Chapter 2: Analytic Geometry – Part B:

Date: Thursday, September 23rd, 2021
Course: MPM2D - Princples of Mathematics

Goal:
- Centriod of a triangle
- Find centriod and prove ratio of 2:1 for all medians intersection midpoint

Review:
1. Assignment #6, Varignon Parallelogram (Project)

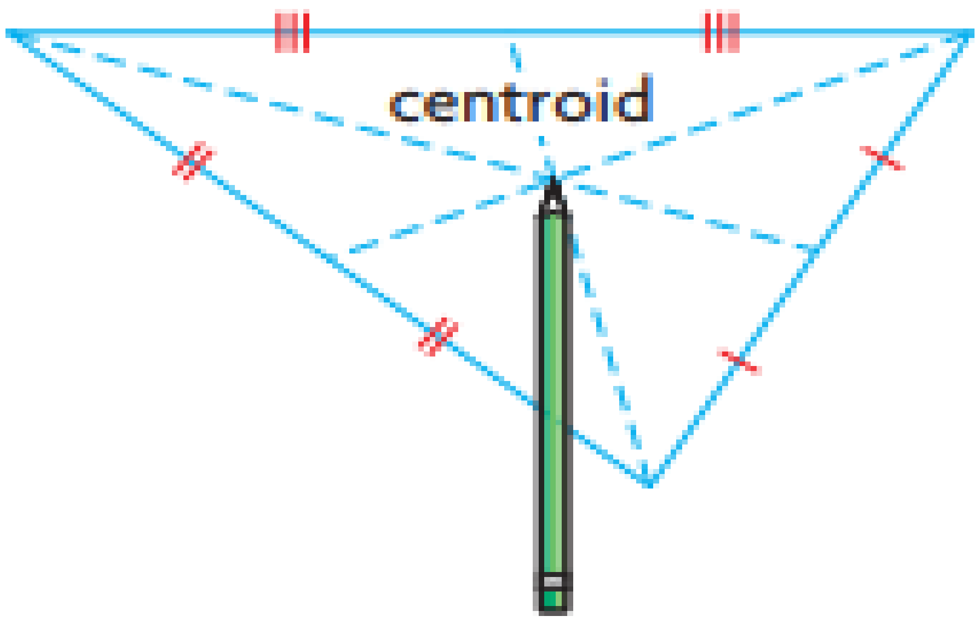
Section 2.6 of Nelson text book.

Centroid: All the medians of a triangle intersect at the same point.

Note: All the median when intersected have a ration of 2:1.

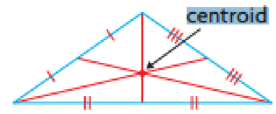
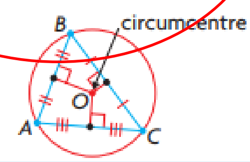
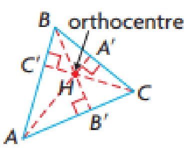
YOU WILL NEED

- grid paper and ruler, or dynamic geometry software



Key Idea

- Some properties of two-dimensional figures are determined by the intersection of lines.

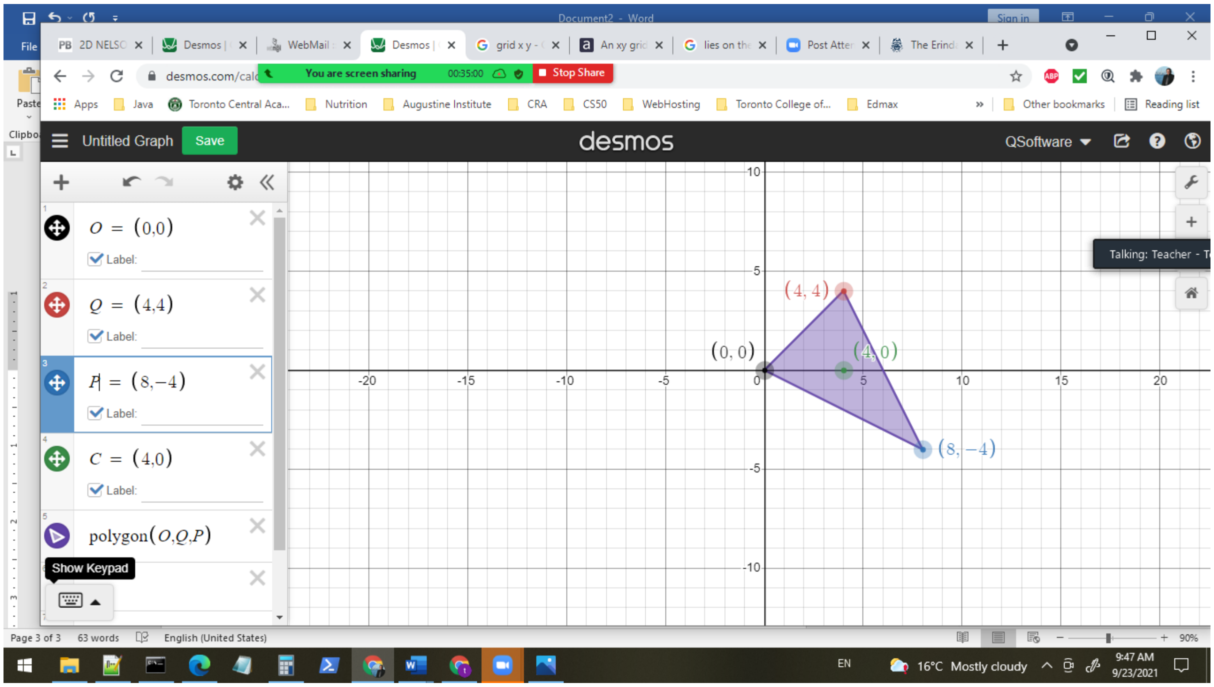
Figure	Properties Determined by the Intersection of Lines or Line Segments	Diagram
triangle	All the medians intersect at the same point, called the centroid.	
	All the perpendicular bisectors intersect at the same point, called the circumcentre. The three vertices of the triangle are the same distance from this point.	
	All the altitudes intersect at the same point, called the orthocentre.	

(continued)

Know this for future examinations.

Example 1: Centroid of a Triangle:

Note 2: The centroid of a triangle in physics, is the point in the triangle that creates a balancing point.



Objective:

1. Verify that point C(4,0) is the centroid of ΔOPQ
2. Verify that the centroid divides each median in ratio of 2:1.

Logic:

1. The point of a triangle is the point of intersection of three medians.
2. A point of intersection is a solution or common point, to three linear equations, line segments or in our case, medians.

What is a median? A median is a midpoint of a line segment. A line segment has two end points.

How do we find the point of any two points?

The formula for midpoint is as follows:

$M(x, y) = (\frac{x1+x2}{2}, \frac{y1+y2}{2})$

The median is a line segment from a vertex to the opposite side that splits the line segment of the opposite into two equal parts.

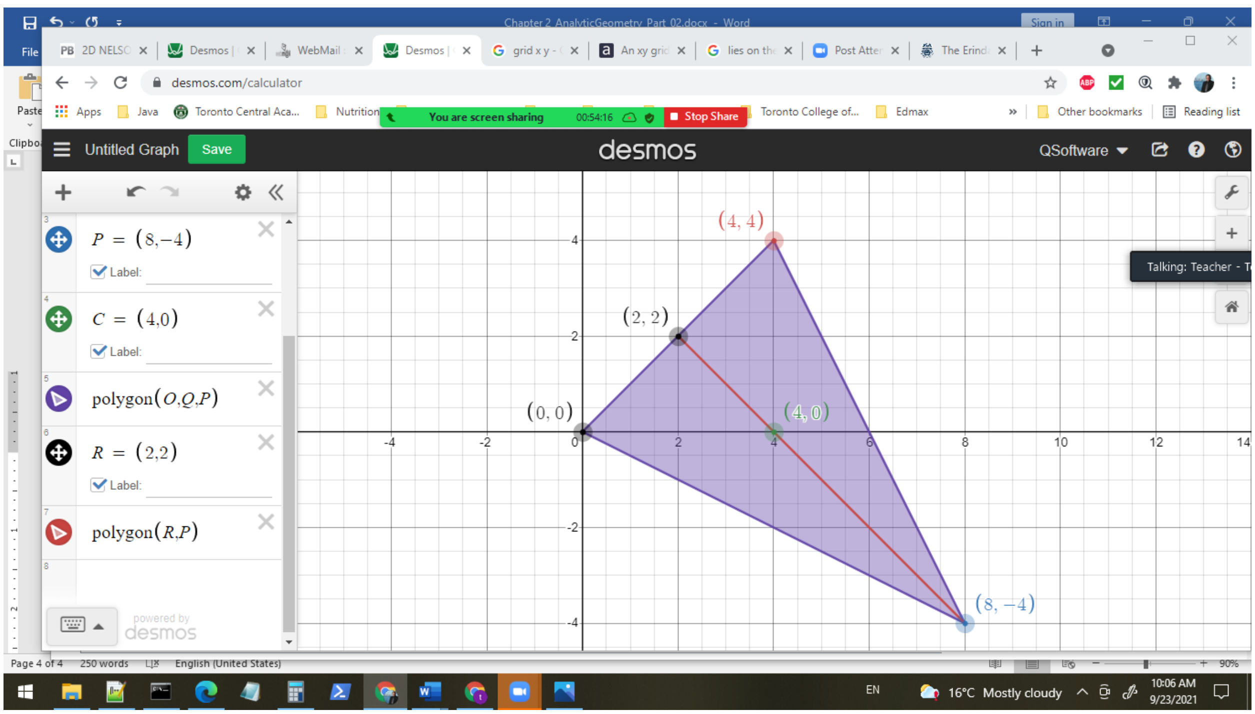
Point P(8, -4), call the midpoint of vertex P, is R.

$\text{Mid}_{pr} = (\frac{x1+x2}{2}, \frac{y1+y2}{2})$

$O(x_1, y_1) = (0, 0)$
$Q(x_2, y_2) = (4, 4)$

$R(x,y)$ = Midpoint of OQ = $(\frac{x1+x2}{2}, \frac{y1+y2}{2}) = (\frac{0+4}{2}, \frac{0+4}{2}) = (\frac{4}{2}, \frac{4}{2}) = (2, 2)$

$\text{Mid}_{pr} = (2, 2)$



Find the slope of line PR, $M_{pr} = \frac{y2-y1}{x2-x1} = \frac{2-(-4)}{2-8} = \frac{6}{-6} = -1$

What is the equation of line segment or line PR, general formula is:

y = mx + b
m = -1
b is determined by substituting any point on the segment PR to find what b is, which is the y-intercept, that is when x = 0.

2 = (-1)(2) + b
2 = -2 + b
2 + 2 = -2 + 2 + b

4 = b
∴ b = 4.

Substitute m = -1, b = 4. The general equation for M_{pr} is equal to: y = -1x + 4

How do we know that a point is a solution to an equation? The answer is when that point falls on the line.

Now substitute point C(4, 0) into the equation for M_{pr} is equal to: y = -1x + 4

Equation 1: PR: : y = -1x + 4	Equation 2: C(4, 0), b = 0 QS: x =4 ; y = mx + b,	Equation 3: y = 0, C(4, 0) OT: y = 0; y = mx + b
L.S. = 0	L.S. = 0	L.S. = 0
R.S. = -1x + 4 R.S = -1(4) + 4 R.S. = -4 + 4 = 0 ∴ L.S. = R.S. = 4 ∴ is on line segment PR.	R.S. = m(0) + 0 R.S. = 0 ∴ L.S. = R.S. = 0 ∴ is on line segment QS.	R.S. = (0)(4) + 0 R.S. = 0. ∴ L.S. = R.S. = 0 ∴ is on line segment OT.

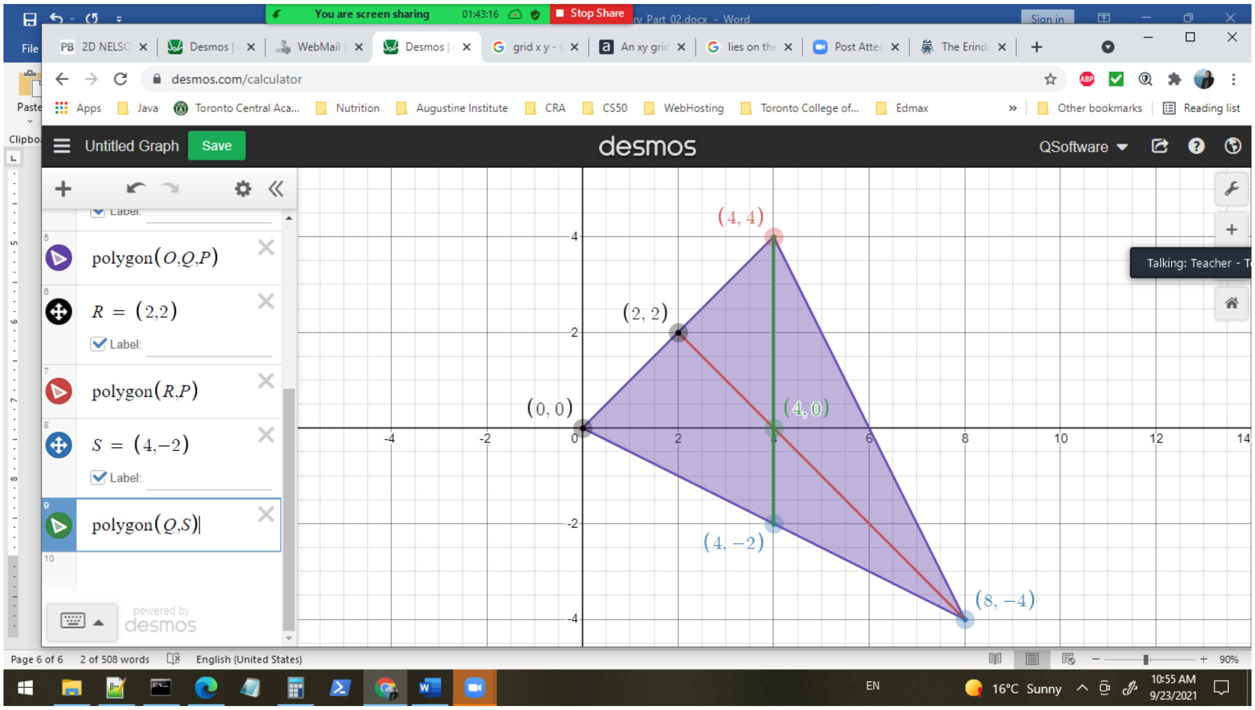
Next, the midpoints for sides OP, and PQ is S, T respectively.

S(x, y) = Mid_{op} = $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

$O(x_1, y_1)$ = (0, 0)
$P(x_2, y_2)$ = (8, -4)

S(x,y) = Midpoint of OP = $(\frac{x1+x2}{2}, \frac{y1+y2}{2}) = (\frac{0+8}{2}, \frac{0+(-4)}{2}) = (\frac{8}{2}, \frac{-4}{2})$ = (4, -2)

Plot point S(x, y) = (4, -2) onto the triangle in Desmos Graphing Calculator. See new line segment below.



Find the slope of line QS, $M_{qs} = \frac{y2-y1}{x2-x1} = \frac{-2-4}{4-4} = \frac{-6}{0}$ = unknown, which is a vertical line.

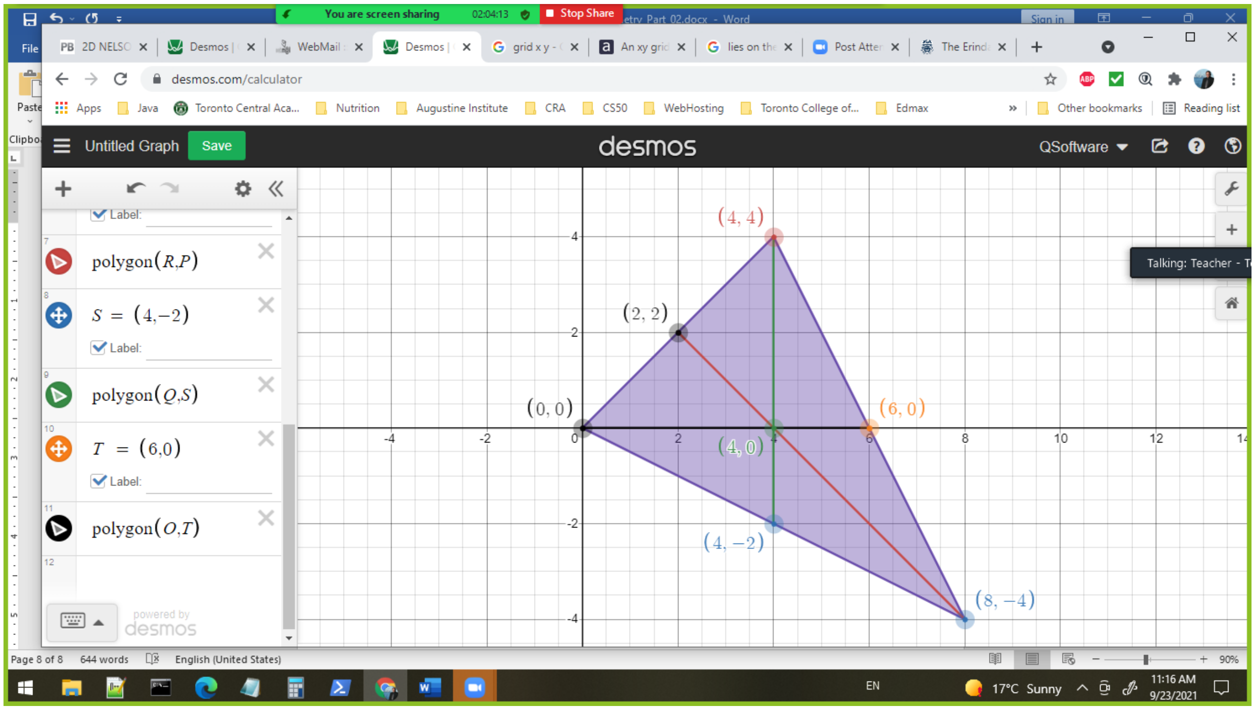
Equation of this line QS is x = 4, we know that there are many values for y, which one values is o.

$\therefore$ C(4,0) is a point of intersection.

$P(x_1, y_1) = (8, -4)$
$Q(x_2, y_2) = (4, 4)$

$T(x, y) = Mid_{pq} = (\frac{8+4}{2}, \frac{-4+4}{2}) = (\frac{12}{2}, \frac{0}{2}) =$ (6, 0)

Plot T(x, y) = (6, 0) onto our graph.



Equation of line segment QT is y = 0.

So since y = 0, is the x – axis, it would reason that point C(4, 0) is on line y = 0 since they are collinear.

Collinear is when two lines superimpose on top of each other and all the points fall on the same two lines.

Summary table:

Equation 1: PR: : y = -1x + 4	Equation 2: C(4, 0), b = 0 QS: x =4 ; y = mx + b,	Equation 3: y = 0, C(4, 0) OT: y = 0; y = mx + b
L.S. = 0	L.S. = 0	L.S. = 0
R.S. = -1x + 4 R.S = -1(4) + 4 R.S. = -4 + 4 = 0 $\therefore$ L.S. = R.S. = 4 $\therefore$ is on line segment PR.	R.S. = m(0) + 0 R.S. = 0 $\therefore$ L.S. = R.S. = 0 $\therefore$ is on line segment QS.	R.S. = (0)(4) + 0 R.S. = 0. $\therefore$ L.S. = R.S. = 0 $\therefore$ is on line segment OT.

$\therefore$ C(4,0) is solves all 3 equations, and therefore is the point of intersection of all line segments: PR, QS, OT.

Verify the ration of 2:1 for all medians on either side of the centroid.

Use the length formula to determine the length, and hence the ratio of the lengths.

Length = $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

Line segment: PR	
PC $P(x_1, y_1) = (8, -4)$ $C(x_2, y_2) = (4, 0)$	RC $R(x_1, y_1) = (2, 2)$ $C(x_2, y_2) = (4, 0)$
$L_{pc} = \sqrt{(4-8)^2 + (0-(-4))^2}$ $L_{pc} = \sqrt{(-4)^2 + (4)^2}$ $L_{pc} = \sqrt{(4)^2 + (4)^2}$ $L_{pc} = \sqrt{2 \text{ x } (4)^2} = \sqrt{(4)^2} \text{ x } \sqrt{2}$ $L_{pc} = 4 \text{ x } \sqrt{2}$	$L_{RC} = \sqrt{(4-2)^2 + (0-(2))^2}$ $L_{RC} = \sqrt{(2)^2 + (-2)^2}$ $L_{RC} = \sqrt{(2)^2 + (2)^2}$ $L_{RC} = \sqrt{2 \text{ x } (2)^2} = \sqrt{(2)^2} \text{ x } \sqrt{2}$ $L_{RC} = 2 \text{ x } \sqrt{2}$
Ratio: 2: 1 PC is two the length of RC	

<table>
<tr><td colspan="2">Line segment: OT
Visually
O(0, 0)
C(4, 0)
T(6, 0)</td></tr>
<tr><td>OC = (X2 – x1)
OC = (4 – 0) =
4</td><td>TC = (X2 – X1)
TC = (6 – 4)
TC = 2</td></tr>
<tr><td colspan="2">Ratio: 2: 1
OC is two the length of TC</td></tr>
</table>

<table>
<tr><td colspan="2">Line Segment: QS
Q(4, 4)
C(4, 0)
S(4, -2)</td></tr>
<tr><td>QC = y2 – y1
QC = 4 – 0
QC = 4</td><td>SC = y2 – y1
SC = 0 – (-2)
SC = 2</td></tr>
<tr><td colspan="2">Ratio: 2: 1
QC is two the length of SC</td></tr>
</table>

$\therefore$ C(4,0) divides each of the medians of ΔOPQ into a ratio of 2:1.

Assignment #7:

Prove that medians of a triangle formed by any 3 points, which form a triangle, form a centroid for the triangle.

Prove also the ratio of the medians are in a ratio of 2:1.

Step 1: Table of Points:

Student Name	Point A	Point B	Point C	Verify
Joanna	(-6, 8)	(-12,0)	(12, 0)	Hayden
Hayden	(-9, 13)	(-11, -7)	(9, -6)	Cody
Cody	(-6, 5)	(-14, -11)	(8, -9)	Sherry
Sherry	(-11, -14)	(4, -13)	(12, 6)	Leon
Wenkang Li	(-3, 1)	(-4, 1)	(5, -1)	Tee
Tee	(-10, 4)	(-10, -12)	(10, -9)	Brian N.
Brian N.	(-9, 6)	(-9, -5)	(12, -11)	Henry T.
Henry T.	(-10, 3)	(-10, -6)	(10, -7)	Roy
Roy	(-9,9)	(-9, -5)	(9, -10)	Kyle
Kyle	(-7,12)	(-10, -11)	(9, -11)	Henry N.
Henry N.	(-4, -2)	(1,-1)	(4,4)	Lavinia
Lavinia	(-5,-6)	(10,-4)	(5,8)	Jason
Jason	(-6,-8)	(5,-10)	(5, 5)	Alex
Alex	(-7, 9)	(-11,1)	(12, 1)	Joanna

Steps:

1. Graph the points above using Demos Graphing Calculator
2. Find the vertices and medians of each vertex.
3. Find the Centroid.
4. Prove the ratio of each median as a ratio of 2:1
5. Use the AR, BS, CT as the line segments for the medians.