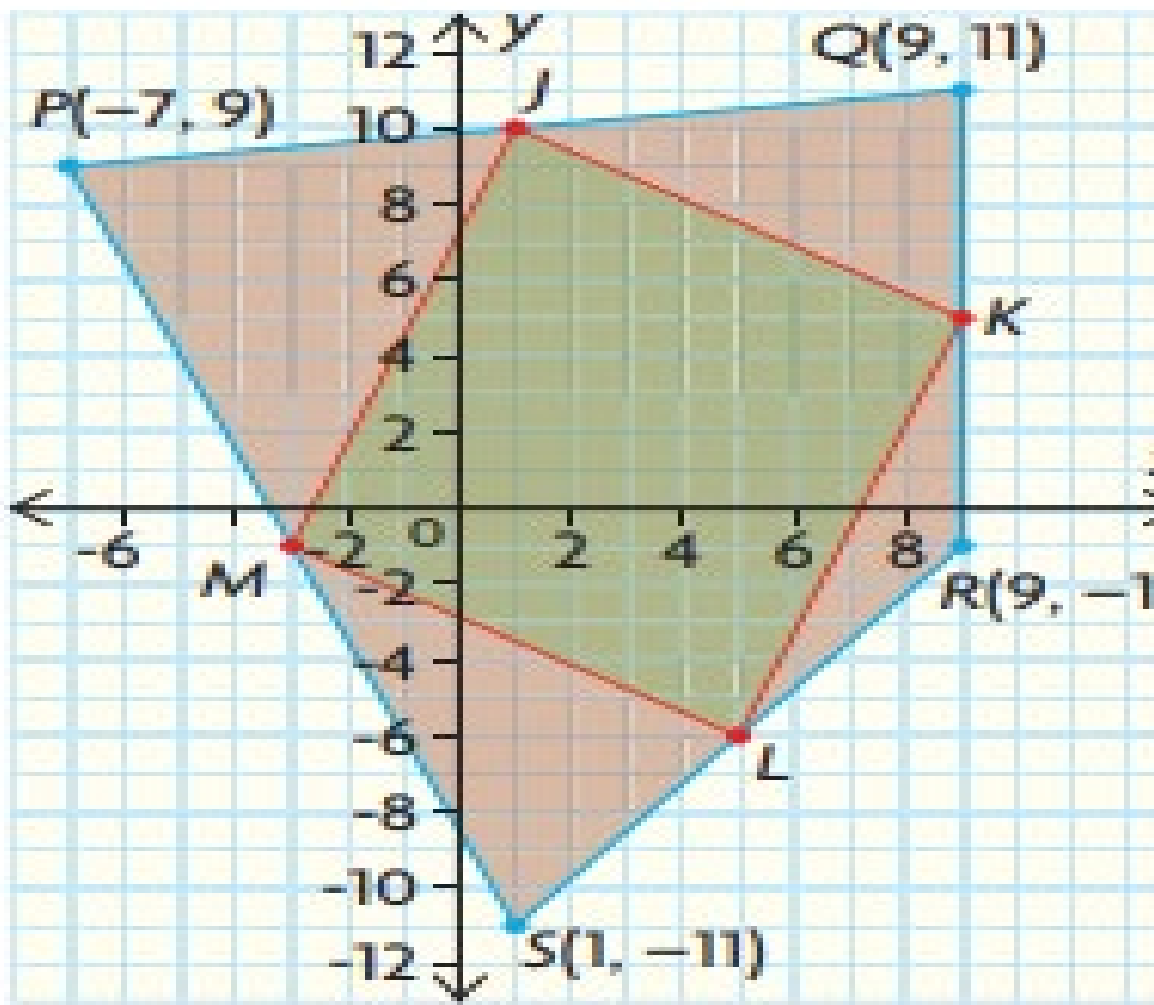
Handwritten notes in red ink include:

- A circled "1" followed by "mid point = $m(x,y) = ((x_1+x_2)/2, (y_1+y_2)/2)$ ".
- A circled "2" followed by "Length = $\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ ".
- A circled "Midpoint" label with a small diagram showing a line segment with endpoints x_1, y_1 and x_2, y_2 , and a midpoint.

At the bottom of the whiteboard, there is a "Teacher - Ton..." button. The bottom of the screen shows the Windows taskbar with the system tray displaying "EN", "21°C Sunny", and the time "11:46 AM 9/20/2021".

YOU WILL NEED

- grid paper and ruler, or dynamic geometry software



EXAMPLE 1

Proving a

Note: The same above with the 4-points is a 4-sided polygon.

P(-7, 9),
Q(9, 11),
R(9,-1),
S(1, -11)

You Can randomly take any 4 points on a grid, and connect the 4 points. If you find the midpoints of the 4-line segments, what happens you create a parallelogram.

Reference: McGraw-Hill Ryserson, ISBN-13: 978-0-07-097332-9, page 132.

This is called a Varignon Parallelogram, which is proven by a French mathematician Pierre Varignon (1654-1722).

Points on the shape (Quadrilateral), MidPoint	Line Segment Name of Line segment / Midpoint	Midpoints (J, K, L, M) $M(x,y) = (x_1 + x_2) / 2, (y_1 + y_2) / 2)$	Slope of Line Midpoints Line segment $M = \frac{y_2 - y_1}{x_2 - x_1}$
<u>P(-7,9), Q(9, 11),</u> <u>J_{pq}</u>	PQ, J(x,y)	$J_{pq} = ((-7 + 9) / 2), (9 + 11) / 2)$ $= (2/2, 20/2) = (1, 10)$	$M_{jk} = \frac{5 - 10}{9 - 1} = \frac{-5}{8}$
<u>Q(9, 11), R(9,-1),</u> <u>K_{QR}</u>	QR, K(x,y)	$K_{QR} = ((9 + 9) / 2), (11 + (-1)) / 2)$ $= (18/2, 10/2) = (9, 5)$	$M_{kl} = \frac{-6 - 5}{5 - 9} = \frac{-11}{4}$ $M_{kl} = \frac{-11}{4} = 2.75$
<u>R(9,-1), S(1,-11),</u> <u>L_{RS}</u>	RS, L(x,y)	$L_{RS} = ((9 + 1) / 2), (-1 + (-11)) / 2)$ $= (10/2, -12/2) = (5, -6)$	$M_{lm} = \frac{-1 - (-6)}{-3 - 5} = \frac{5}{-8}$ $M_{rs} = -\frac{5}{8}$
<u>S(1,-11), P(-7,9),</u> <u>M_{sp}</u>	SP, M(x,y)	$M_{sp} = ((1 + (-7)) / 2), (-11 + (9)) / 2)$ $= (-6/2, -2/2) = (-3, -1)$	$M_{sp} = \frac{-5 - 10}{-5 - (-20)} = \frac{-15}{-25}$ $M_{sp} = \frac{3}{5}$

Midpoint Line of Segments	Midpoint Line Segments	Slopes of MidPoints
$P(-7,9), Q(9, 11): \underline{J(1,10)},$ $Q(9, 11), R(9,-1): \underline{K(9,5)}$	$\underline{J_{PQ}}(1,10), \underline{K_{QR}}(9,5)$	$M_{jk} = \frac{5-10}{9-1} = \frac{-5}{8}$
$Q(9, 11), R(9,-1): \underline{K(9,5)},$ $R(9,-1), S(1,-11): \underline{L(5,-6)}$	$\underline{K_{QR}}(9,5), \underline{L_{RS}}(5,-6)$	$M_{kl} = \frac{-6-5}{5-9} = \frac{-11}{-4}$ $M_{kl} = \frac{11}{4} = 2.75$
$R(9,-1), S(1,-11): \underline{L(5,-6)}$ $S(1,-11), P(-7,9): \underline{M(-3,-1)}$	$\underline{L_{RS}}(5,-6), \underline{M_{SP}}(-3,-1)$	$M_{lm} = \frac{-1-(-6)}{-3-5} = -\frac{5}{8}$
$S(1,-11), P(-7,9): \underline{M(-3,-1)}$ $P(-7,9), Q(9, 11): \underline{J(1,10)},$	$\underline{M_{SP}}(-3,-1), \underline{J_{PQ}}(1,10)$	$M_{SP} = \frac{10-(-1)}{1-(-3)} = \frac{11}{4}$ $M_{SP} = 2.75$

Method 1: Proof that midpoints line segments of a quadrilateral form a parallelogram.

Conclusion:

$M_{jk} = M_{lm} = \frac{5}{8}$; Therefore, these two-line segments for midpoints are parallel.

$M_{kl} = M_{js} = 2.75$; Therefore, the other two-line segments for the midpoints are parallel as well.

Therefore, the midpoints of any quadrilateral generate a parallelogram, which is called: Varignon Parallelogram.

Method 2: Diagonals of a parallelogram having same midpoints.

The diagonals of line segments JL(3, 2), and KM (3,2) proves that JKLM is a parallelogram. Properties a parallelogram. (Student reviews themselves).

Assignment #6:

We will create an assignment for Varignon Parallelogram, which will be posted on Moodle.

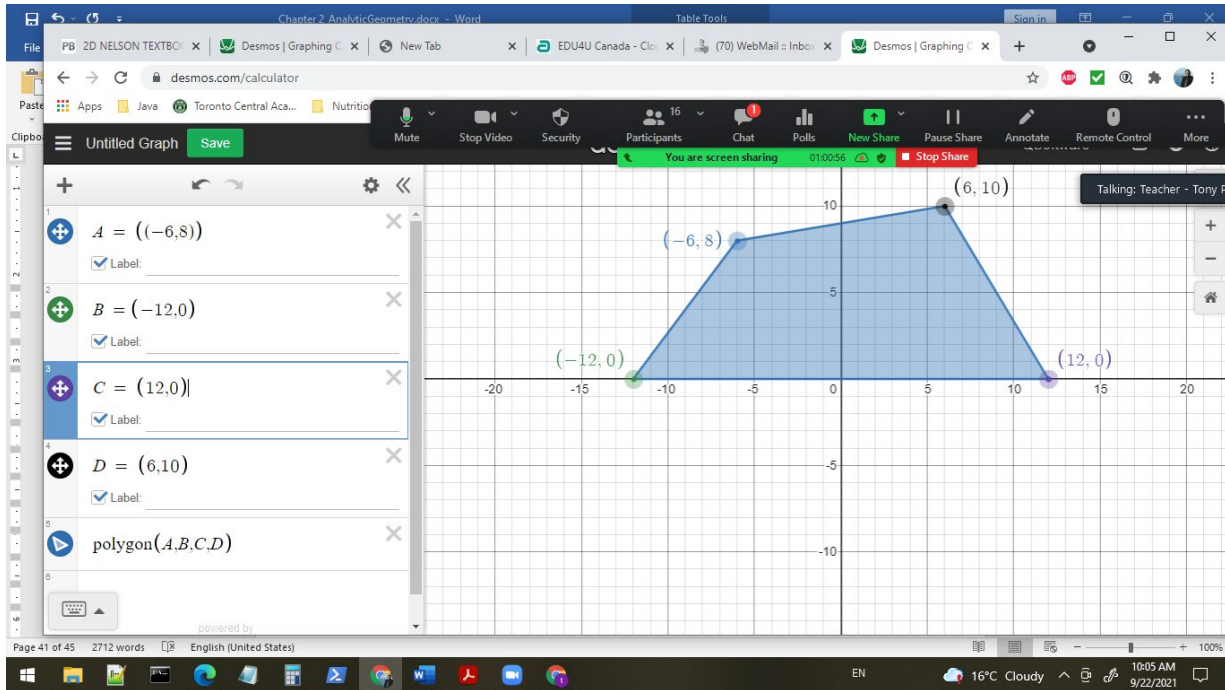
Proof of a parallelogram, Varignon parallelogram, inside any random 4-sided polygon, i.e. a quadrilateral, where their midpoints of the 4-side polygon, when connected produce a parallelogram.

Step 1: Table of Points:

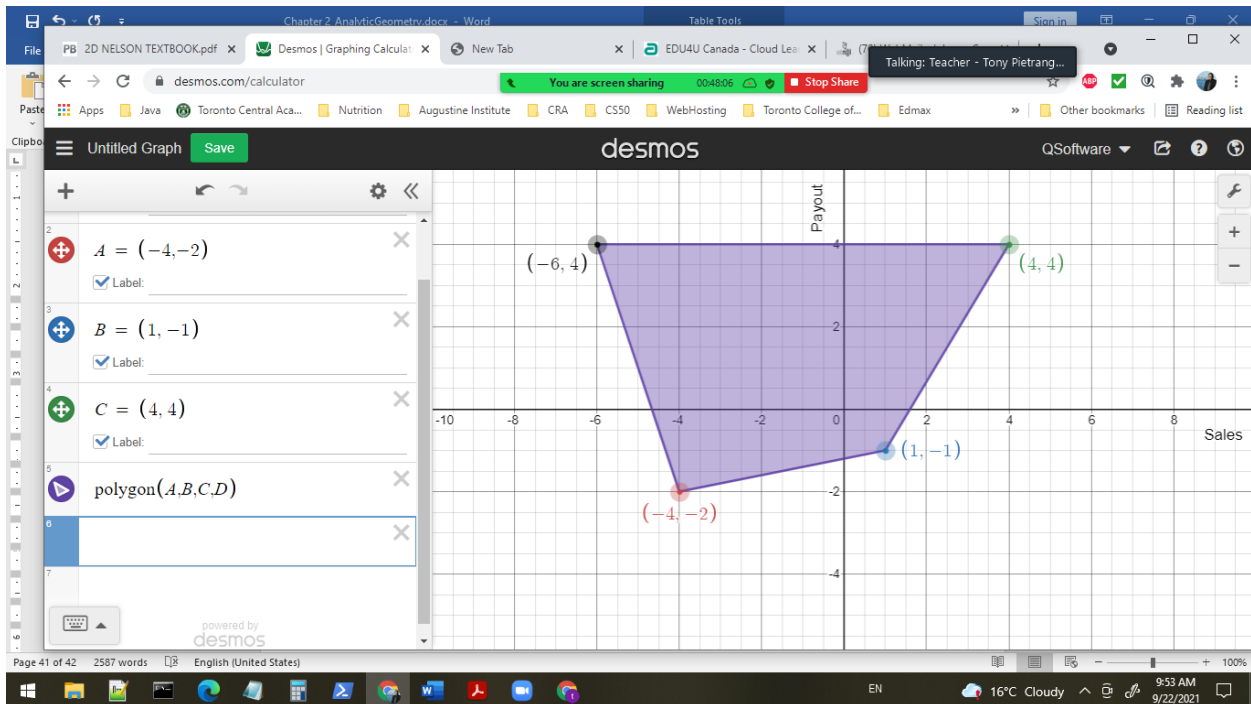
Student Name	Point A	Point B	Point C	Point D
Joanna	(-6, 8)	(-12,0)	(12, 0)	(6, 10)
Hayden	(-9, 13)	(-11, -7)	(9, -6)	(12,10)
Cody	(-6, 5)	(-14, -11)	(8, -9)	(5, 10)
Sherry	(-11, -14)	(4, -13)	(12, 6)	(-11, 12)
Wenkang Li	(-3, 1)	(-4, 1)	(5, -1)	(1, 4)
Tee	(-10, 4)	(-10, -12)	(10, -9)	(0, 14)
Brian N.	(-9, 6)	(-9, -5)	(12, -11)	(7, 10)
Henry T.	(-10, 3)	(-10, -6)	(10, -7)	(1, 13)
Roy	(-9,9)	(-9, -5)	(9, -10)	(6, 14)
Kyle	(-7,12)	(-10, -11)	(9, -11)	(9, 12)
Henry N.	(-4, -2)	(1,-1)	(4,4)	(-6,4)
Lavinia	(-5,-6)	(10,-4)	(5,8)	(-7,4)
Jason	(-6,-8)	(5,-10)	(5, 5)	(-1,5)
Alex	(-7, 9)	(-11,1)	(12, 1)	(7, 9)

Example: Quadrilaterals

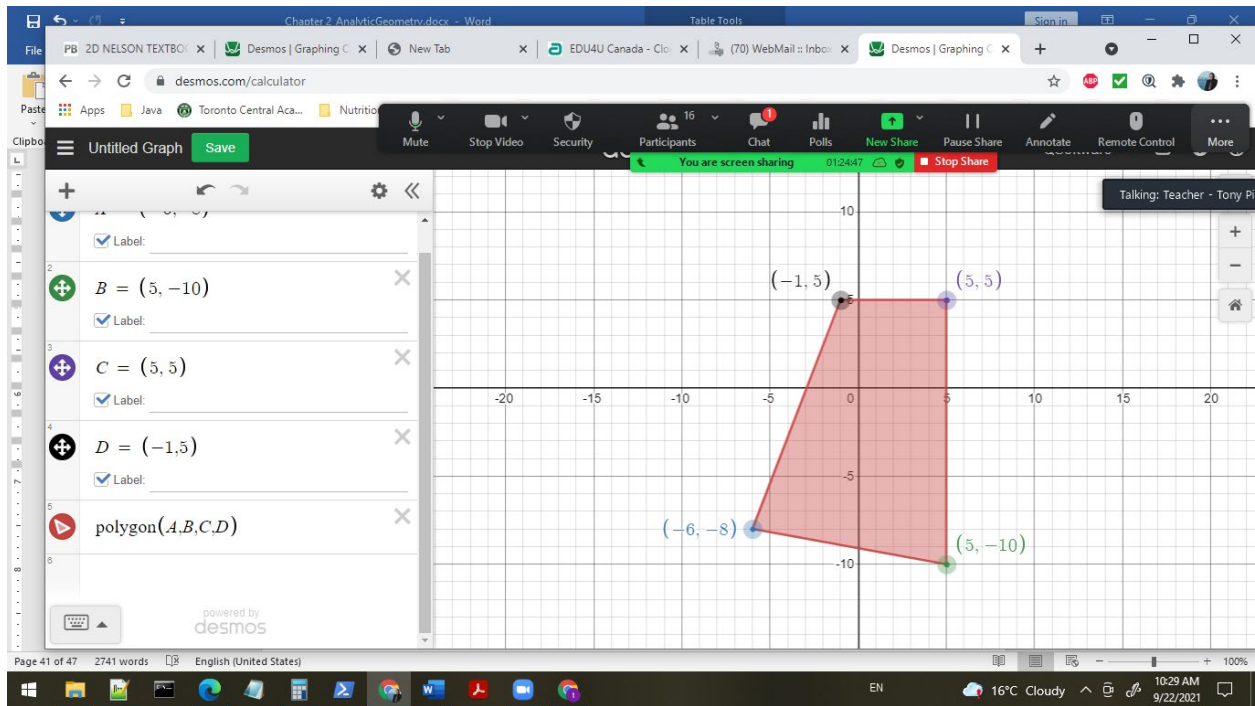
Joanna's 4-sided polygon:



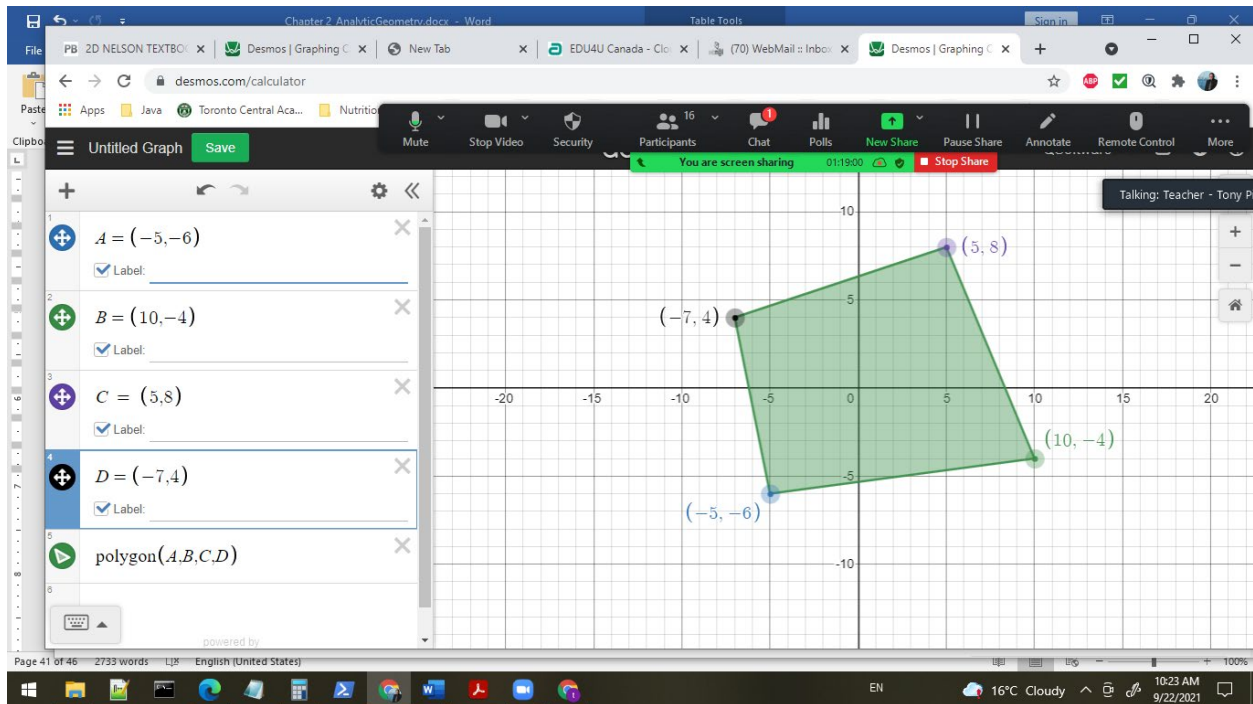
Henry N's 4-sided polygon:



Jason's quadrilateral:



Lavinia – 4-sided Polygon, quadrilateral



Step 2: Table of MidPoints:

Student Name	MidPoint E(X,Y) is for AB	MidPoint F(X,Y) is for BC	MidPoint G(X,Y) is for CD	MidPoint H(X,Y) is for DA	Verified by
Joanna					Hayden
Hayden					Cody
Cody					Sherry
Sherry					Leon
Wenkang Li					Tee
Tee					Brian N.
Brian N.					Henry T.
Henry T.					Roy
Roy					Kyle
Kyle					Henry N.
Henry N.					Lavinia
Lavinia					Jason
Jason					Alex
Alex					Joanna

Step 3: Table of Lengths:

Student Name	Length Of (AE, BE)	Length Of (BF, CF)	Length Of (CG, DG)	Length Of (AH, DH)	Verified by (V1)
Joanna					Hayden
Hayden					Cody
Cody					Sherry
Sherry					Leon
Wenkang Li					Tee
Tee					Brian N.
Brian N.					Henry T.
Henry T.					Roy
Roy					Kyle
Kyle					Henry N.
Henry N.					Lavinia
Lavinia					Jason
Jason					Alex
Alex					Joanna

Step 4 & 5: Table of Slopes:

Student Name	Slopes (M) EH, FG	Slopes(M) EF, HG	— Verified by
Joanna			Hayden
Hayden			Cody
Cody			Sherry
Sherry			Leon
Wenkang Li			Tee
Tee			Brian N.
Brian N.			Henry T.
Henry T.			Roy
Roy			Kyle
Kyle			Henry N.
Henry N.			Lavinia
Lavinia			Jason
Jason			Alex
Alex			Joanna

You need a conclusion:

The line segments have the same slope; therefore, the shape is a parallelogram.

Grid of the Internet or a better one for those that can not access Desmos Graphing software

