Date: Tuesday, September 28th, 2021
Course: MPM2D - Principles of Mathematics.

Covered;
1. System of Linear Equations. (Chapter 1)
2. Analytic Geometry. (Chapter 2).

Goal:
1. Quadratic Relations.
   Covering Parabolas:
- 3 equations:
y = ax^2 + bx + c
y = a(x - h)^2 + k
y = a(x - s)(x - r)

Simple quadratric relation is:
y =x^2.

$Y = x^2$

Quick Review:

Chapter 1: - System of Linear Equations:

Y = mx + b

Multiple linear equations, we have solution when the system of linear equations intersects at a point.

If there is no point of intersection, there is no solution to the linear equations or system of linear equations.

If the lines are collinear, that means, they are superimposed on top of each other, there is an infinite number of solutions since all the points on one line satisfy the equation for the other line.

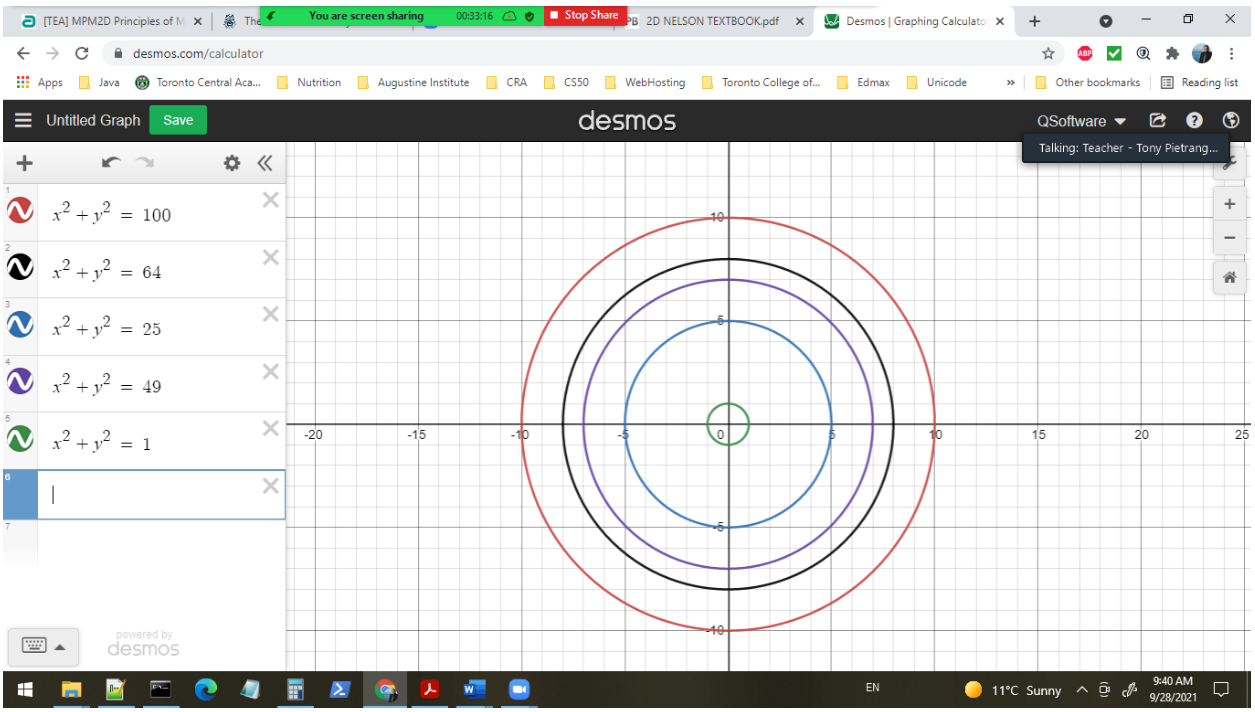
Chapter 2: Analytic Geometry:

General equation of a line: y = mx + b, where me is the slop.

Equation of line segment:- The connection of two points
Midpoints of a line segment – m(x,y) = ((x1 + x2) / 2, (y1 + y2) / 2)

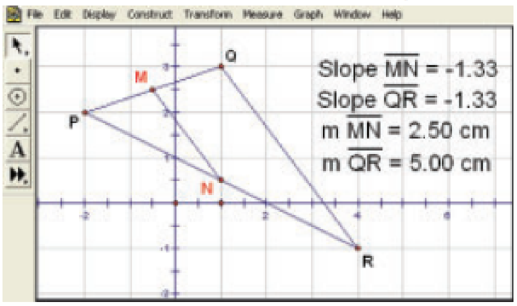
Equation of circle: x^2 + y^2 = r^2

$X^2 + y^2 = 100$, here is a circle of a radius of 10 units.



Review:
Nelson Textbook,
Examples 2, 3: pages 106 – 108.

Example 2: Slopes and lengths of line segments.



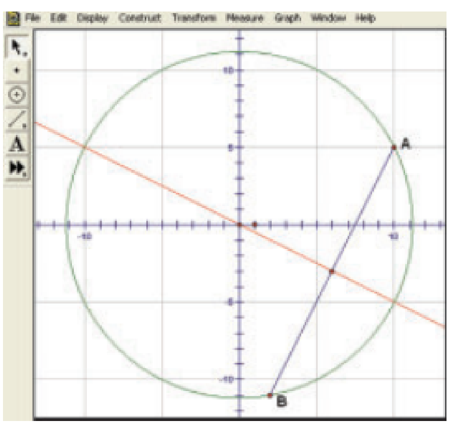
I verified my calculations using dynamic geometry software. I chose a scale where 1 unit = 1 cm. I constructed the triangle and the midsegment *MN*. Then I measured the lengths and slopes of *MN* and *QR*. My calculations were correct.

Example 3: Lines segments verifying properties of a circle

Two points on a circle for a chord, and a midpoint on the chord, if we get a line parallel to the chord, we always get a line that passes through the origin of a circle.

chord *AB* is $y = -\frac{1}{2}x$. The y-intercept is 0.

of the circle.



I verified my calculations using dynamic geometry software. I constructed the circle, the chord, and the perpendicular bisector of the chord. The sketch confirmed that the perpendicular bisector passes through the centre of the circle.

In Summary

Key Idea

Shapes: Triangle.

1. Isosceles Triangle – which two sides or inner angles that are equal.

2. Right Angle Triangle – Pythagorean theorem – L = $\sqrt{(\mathbf{x2}-\mathbf{x1})^2+(\mathbf{y2}-\mathbf{y1})^2}$

Parallelogram – when opposites sides have the same slope.
Square, Rectangle, rhombus, trapezoid, etc.

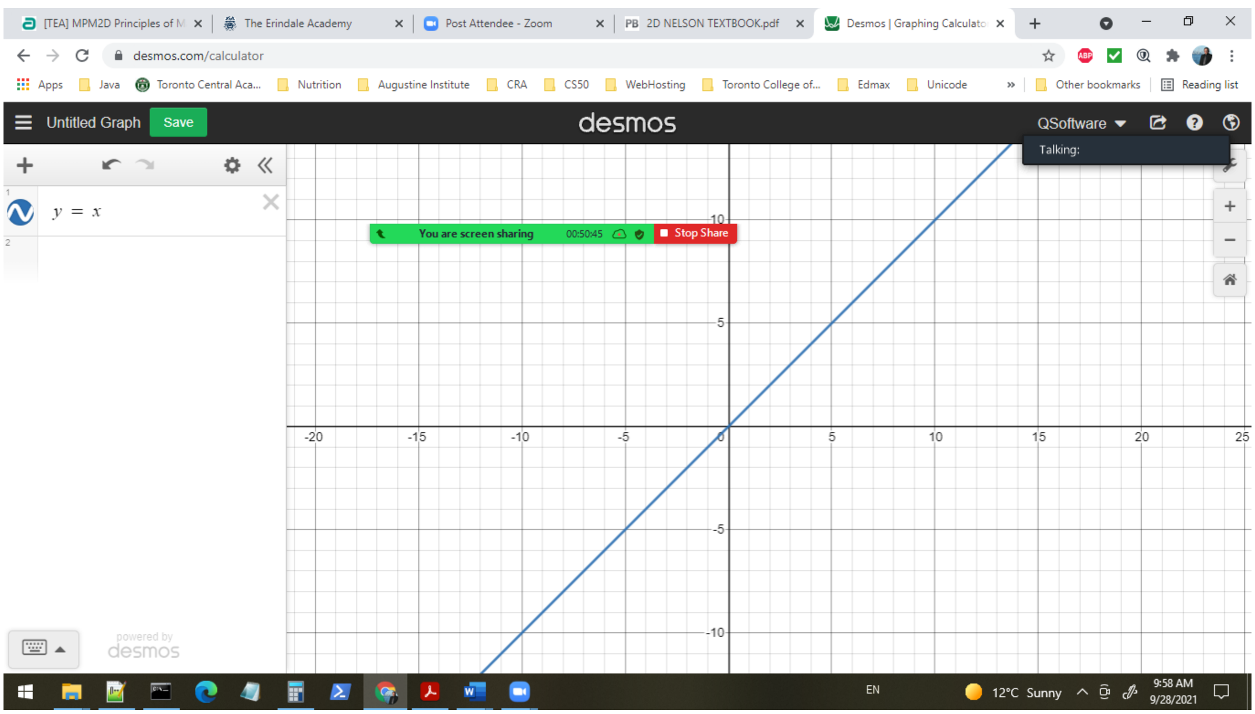
To find any general parallelogram, that when we find all the midpoints of a parallelogram, and join the midpoints, we get Vargnion Parallegram.

Chapter 3: - Graphs of Quadratic Relations:

Terminology:

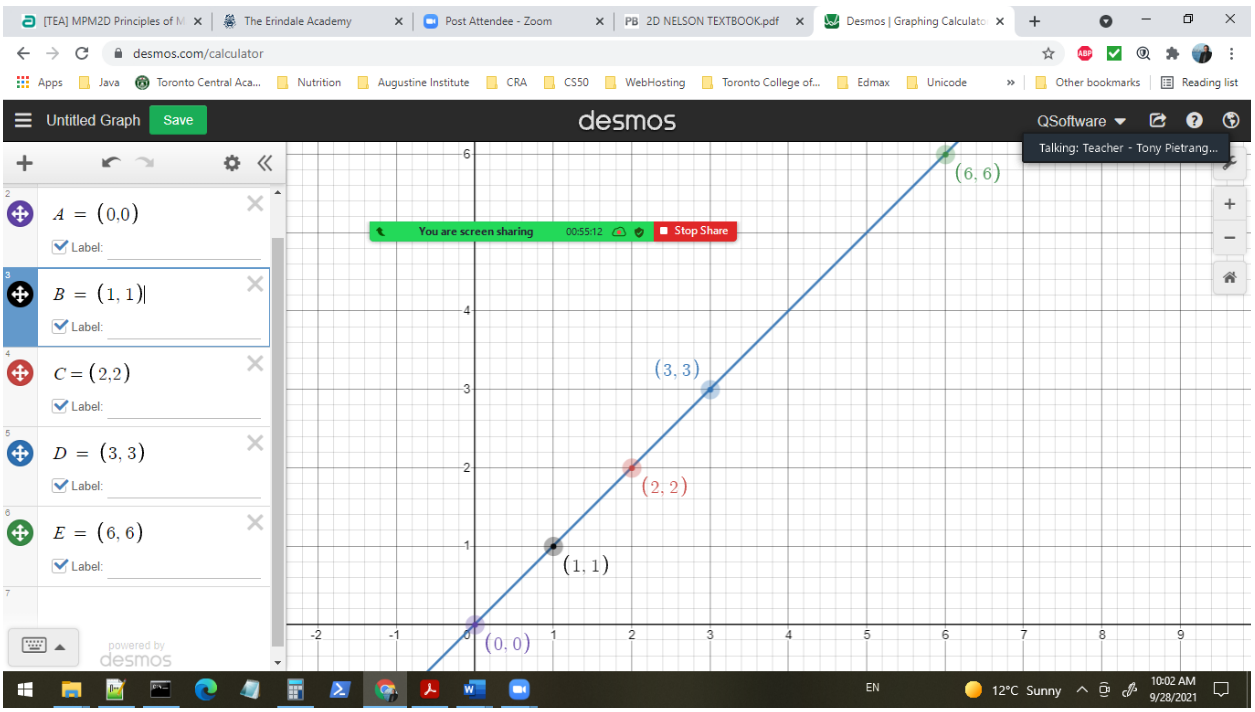
1. **Non-linear relation**
2. **Curve of best fit**
3. **Quadratic relation**
4. **Parabola**
5. **Vertex**
6. **Axis of Symmetry**
7. **Finite differences**
8. **Zero (Zeroes)**

linear Relation is where the dependent variable increases or decreases at a rate where the differences between any two points in the relation is a constant. That is, the first difference in the relation is a constant.



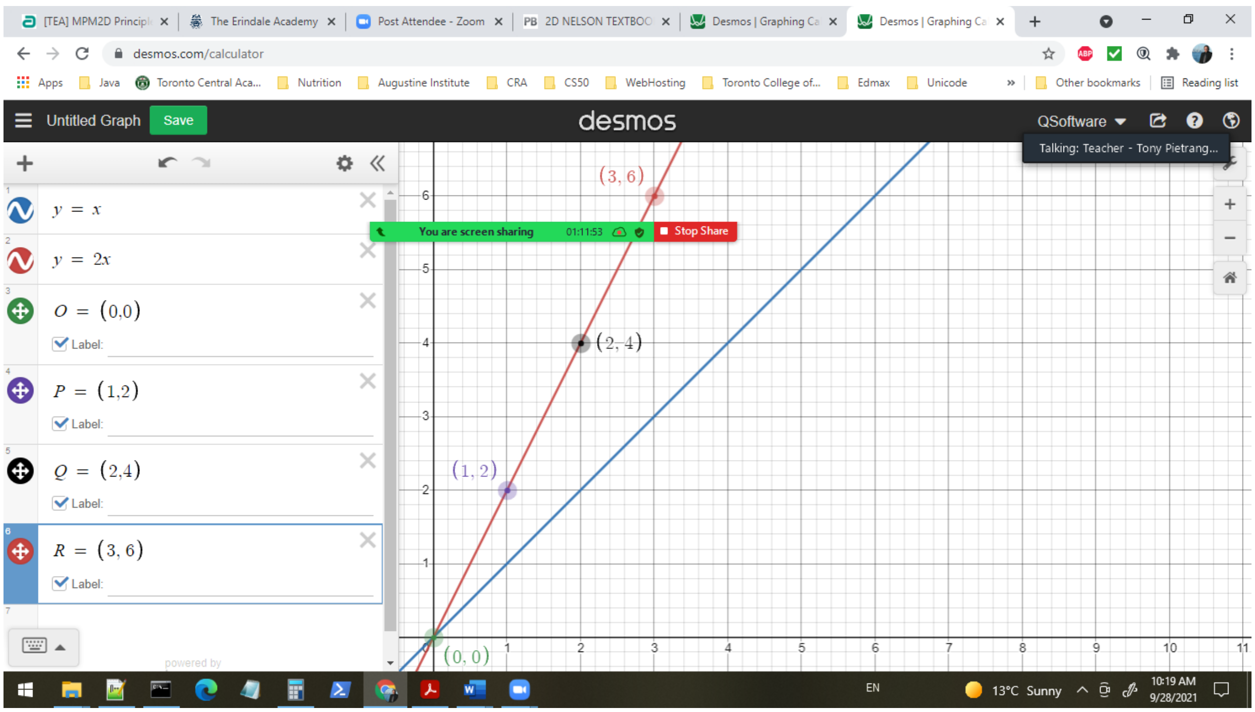
Graph points on a line: y = x

Point(x,y)	Point 2 (x2, y2)	Point 1 (x1, y1)	First Difference. (Slope between two points) Note: Constant. (1)
A(0, 0)	B(1, 1)	A(0,0)	AB = $\frac{y2-y1}{x2-x1} = \frac{1-0}{1-1} = \frac{1}{1} = 1$
B(1, 1)	C(2, 2)	B(1, 1)	BC = $\frac{y2-y1}{x2-x1} = \frac{2-1}{2-1} = \frac{1}{1} = 1$
C(2, 2)	D(3,3)	C(2,2)	CD = $\frac{y2-y1}{x2-x1} = \frac{3-2}{3-2} = \frac{1}{1} = 1$
D(3, 3)	E(6, 6)	D(3,3)	DE = $\frac{y2-y1}{x2-x1} = \frac{6-3}{6-3} = \frac{3}{3} = 1$
E(6, 6)			



Graph points on a line: y = 2x

Point(x,y)	Point 2 (x2, y2)	Point 1 (x1, y1)	First Difference. (Slope between two points) Note: Constant. (2)
O(0, 0)	P(1, 2)	O(0,0)	OP = $\frac{y2-y1}{x2-x1} = \frac{2-0}{1-0} = \frac{2}{1} = 2$
P(1, 2)	Q(2, 4)	P(1, 2)	PQ = $\frac{y2-y1}{x2-x1} = \frac{4-2}{2-1} = \frac{2}{1} = 2$
Q(2, 4)	R(3, 6)	Q(2,4)	QR = $\frac{y2-y1}{x2-x1} = \frac{6-4}{3-2} = \frac{2}{1} = 2$
R(3, 6)			



∴ all the differences between all the points is a constant.

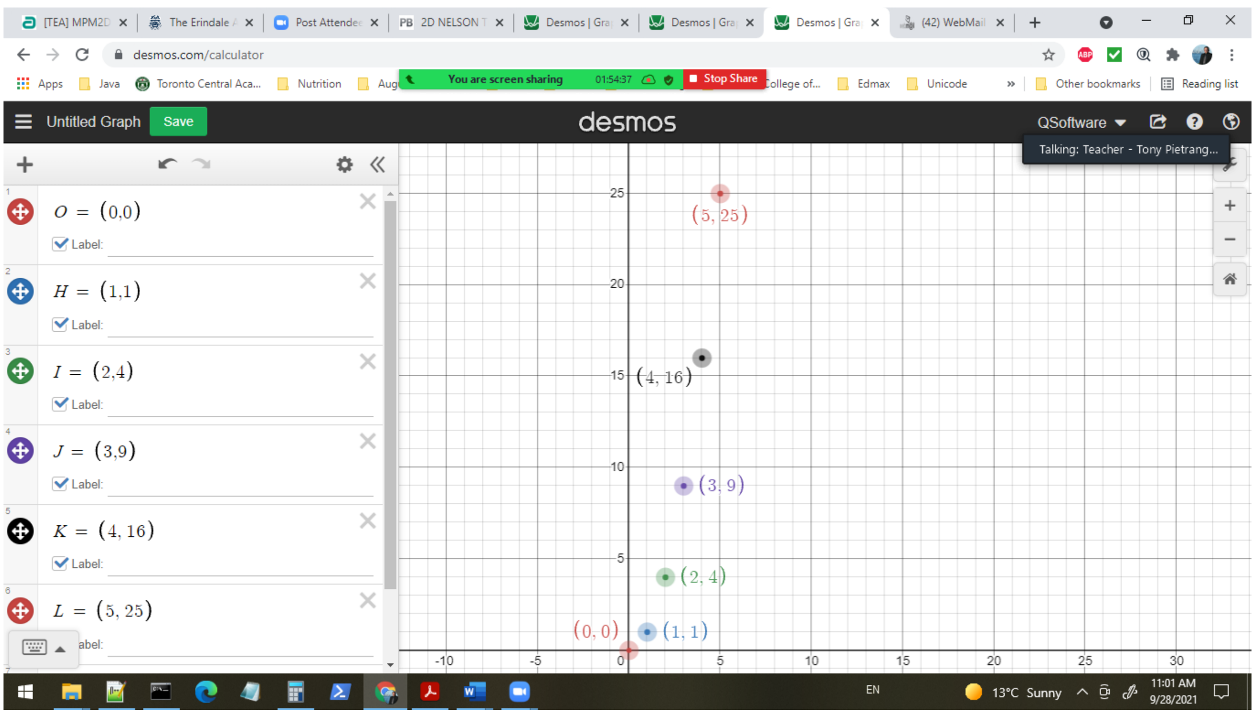
∴ y = 2x is linear. Set of points (O, P, Q, R) – first difference is 2.

∴ y = x is linear. Set of points (A, B, C, D E) – first difference is 1.

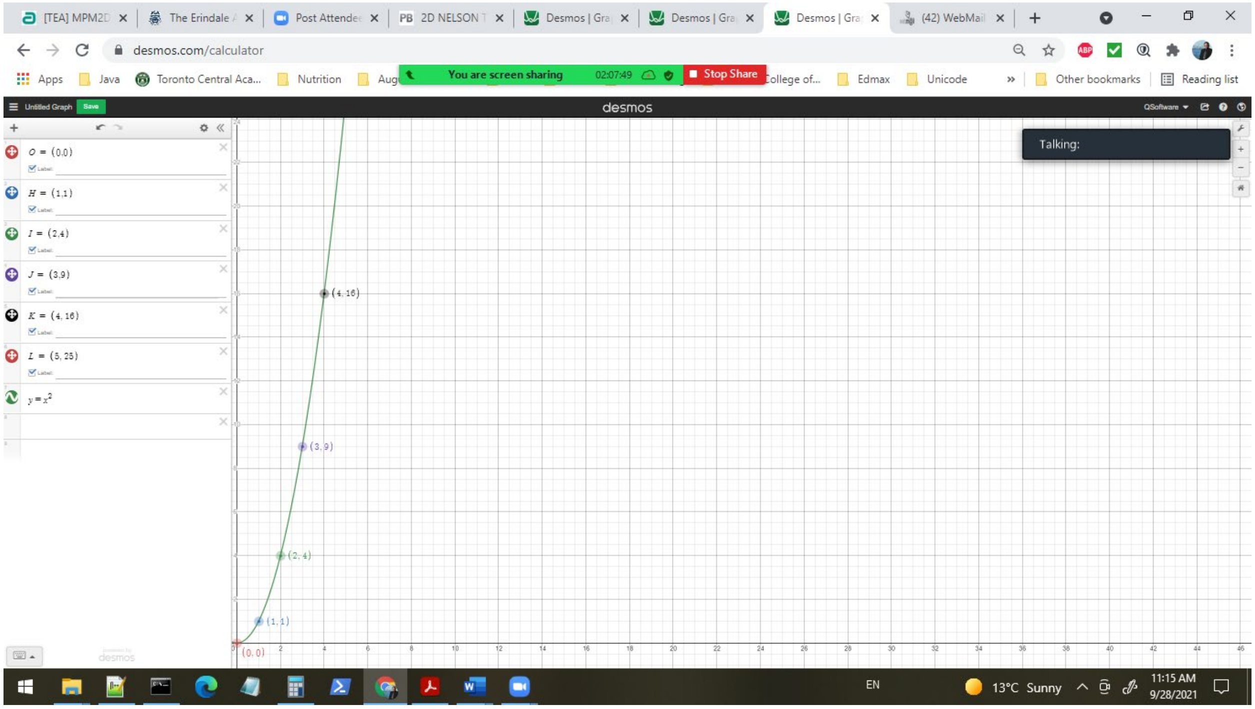
Quadratic Relation

Graph

Point(x,y)	Point 2 (x2, y2)	Point 1 (x1, y1)	First Difference. (Slope between two points) Note: NOT Constant.
O(0, 0)	H(1, 1)	O(0,0)	OH = $\frac{y2-y1}{x2-x1} = \frac{1-0}{1-0} = \frac{1}{1} = 1$
H(1, 1)	I(2, 4)	H(1, 1))	HI = $\frac{y2-y1}{x2-x1} = \frac{4-1}{2-1} = \frac{3}{1} = 3$
I(2, 4)	J(3, 9)	I(2, 4)	IJ = $\frac{y2-y1}{x2-x1} = \frac{9-4}{3-2} = \frac{5}{1} = 5$
J(3, 9)	K(4, 16)	J(3,9)	JK = $\frac{y2-y1}{x2-x1} = \frac{16-9}{4-3} = \frac{7}{1} = 7$
K(4, 16)	L(5, 25)	K(4, 16)	LK = $\frac{y2-y1}{x2-x1} = \frac{25-16}{5-4} = \frac{9}{1} = 9$
L(5, 25)			



Join the points to see the shape:



The relationship is non-linear since the first difference is not a constant.

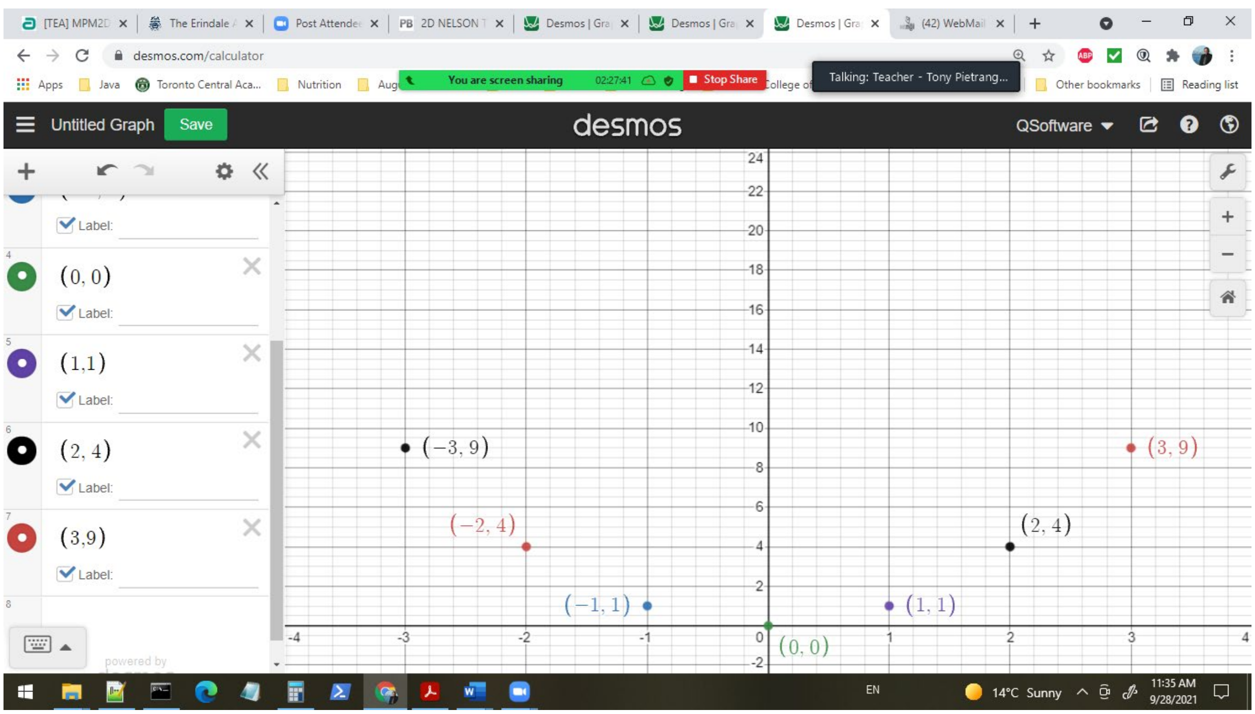
Activity:

Make a table of values from a domain of x integer values from -3 to +3 and then graph the points on a graph. Join the points trying to make a smooth curve as best as possible.

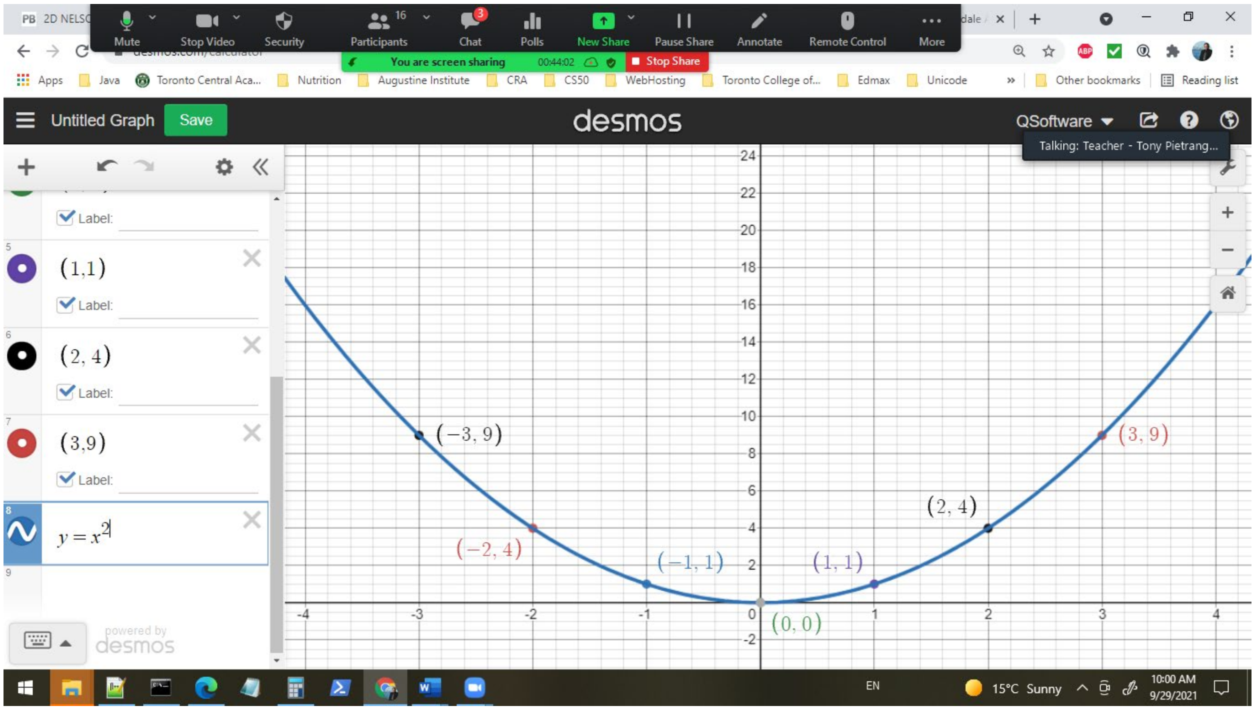
a) $y = x^2$

X value	$y = x^2$	**Y Value**	**Point(x,y)**
-3	**$Y = (-3)^2$**	**9**	**(-3,9)**
-2	**$Y = (-2)^2$**	**4**	**(-2,4)**
-1	**$Y = (-1)^2$**	**1**	**(-1, 1)**
0	**$Y = (0)^2$**	**0**	**(0, 0)**
1	**$Y = (1)^2$**	**1**	**(1, 1)**
2	**$Y = (2)^2$**	**4**	**(2, 4)**
3	**$Y = (3)^2$**	**9**	**(3, 9)**

Plot on a grid:



The outcome of the analysis is the points best fit the **equation $y = x^2$**



Activity for student to do:

1. Make a table of values for the following equations:
2. Plot the points on the Desmos graphing software.

a) $y = x^2$ 🡐 done by instructor.

b) $y = 2x^2$

c) $y = -x^2$

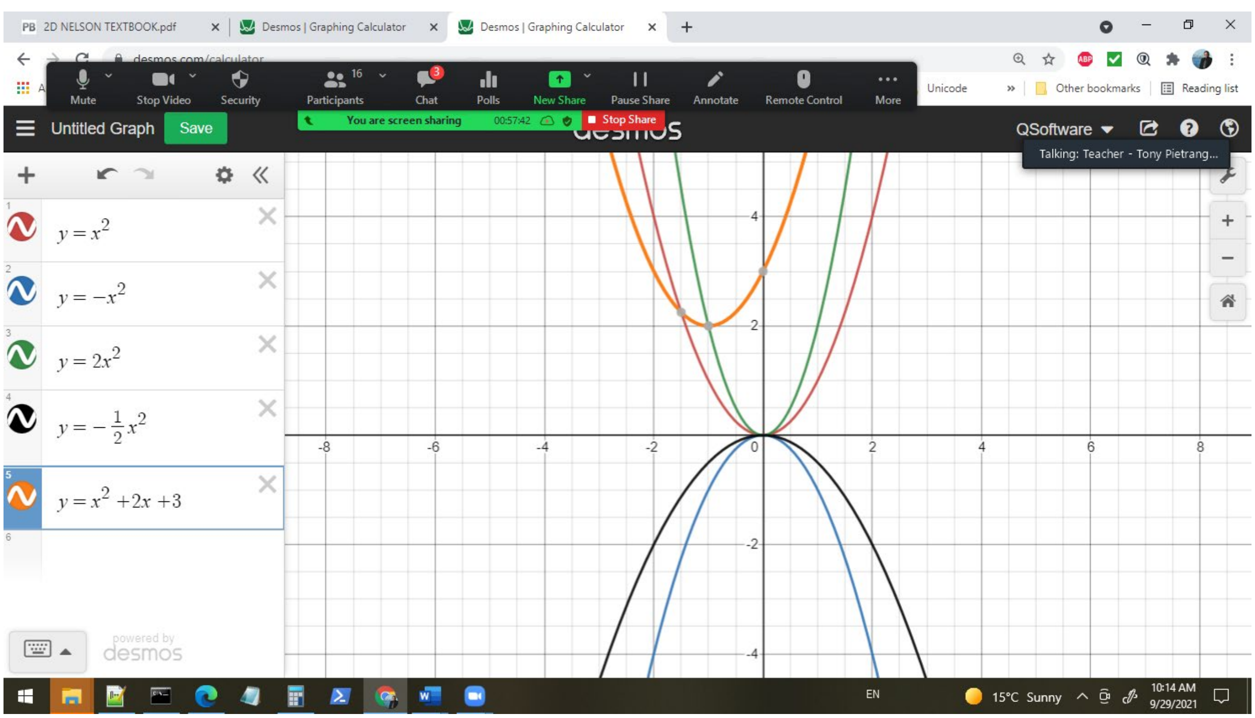
d) $y = -\frac{1}{2}x^2$

e) $y = x^2 + 2x + 3$

Your work:

1. Show me a table of points from x integer values ($-5 \le x \le 5$) and get the values for y.
2. Plot these points on a graph by themselves first.
3. Plot the equation on the points above.
4. Take a screen snapshot and place into your document for that equation.
5. After all the individual points, and equations are plotted individually, place all the equations together on the same graph.

This should be your result:



Simply start like this, and create your Y values, and points.

b) $y = 2x^2$

X value	$y = 2x^2$	Y Value	Point (x, y)
-3	**$Y= 2(-3)^2$**		
-2	**$Y=2 (-2)^2$**		
-1	**$Y= 2(-1)^2$**		
0	**$Y= 2(0)^2$**		
1	**$Y= 2(1)^2$**		
2	**$Y= 2(2)^2$**		
3	**$Y= 2(3)^2$**		

Date: Wednesday, September 29th, 2021
Course: MPM2D - Principles of Mathematics

Chapter 3: Quadratic Relations. (Parabolas y = x^2, and other forms.
Goal:
Parabolas:
1. y = ax^2+bx + c <== standard form for paraboloas.
2. y = a(x - h)^2 + k <= explore this form.
3. y = a(x -r)(x - s)

Yesterday:
1. First Difference - If it's a constant, then it is a linear relationship. (y = x, y = 2x)
   y = x, the first difference, it was a constant of 1 .
   y = 2x, the first difference, it was a constant of 2.

   The first difference is really the slope of the relationship
2. First Difference: for an non-linear relationship.
   First difference was increasing as well anlyzed the points y = x^2
   Domain set x integer values from -3 to +3
3. We generated tables of values x, y, and plotted points of (x,y)
4. Calculated first difference.

Activity:
5 equations;
You: student to
create a table of values x, y, plot the points on a graph using desmos graphing calculator.

Verify students Activity:

Activity for student to do (Continued):

1. Make a table of values for the following equations:
2. Plot the points on the Desmos graphing software.

a) $y = x^2$ 🡐 done by instructor.

b) $y = 2x^2$

c) $y = -x^2$

d) $y = -\frac{1}{2}x^2$

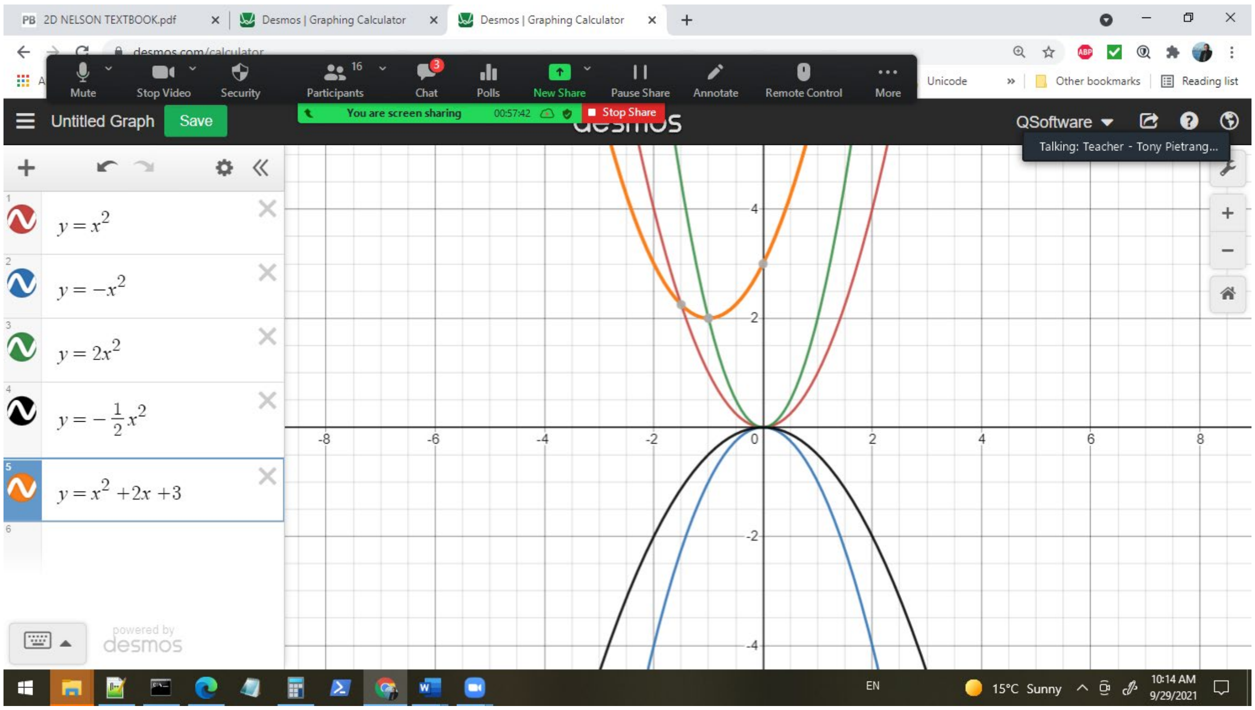
e) $y = x^2 + 2x + 3$

Your work:

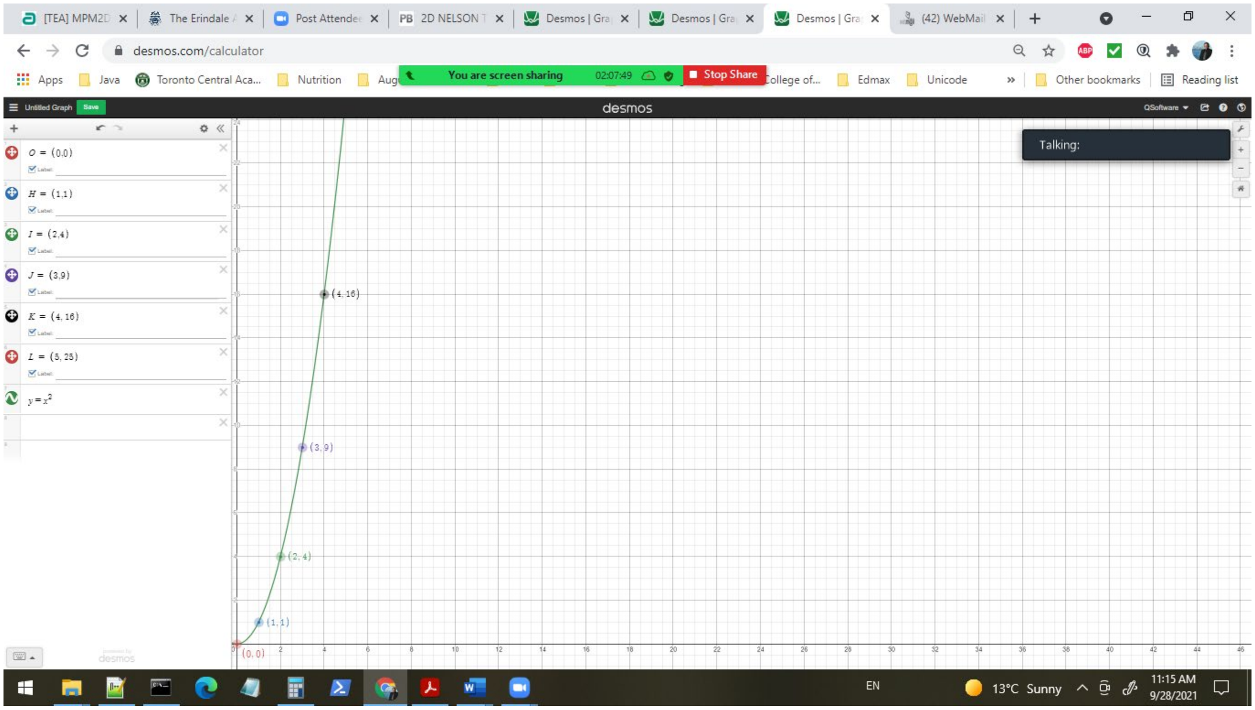
3. Show me a table of points from x integer values ($-5 \le x \le 5$) and get the values for y.
4. Plot these points on a graph by themselves first.
5. Plot the equation on the points above.
6. Take a screen snapshot and place into your document for that equation.

7. After all the individual points, and equations are plotted individually, place all the equations together on the same graph.

This should be your result:



Join the points to see the shape:



The relationship is non-linear since the first difference is not a constant.

Date: Friday, January 26, 2024
Course: MPM2D – Principles of Mathematics

Review of Plotting of the following equations:
$y=x^2$
$y=2x^2$
$Y=-\frac{1}{2}x^2$
$y=x^2+2x+3$

Create:
1. Table for each equation from x integer values form -5 <= x <= +5. Text book has -3 to +3 range.
2. Plot all the points only.
3. Plot the actual equation points.
4. Graph all the equations together in another window.

Goal:
1. Key terminology
2. Transformations:
$y=a(x-h)^2 + k$

Investigate First Difference, second difference and finite Difference.

Definitions:
finite differences: - differences found from the y-values with evenly spaced x-values.

First differences: - are the differences between consecutive y-values.
Second differences: - are the differences between consecutive first differences, and so on.

Analyze First Differences:

y=2x - 5

X	y	First differences
-2	-9	
		-7-(-9) = 2
-1	-7	
		-5-(-7) = 2
0	-5	
		-3-(-5) = 2
1	-3	
		1 – (-3) = 2
2	1	

Note: First difference is 2, and constant so this is linear relationship.

y=-6x + 2

X	y	First differences
-2	14	
		8 - 14 = -6
-1	8	
		2 – 8 = -6
0	2	
		-4 - 2 = -6
1	-4	
		-10 –(-4) = -6
2	-10	

Note: First difference is -6, and constant so this is linear relationship.

Analyze Second Differences:

$y = x^2 - 4$

X	y	First differences	Second Difference
-2	0	-3 – 0 = -3	-1 – (-3) = 2
-1	-3		
		-4 – (-3) = -1	1 – (-1) = 2
0	-4		
		-3-(-4) = 1	3 – 1 = 2
1	-3		
		0 – (-3) = 3	
2	0		

Note: Second difference is 2 is a constant, which means this is a quadratic of degree 2.

$y = 2x^2 + 3x - 1$

X	y	First differences	Second Difference
-2	1	-2 – 1 = -3	1 – (-3) = 4
-1	-2		
		-1 – (-2) = 1	5 -1 = 4
0	-1		
		4-(-1) = 5	9 – 5 = 4
1	4		
		13 - 4 = 9	
2	13		

Note: Second difference is 4, constant which means this is a quadratic of degree 2.

Definitions:

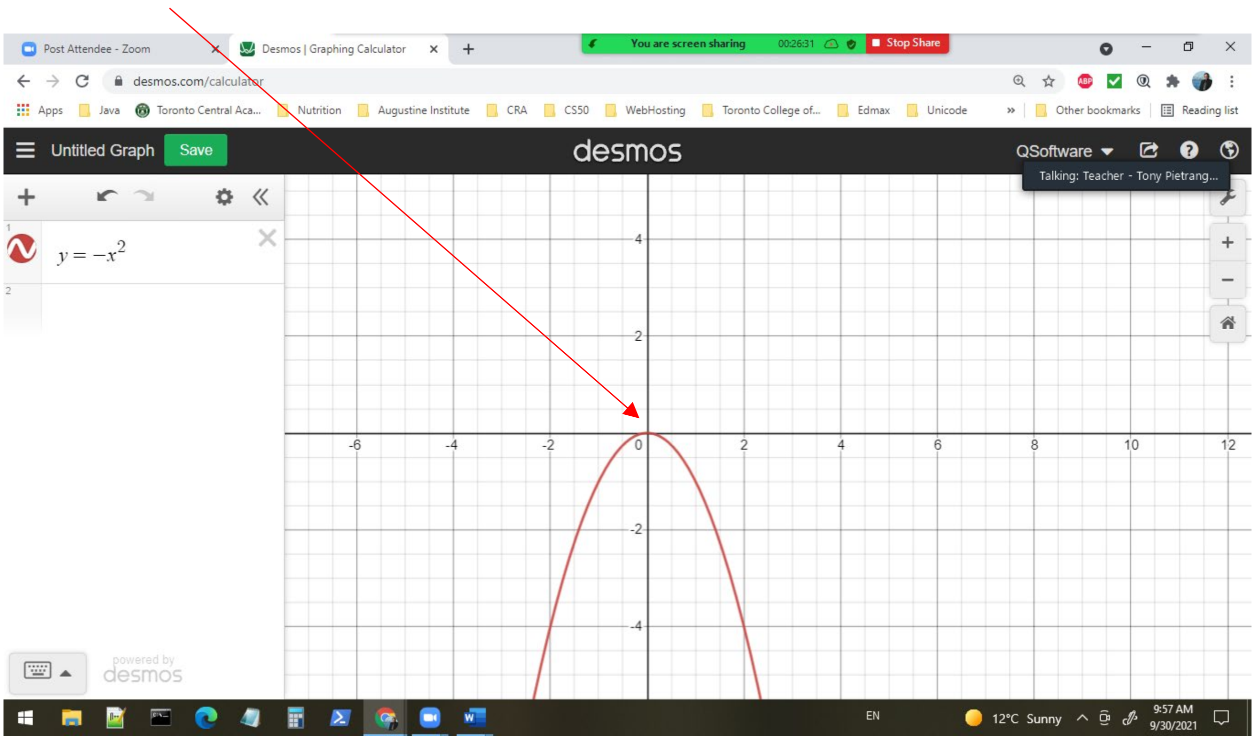
Quadratic relation: - is a relation whose equation is in the form of $y = ax^2 + bx + c$, where a, b, c are real numbers and $a \neq 0$.

Other 2 forms are:
$y = a(x - h)^2 + k$
$y = a(x - r)(x - s)$

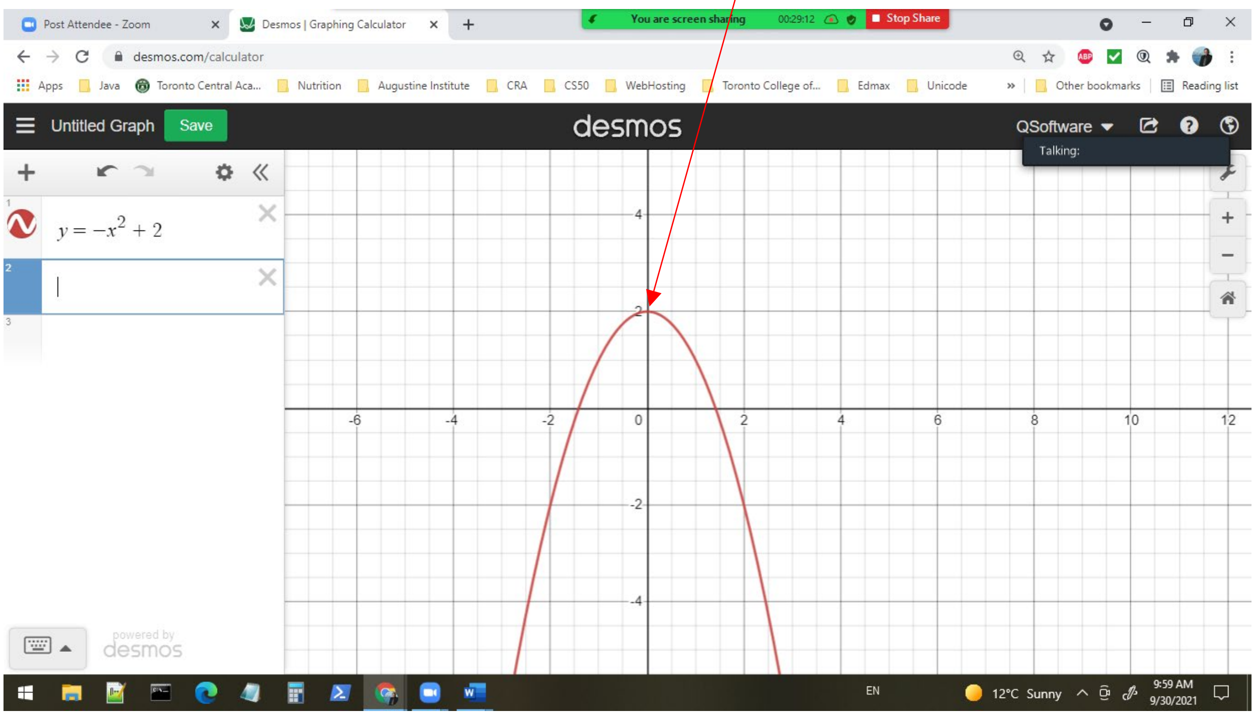
Parabola: - a graph of a quadratic relation, which is a U-shaped and symmetrical.

Vertex: -

- The point on a parabola where the curve changes direction.
- The maximum point exists if the parabola opens downwards.
  Example 1: $y = -x^2$, example maximum point is at point (0, 0).

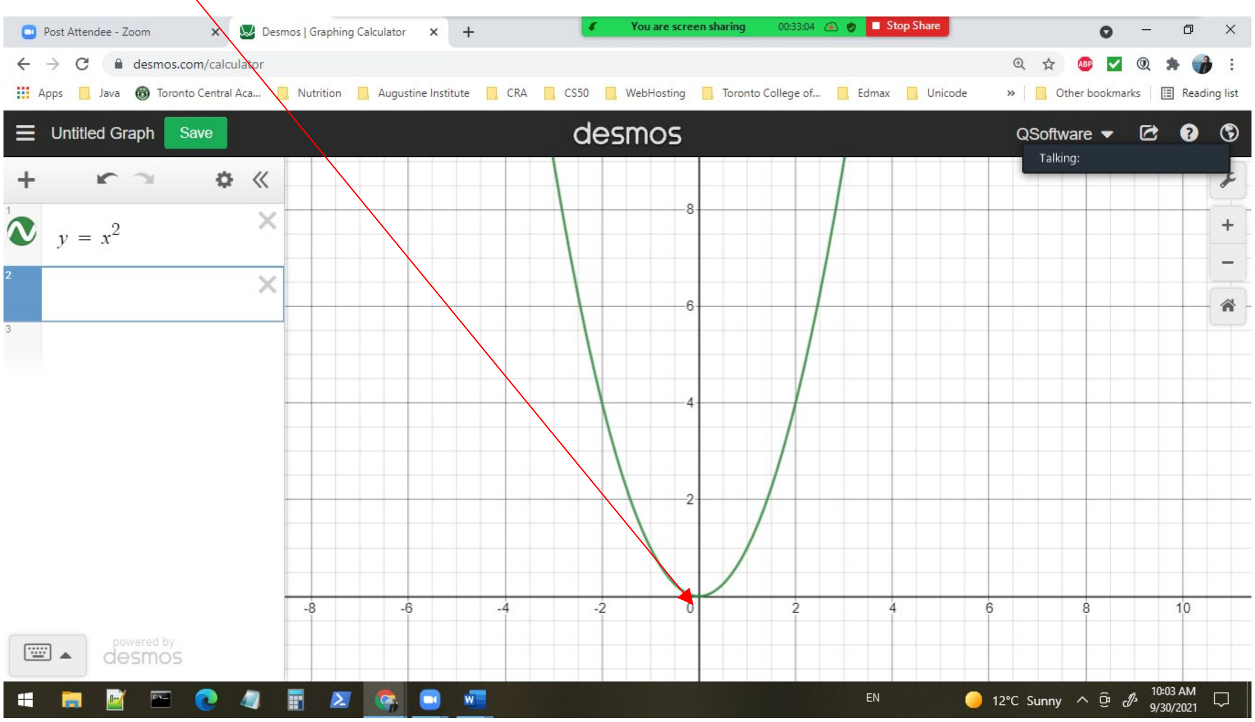


Example 2: $y = -x^2 + 2$. Here is a new vertex, which has a maximum point of P(x, y) = (0, 2)

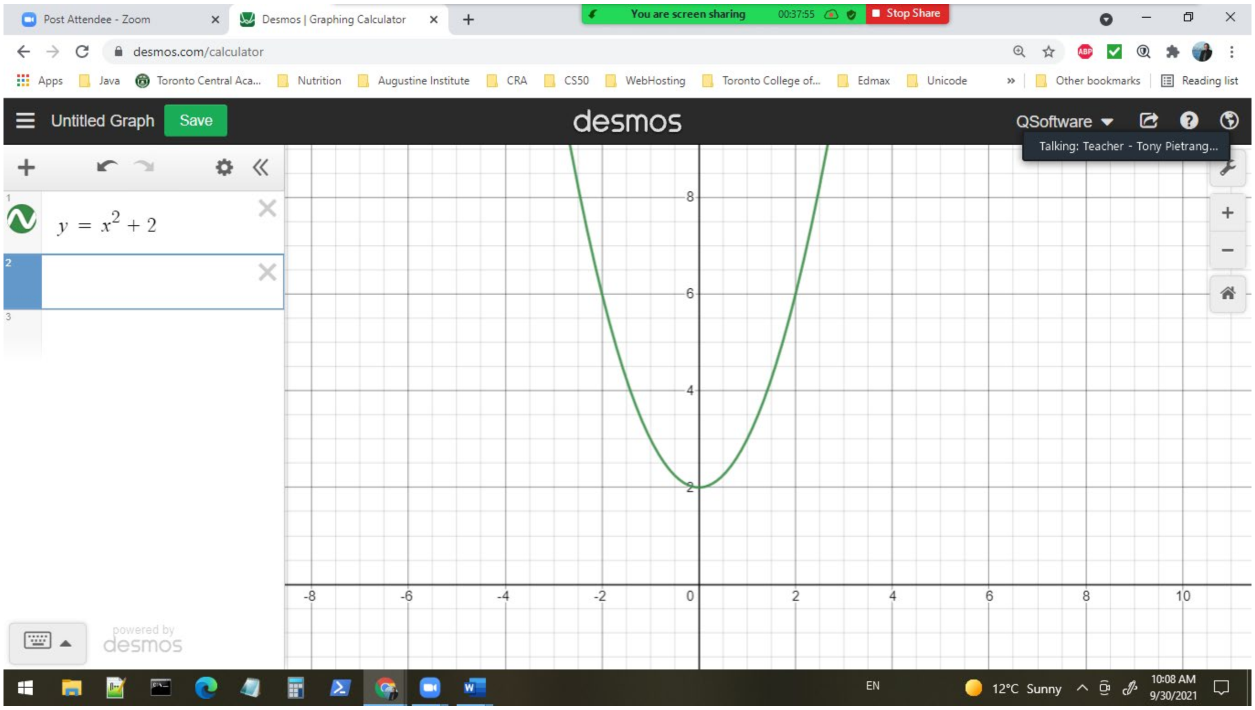


- **Example 3: Minimum**

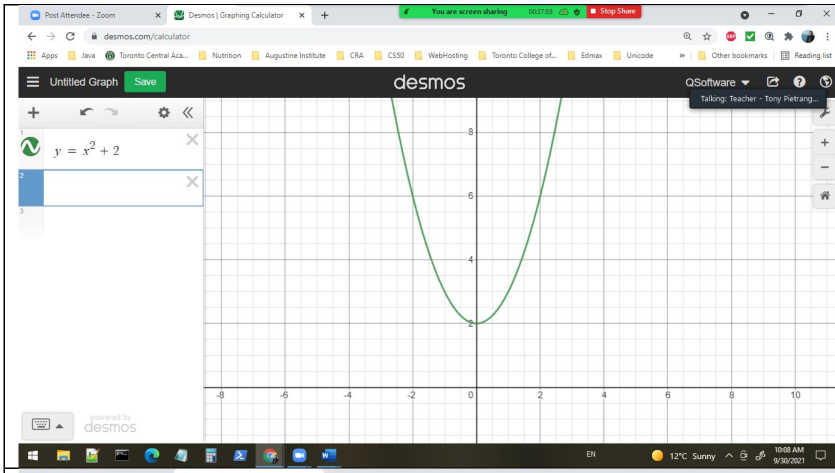
  The minimum point exists if the parabola: $y = x^2$ opens upwards, with a minimum point of P(x, y) = (0, 0), which is the vertex



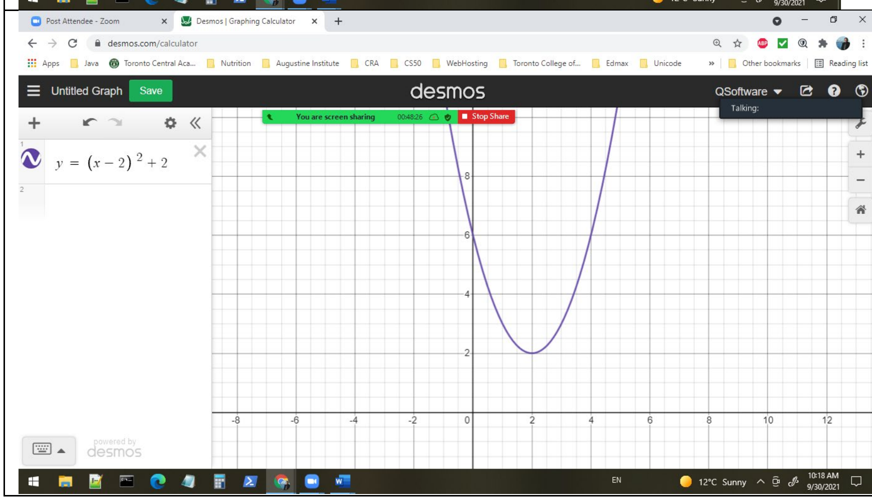
- Example 4: $y = x^2 + 2$
  The minimum point exists if the parabola: $y = x^2 + 2$ opens upwards, with a minimum point of P(x, y) = (0, 2), which is the vertex



Axis of Symmetry: - the line that divides a figure into two congruent (equal) parts. In the example above, the axis of Symmetry is x = 0, which is our y-axis, in this example.



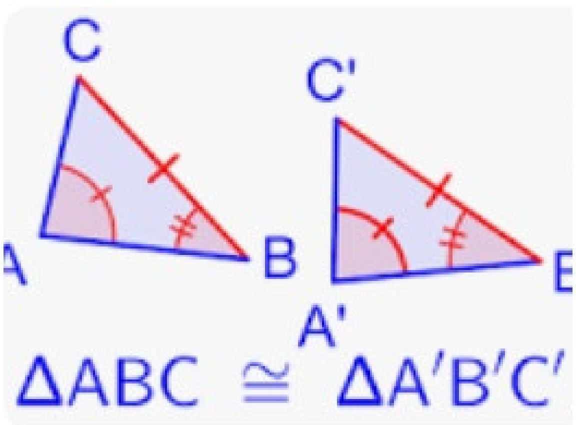
Axis of symmetry is x = 0. Here it happens to be the y-axis.



Axis of Symmetry is x = 2

congruent in geometry definition:- In geometry, two figures or objects are **congruent if they have the same shape and size**, or if one has the same shape and size as the mirror image of the other.

Example of two congruent triangles below:



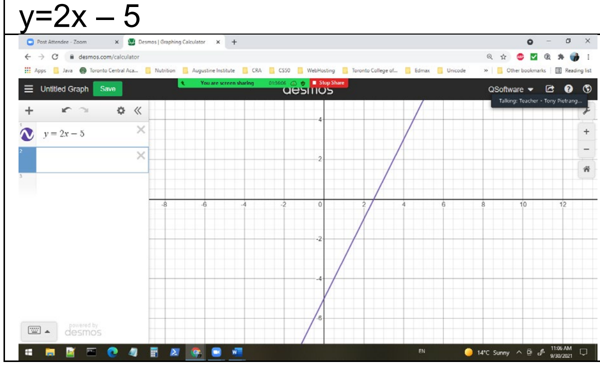
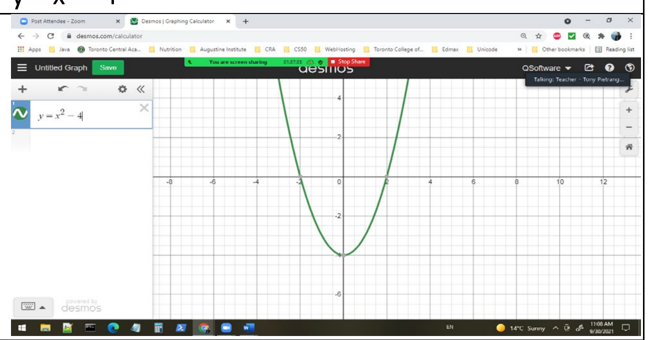
finite differences:- the differences found from the y-values in tables with evenly spaced x-values.

Example: y=2x – 5. If **first difference** is a constant, then the relationship is linear.

X Values	Y Values	First Difference Δy = y2 – y1	Second Difference change Δy
-2	y=2(-2) – 5 = -9	-7 – (-9) = 2	2 - 2 = 0
-1	y=2(-1) – 5 = -7	-5 – (-7) = 2	2 – 2 = 0
0	y=2(0) – 5 = -5	-3 – (-5) = 2	2 – 2 = 0
1	y=2(1) – 5 = -3	-1 – (-3) = 2	
2	y=2(2) – 5 = -1		

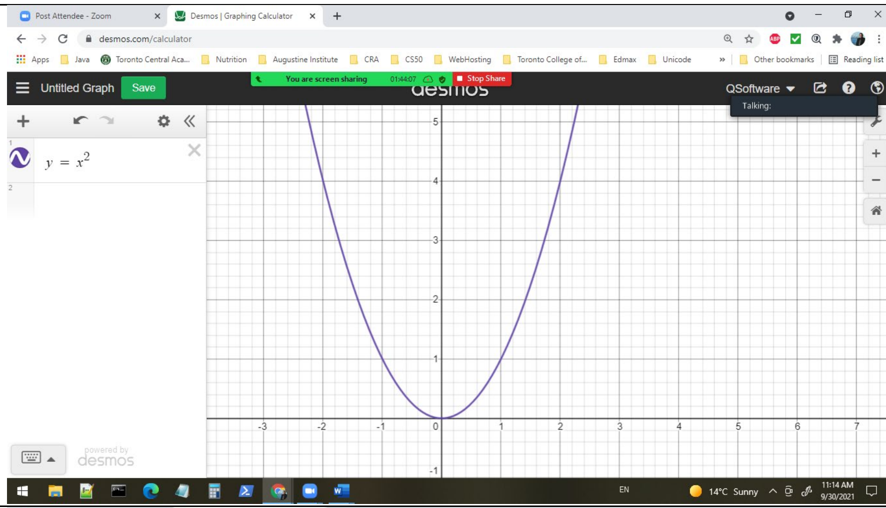
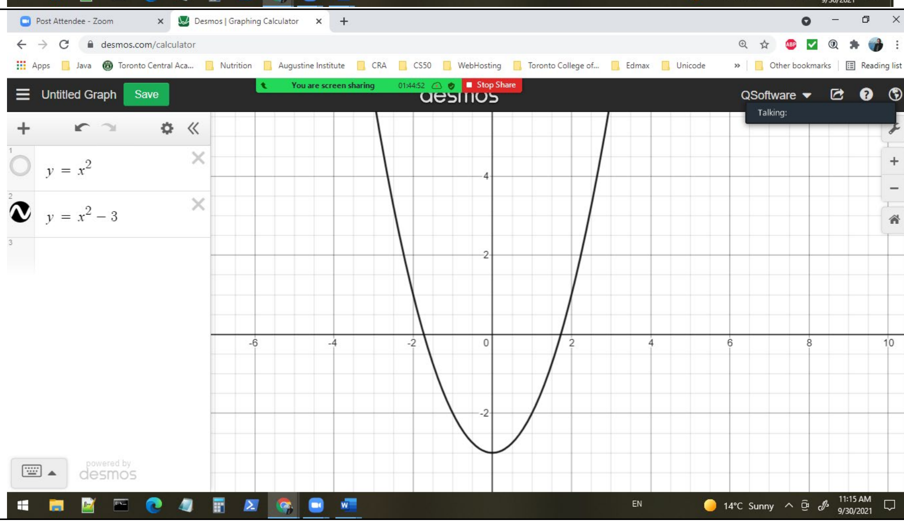
Example: $y = x^2 - 4$, for second difference.

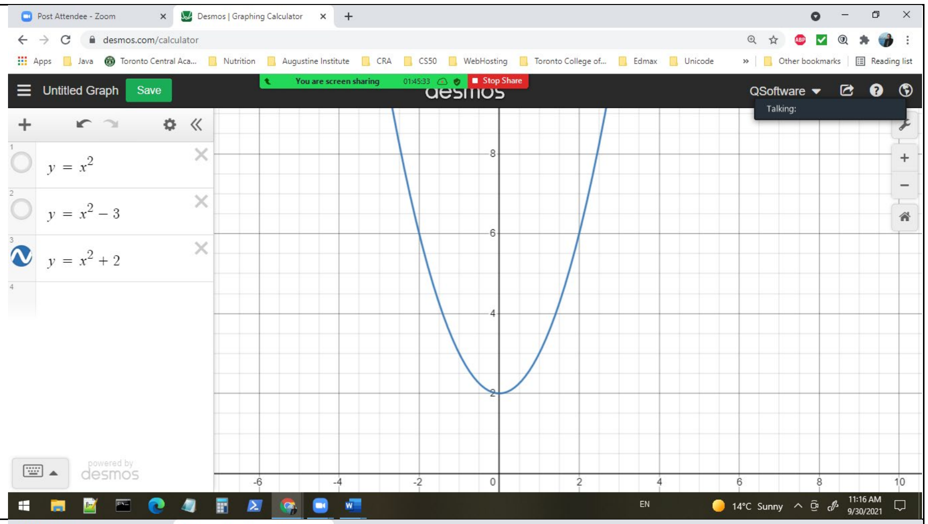
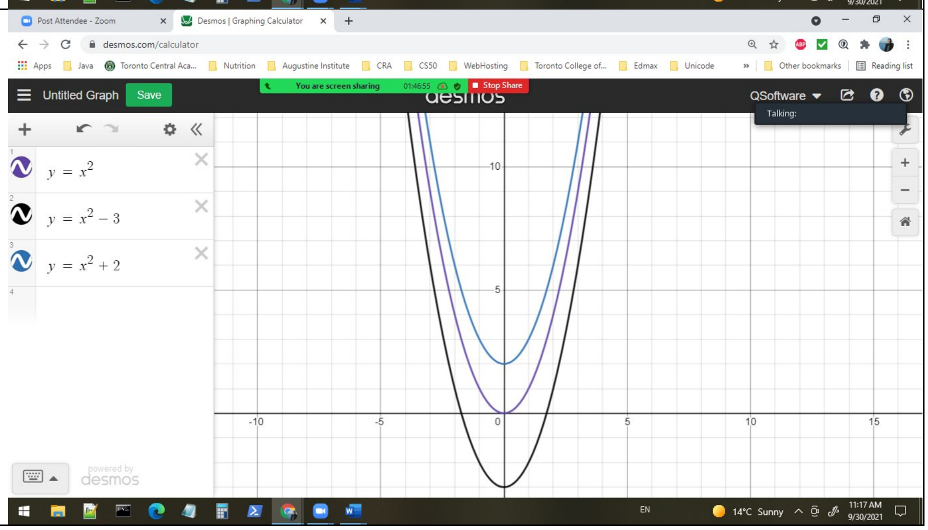
X Values	Y Values	First Difference Δy = y2 – y1	Second Difference change Δy
-2	$y=(-2)^2 - 4 = 0$	-3 - 0 = -3	-1 – (-3) = 2
-1	y=(-1)2 – 4 = -3	-4 – (-3) = -1	1 – (-1) = 2
0	$y=(0)^2 - 4 = -4$	-3 – (-4) = 1	3 – (1) = 2
1	$y=(1)^2 - 4 = -3$	0 – (-3) = 3	
2	$y=(2)^2 - 4 = 0$		

y=2x – 5	$y = x^2 - 4$
	

Investigate Transformations:

$y = x^2$ and transforming into $\mathbf{y = x^2 + k}$, where k is constant.

<table>
<tr><th>Formula and Transformation</th><th>Graph</th></tr>
<tr><td>$y = x^2$ where k = 0</td><td>
</td></tr>
<tr><td>$y = x^2 + k$, where k < 0; k = -3</td><td>
</td></tr>
</table>

<table>
<tr><td>$y = x^2 + k$,
where k > 0;
k = 2</td><td>
</td></tr>
<tr><td><u>All on same graph</u></td><td>
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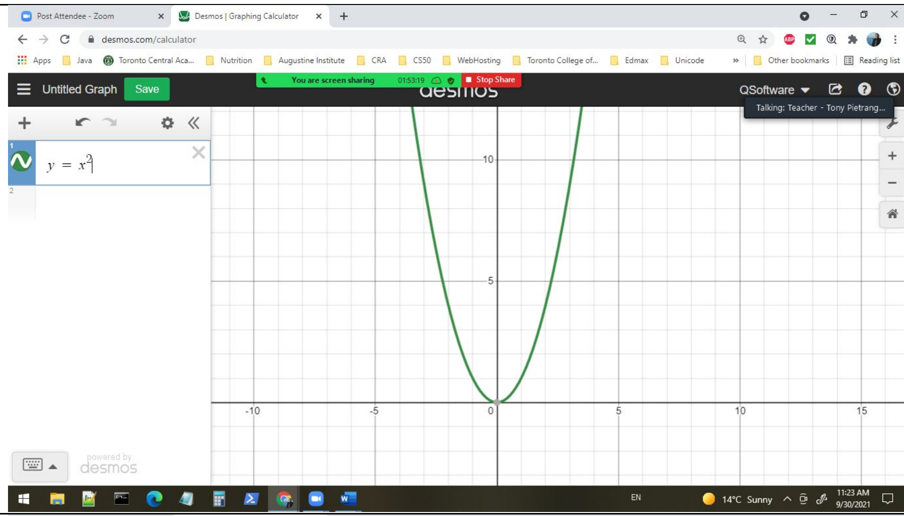
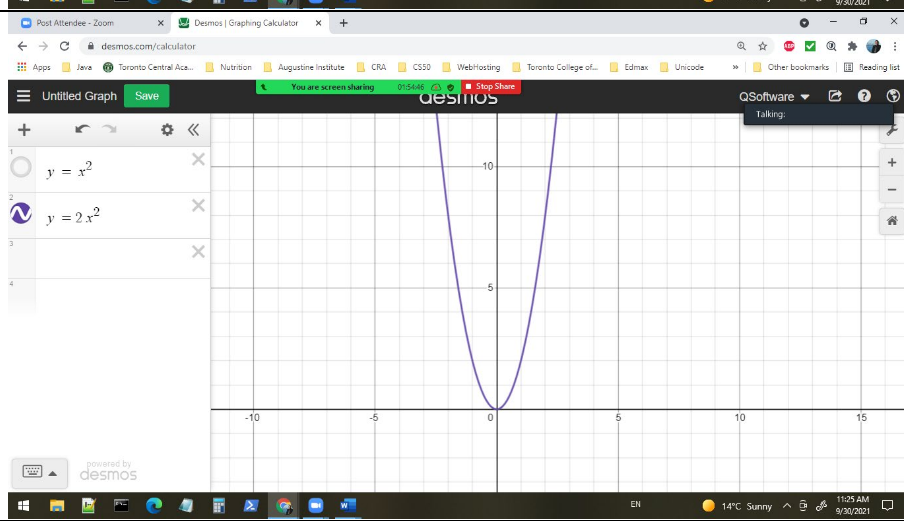
Investigate the coefficient variable (a) in front x^2, where a ≠ 0

Type 1: for a >= 1

$y = x^2$ and transforming into **$y = ax^2$**, where a is real number **and a >= 1**

<table>
<tr><th>Formula and Transformation</th><th>Graph</th></tr>
<tr><td>$y = ax^2$
where a = 1</td><td>
</td></tr>
<tr><td>$y = ax^2$,
where a = 2
$y = 2x^2$,</td><td>
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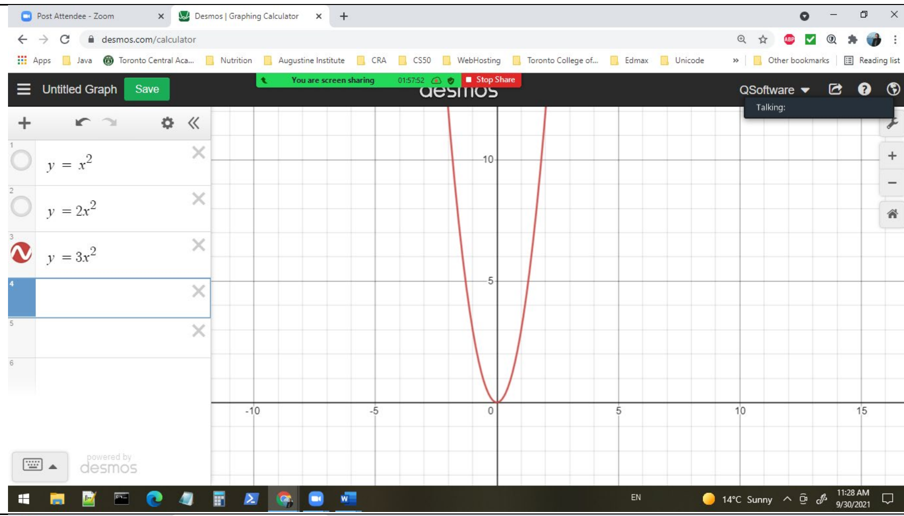
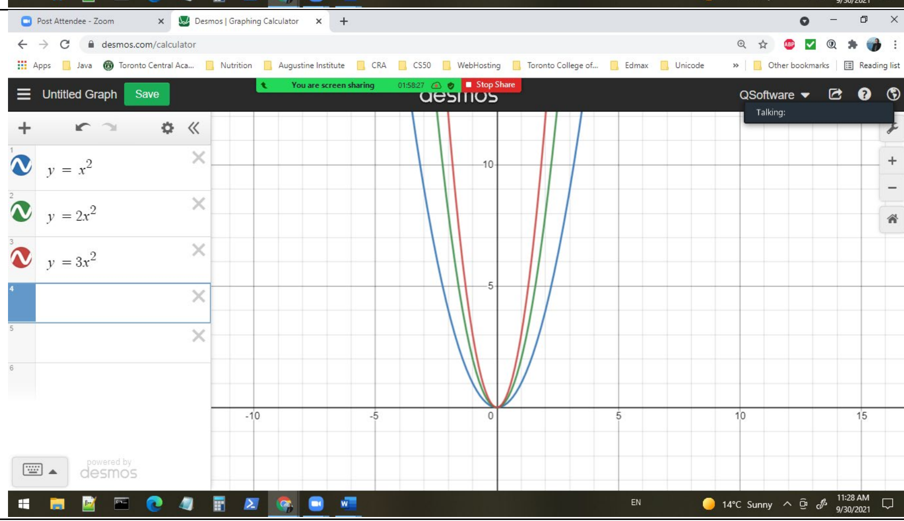
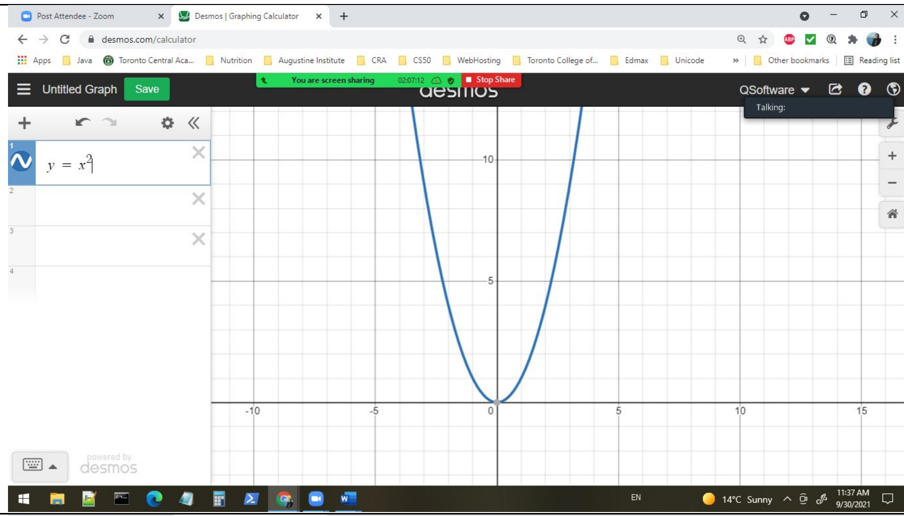
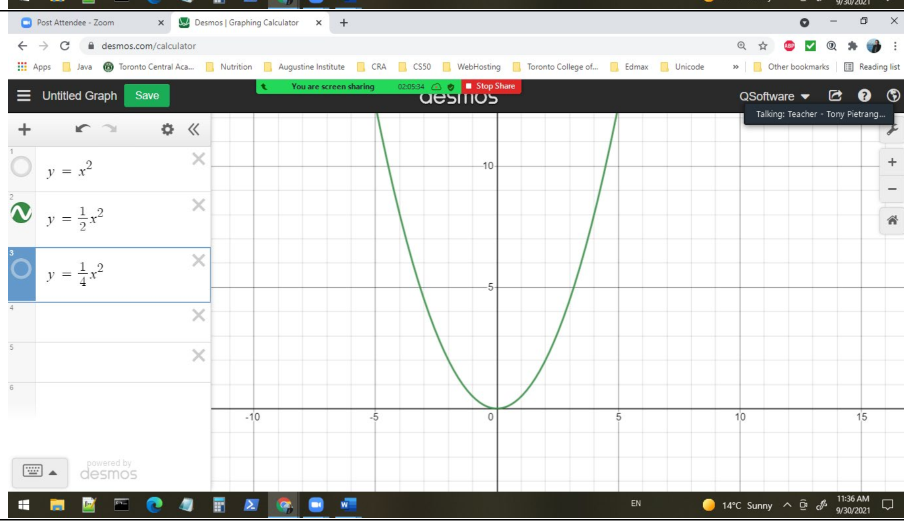
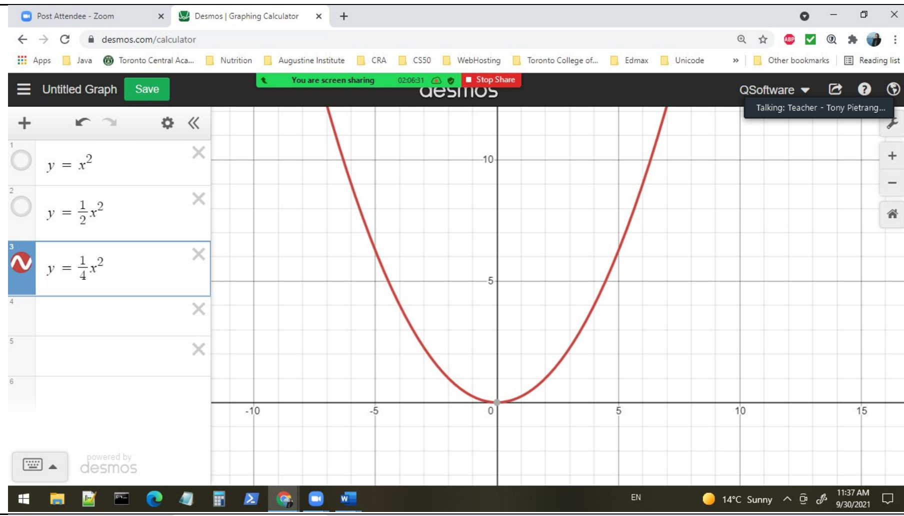
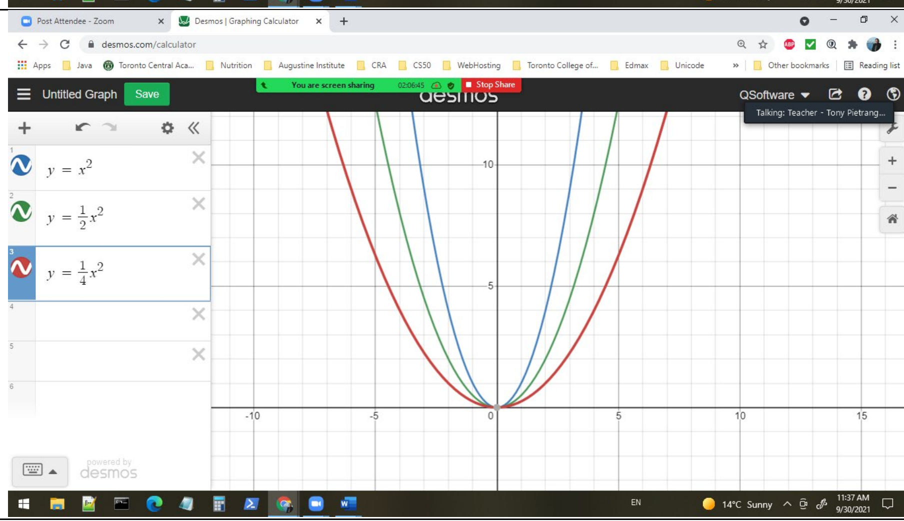
<table>
<tr>
<td>$y = ax^2$,
where a = 3
$y = 3x^2$,</td>
<td>
</td>
</tr>
<tr>
<td><u>All on same graph</u></td>
<td>
</td>
</tr>
</table>

Type 2: for 0 < a < 1; a must be a positive fraction

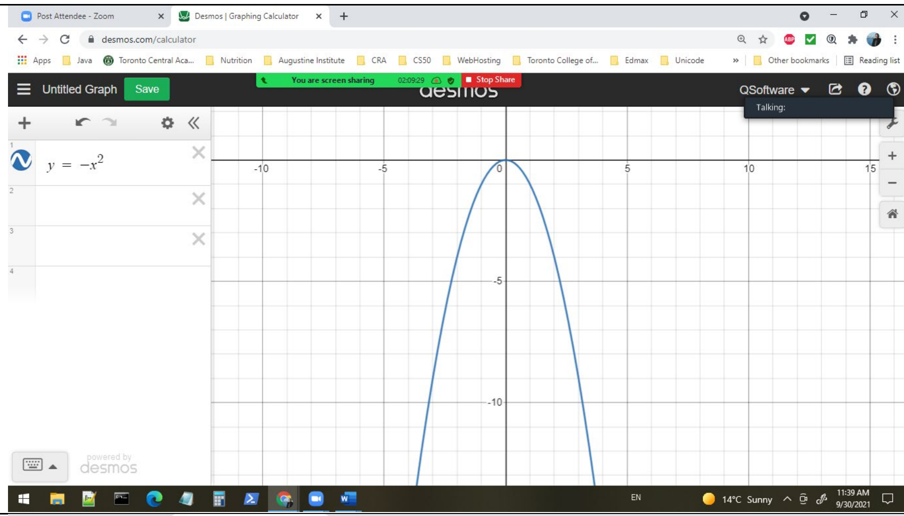
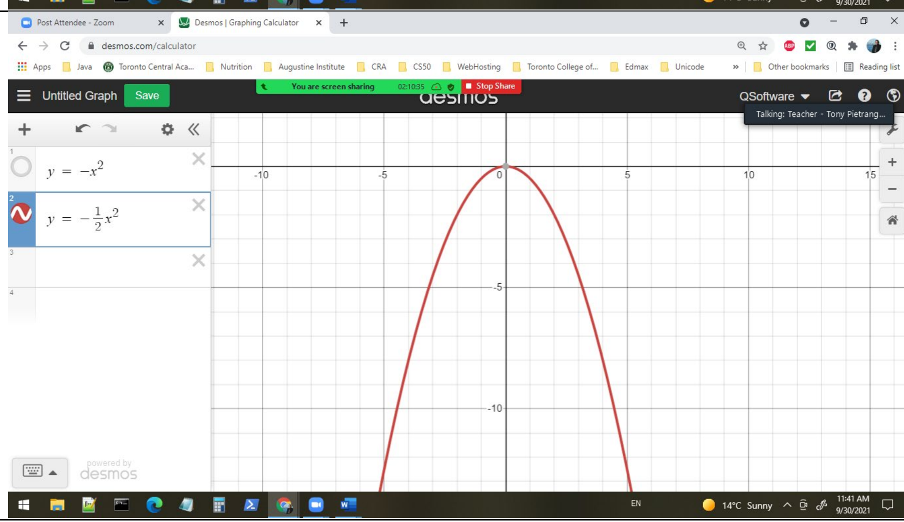
y = x² and transforming into **y = ax²**, where a is positive fraction

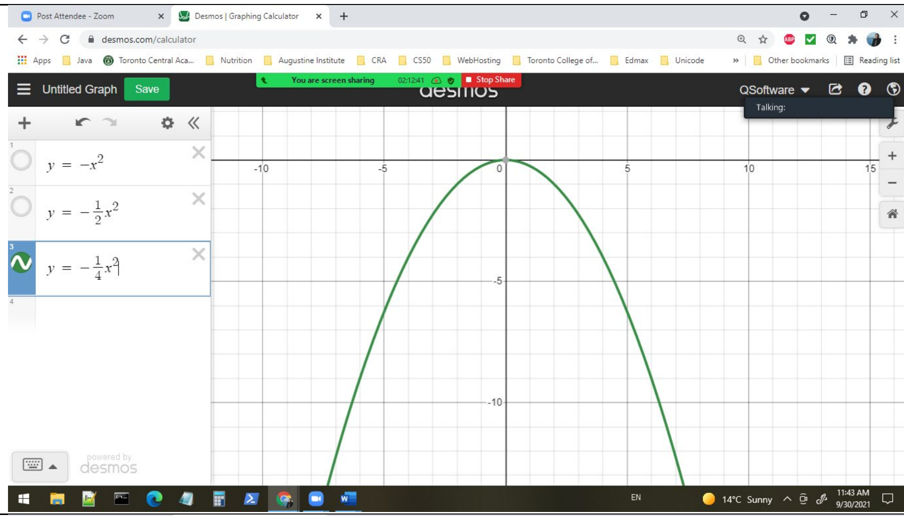
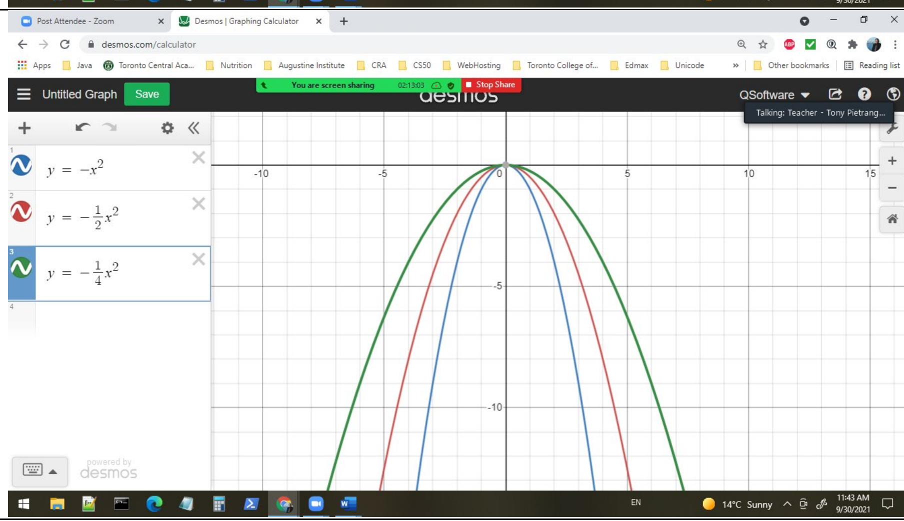
Formula and Transformation	**Graph**
y = ax² where a = 1	
y = ax², where a = $\frac{1}{2}$ **y =** $\frac{1}{2}$**x²**,	

$y = ax^2$, where $a = \frac{1}{4}$ **$y = \frac{1}{4}x^2$,**	
<u>All on same graph</u>	

Type 3: for -1 < a < 0; a must be a negative fraction

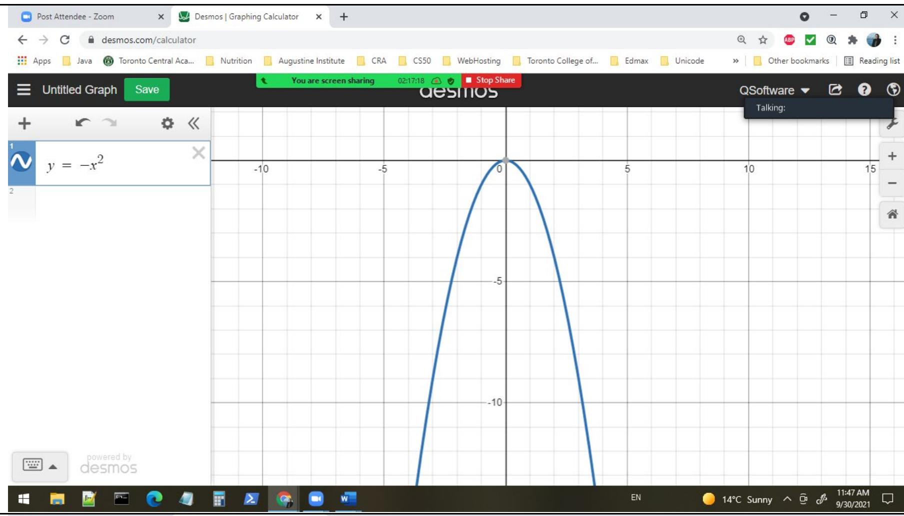
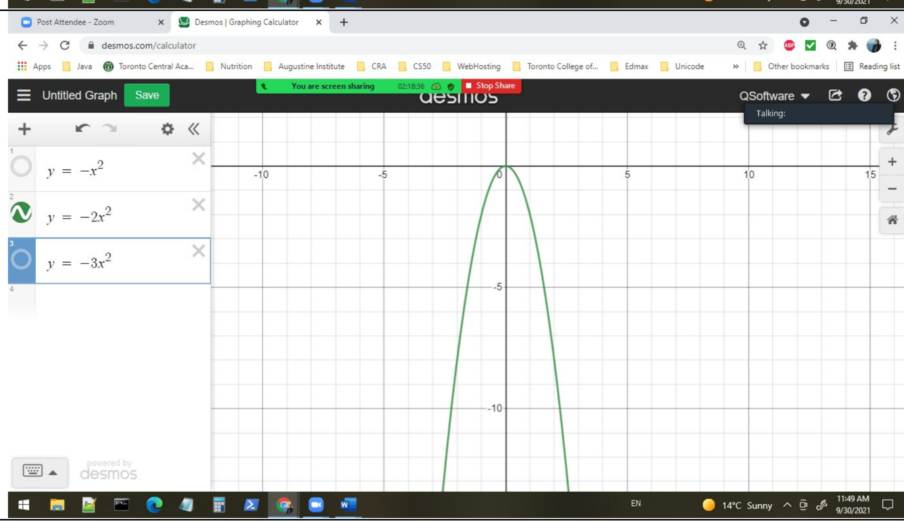
$y = x^2$ and transforming into **$y = ax^2$**, where a is negative fraction

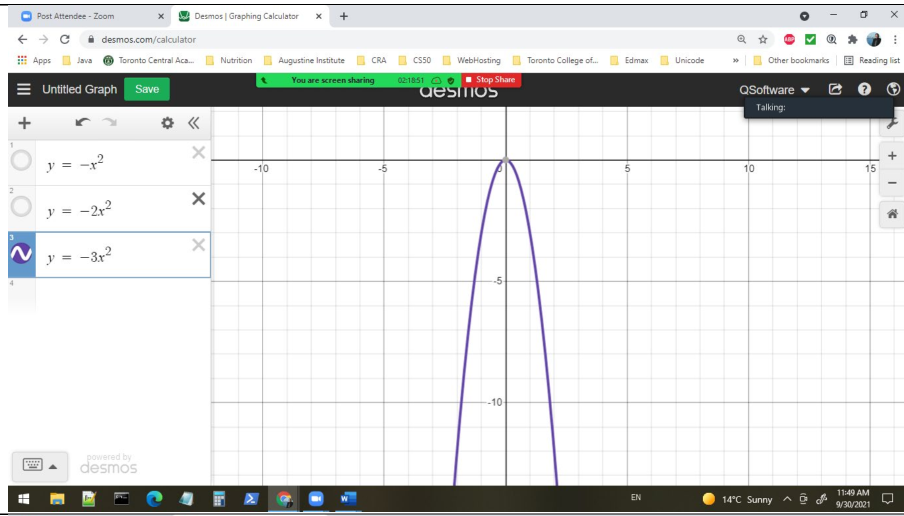
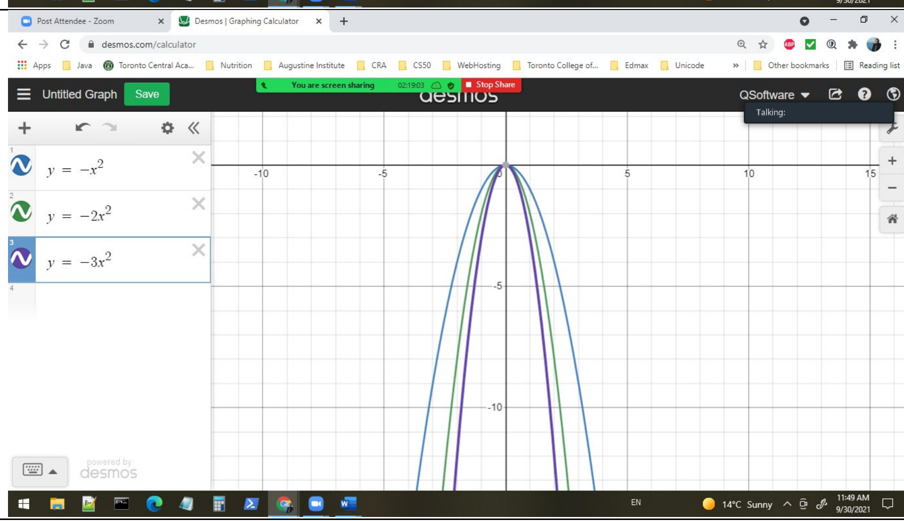
Formula and Transformation	**Graph**
$y = ax^2$ where a = -1	
$y = ax^2$, where $a = -\frac{1}{2}$ **$y = -\frac{1}{2}x^2$**,	

$y = ax^2$, where $a = -\frac{1}{4}$ **$y = -\frac{1}{4}x^2$**,,	
<u>All on same graph</u>	

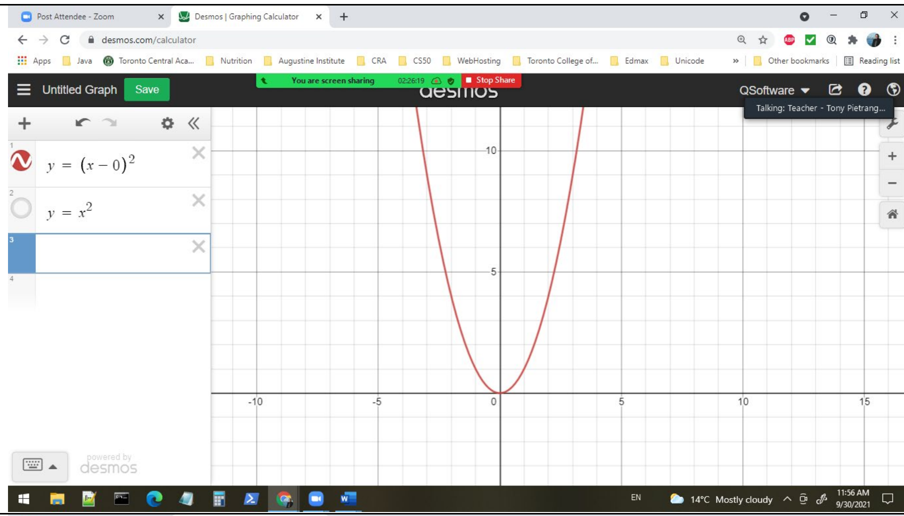
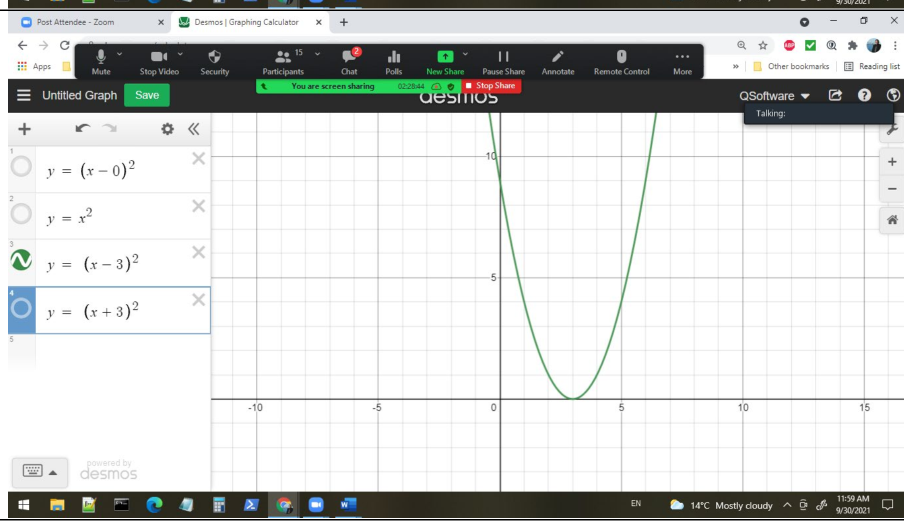
Type 4: for a <= -1

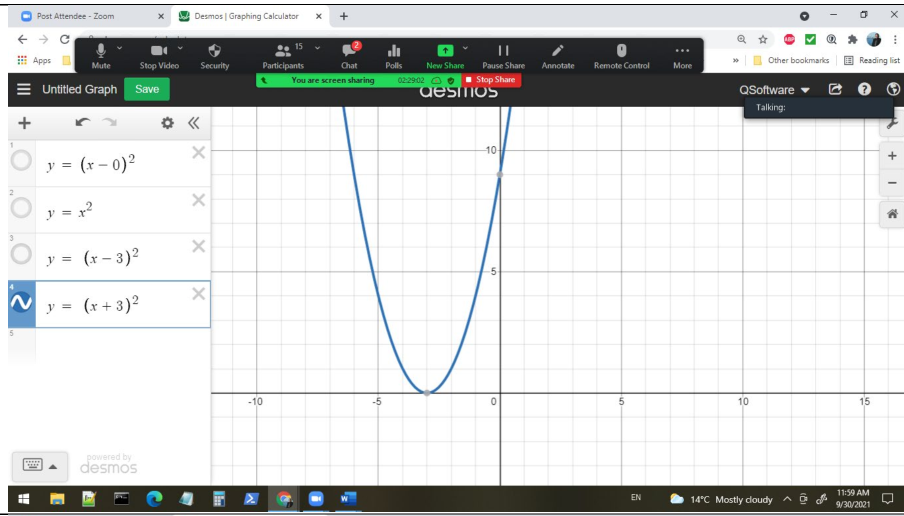
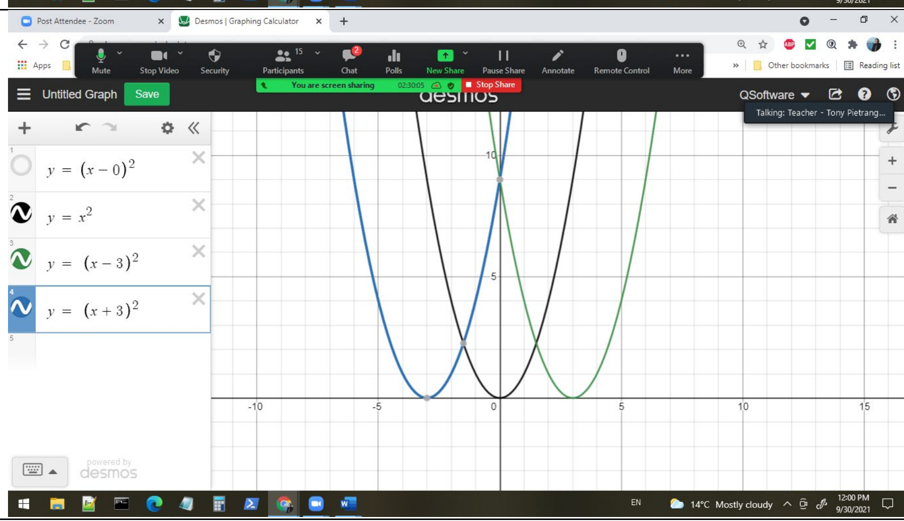
$y = x^2$ and transforming into **$y = ax^2$**, where a is real number **and a= -1**

Formula and Transformation	**Graph**
$y = ax^2$ where a = -1	
$y = ax^2$, where a = -2 **$y = -2x^2$**,	

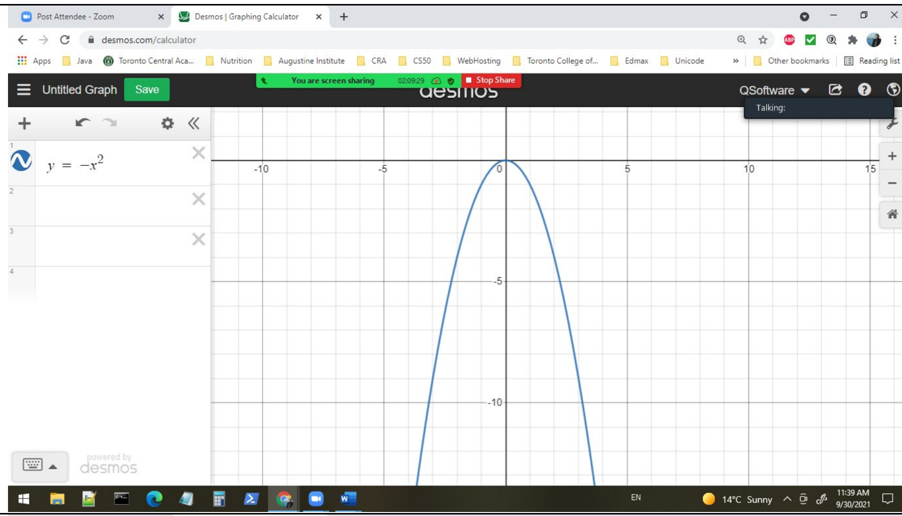
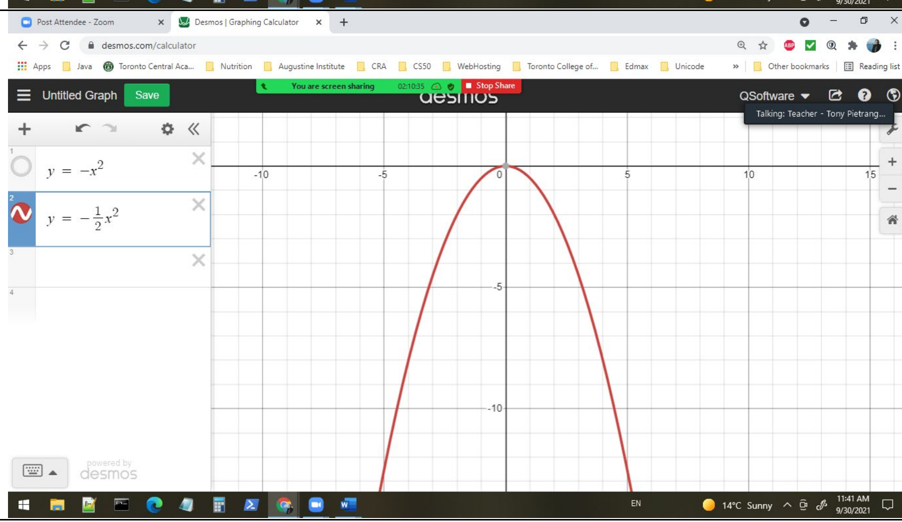
<table>
<tr>
<td>$y = ax^2$,
where a = -3
$y = -3x^2$,</td>
<td>
</td>
</tr>
<tr>
<td><u>All on same graph</u></td>
<td>
</td>
</tr>
</table>

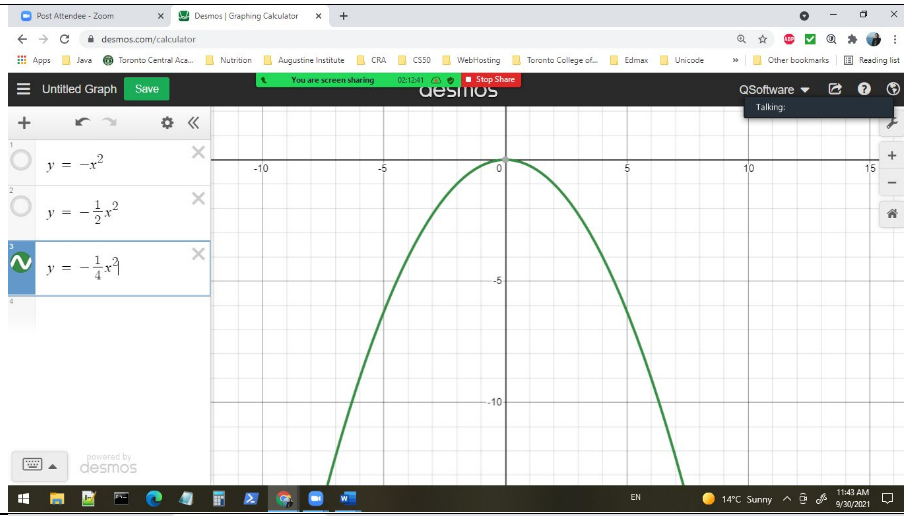
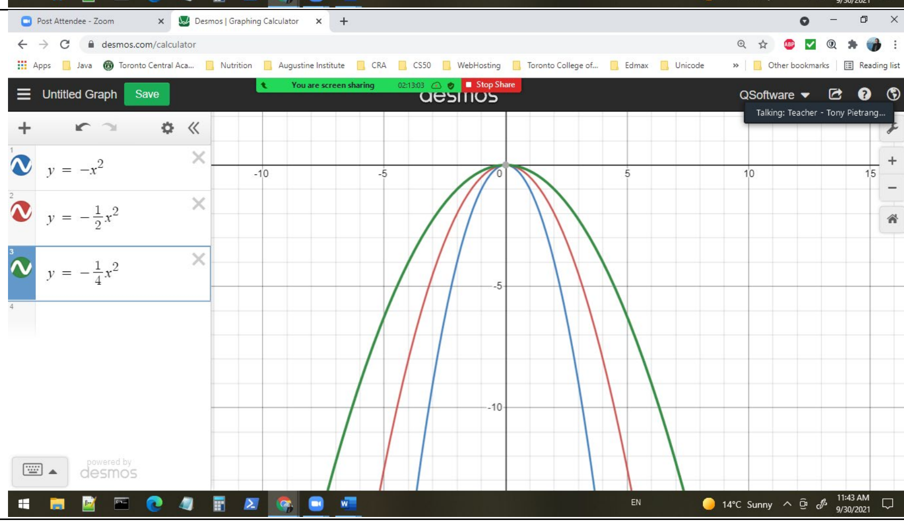
$y = x^2$ and transforming into **$y = (x – h)^2$.** where h is constant.

Formula and Transformation	**Graph**
$y = (x – h)^2$ where h = 0 **$y = x^2$**	
$y = (x – h)^2$ where h = 3 **$y = (x – 3)^2$**	

<u>$y = (x - h)^2$</u> where h = -3 **<u>$y = (x + 3)^2$</u>**	
<u>All on same graph</u>	

$y = x^2$ and transforming into **$y = ax^2$**, where a is positive fraction

Formula and Transformation	**Graph**
$y = ax^2$ where a = -1	
$y = ax^2$, where $a = -\frac{1}{2}$ **$y = -\frac{1}{2}x^2$**,	

$y = ax^2$, where $a = -\frac{1}{4}$ **$y = -\frac{1}{4}x^2$**,,	
<u>All on same graph</u>	

Activity 2 / Home: (Quadric Equations)

1. Create a table of points for the following equations (x values from -9 to 9)

a) $y = -3x^2$
b) $y = \frac{1}{4}x^2$
c) $y = -(\frac{1}{4})x^2$

Graph the following equations

1. **Create a table of points for the following equations (x values from -9 to 9)**
   a. $Y = x^2$
   b. $Y = (x - 9)^2$
   c. $Y = (x + 2)^2$
   d. $Y = (x - 5)^2$

2. **Create a table of points for the following equations (x values from -9 to 9)**
   a. $Y = x^2 + 8$
   b. $Y = x^2 - 5$
   c. $Y = x^2 - 10$

Date: Friday, October 1st, 2021
Course: MPM2D - Principles of Mathematics

Yesterday:
$y = a(x - h)^2 + k$,
we look the transformations what happens to the original parent equation of: $y = x^2$

What k does, with specific values: { k: -3, -2, -1, 1, 2, 3 }
What h does, with specifc values: { h: in your activity }
What a does, with specific values: { a: -3, -2, -1, -1/2, -1/4, 1, 1/2, 1/4, 2, 3

Goal:
Summarize all of the tranformations of:
$y = a(x - h)^2 + k$

Add also:
Transformations of variable h:
h shifts the graph, or parabola to the left or right.

if $h > 0$, then $y = (x - h)^2$, shifts to the right.
if $h < 0$, then $y = (x - h)^2$, shifts to the left.

Create a summary table:

Activity 3:

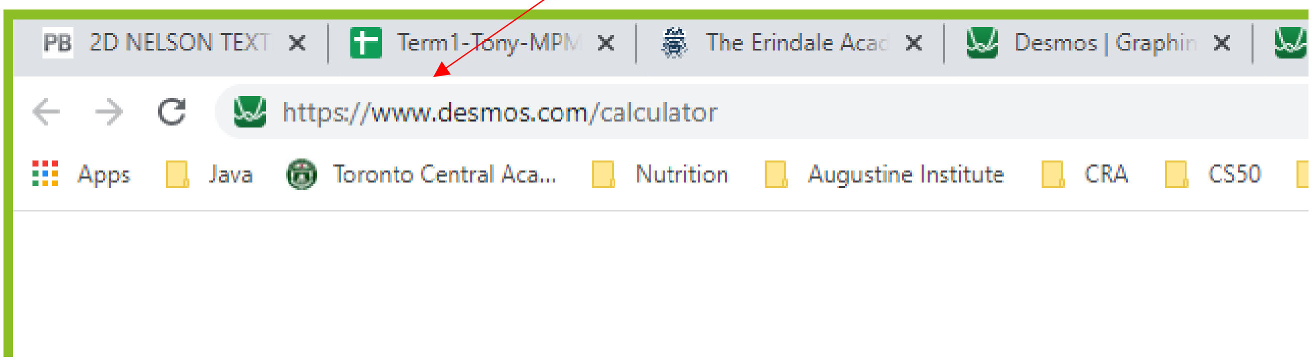
Set up Demos Graphing Software to handle all the 3 variables, a, h, k in formula:

$y = a(x - h)^2 + k$

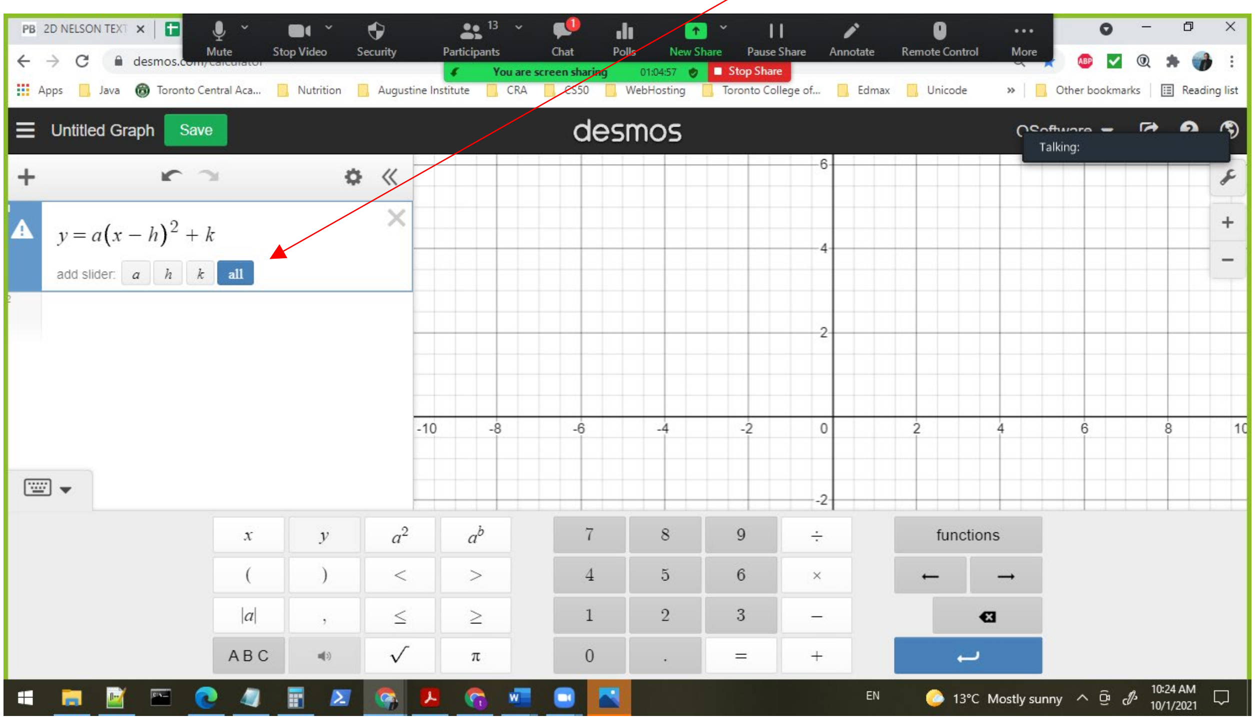
Step 1: launch a browser, instructor uses google chrome.

Step 2: type in url https://www.desmos.com/calculator

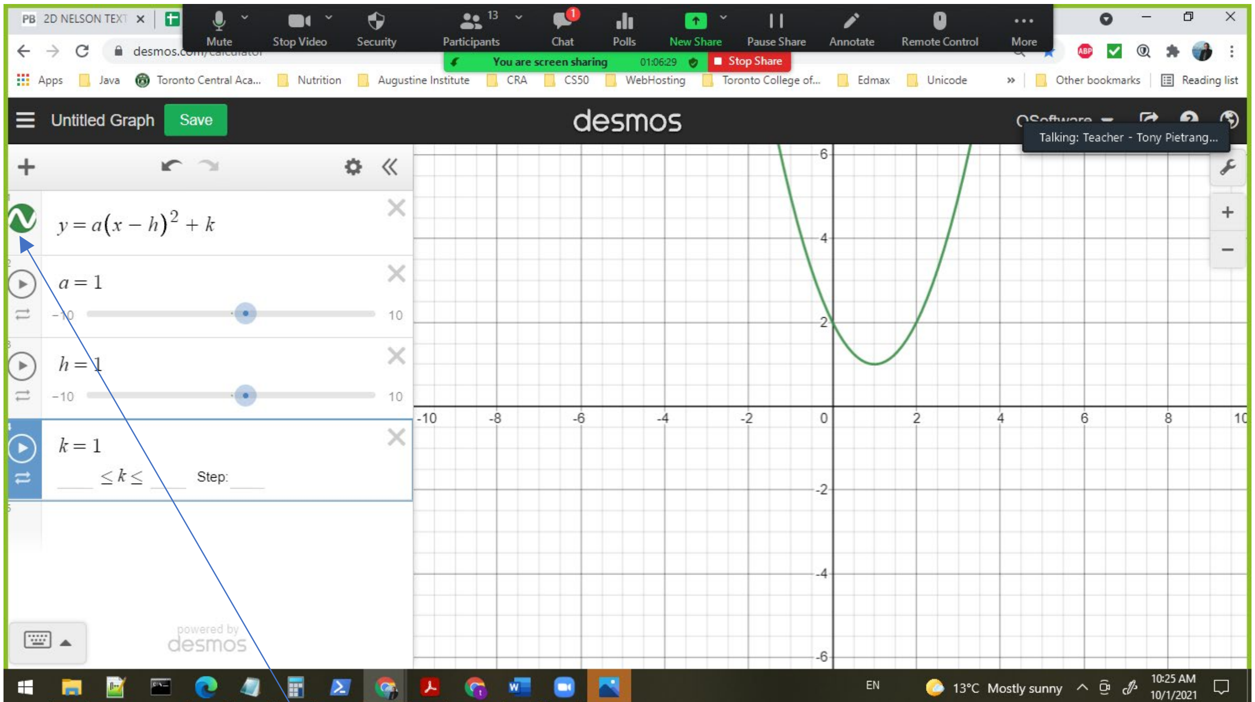
c



Step 3: type in formula: $y = a(x - h)^2 + k$, click on add all sliders



Step 4: The following slides should be present in your equation transformations:



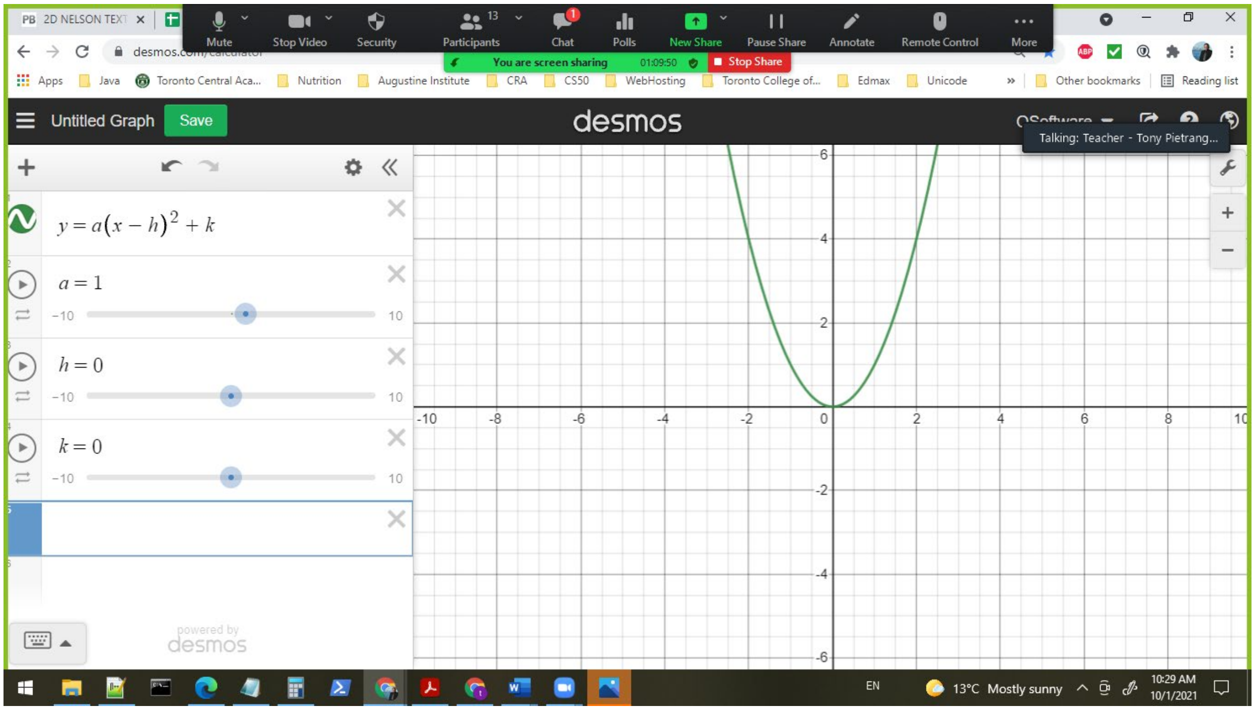
Actions:

1. Change colour to green, by holding cursor over the icon, and select green.
2. Step k = 0, h = 0.

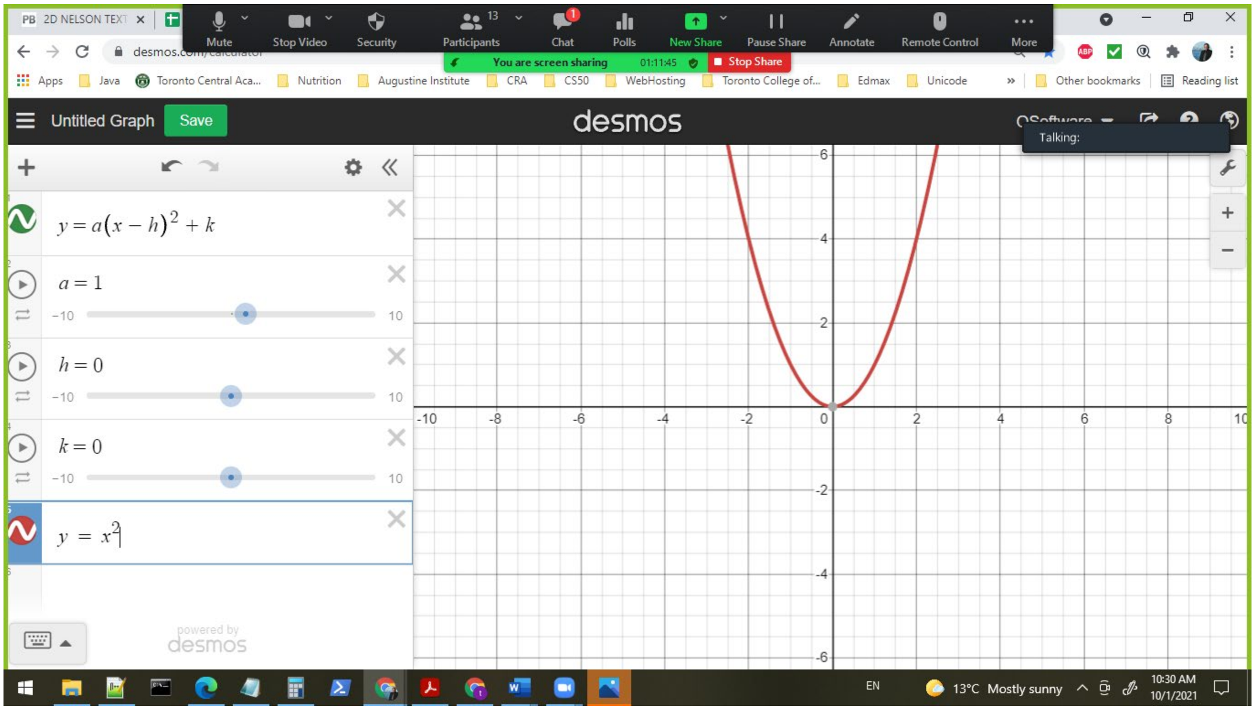
If done correctly, you will see the graph as below:

Results:

You should see Demos graphing software as below:



Step 5: Add parent equation y = x^2 and change colour of parabola to red.



Result of action: y = x^2 should hid, or superimpose itself on top of the green parabola.

$y = a(x - h)^2$ $+ k$

Substitute k = 2, h = 0, and a = 0.

What is the value for y?

$Y = (0)(x - 0)^2 + 2$

Y = 2; 🡸 what does this on an x, y-axis? This means for all the x values, the value y is always 2.

$P(x_{1,}y_1) = (-5, 2)$
$P(x_{2,}y_2) = (-4, 2)$
and so on all the values of x.

The x-axis is when y = 0

$y = a(x - h)^2$ $+ k$

substitute when h = 8, a = 1, k = 0

$y = 1(x - 8)^2$
$y = (x - 8)^2$

Activity 3: Set up Demos Graphing Software to handle all the 3 variables, a, h, k in formula:

Explain in English what are the effects of a, h, k to the formula. Take screen snap shots through our explanation to prove the effects the changing variables.

$y = a(x - h)^2 + k$

If k > 0, the original graph is vertical shift upwards.

If k < 0, the original graph has vertical shift, but downwards.

if h > 0, the original graph has a horizonal shift to the right.

If h < 0, the original graph has a horizonal shift to the left.

Please explain the effects of a under the conditions below:

Type 1: when a >= 1
Type 2: when 0 < a < 1
Type 3: when a <= -1
Type f: when -1 < a < 0;

Date: Monday, October 4th, 2021
Course: MPM2D – Principles of Mathematics.

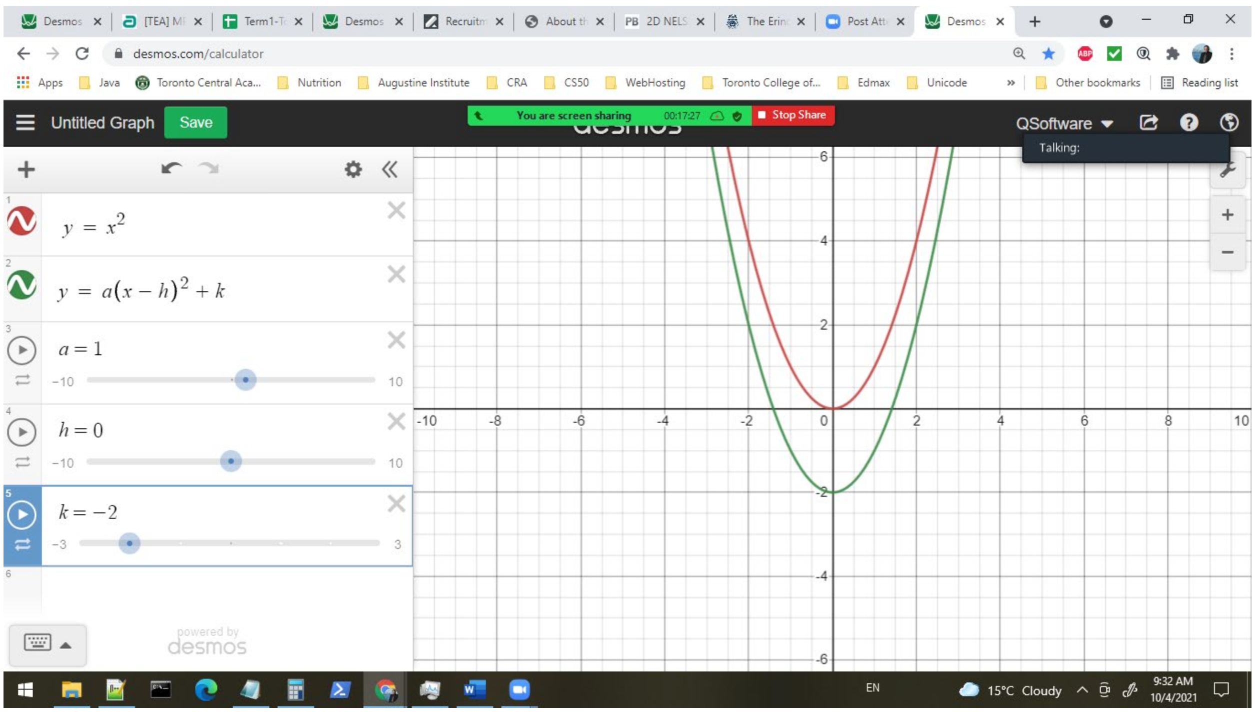
Create a summary table for formula:

$y = a(x - h)^2 + k$

Place constants: a = 1, h = 0, k = 0,

$y = 1(x - 0)^2 + 0$
$y = (x)^2 = x^2$

see the effects of k = 3, 2, 1, 0, -1, -2, -3



If k > 0, the transformation is upwards.

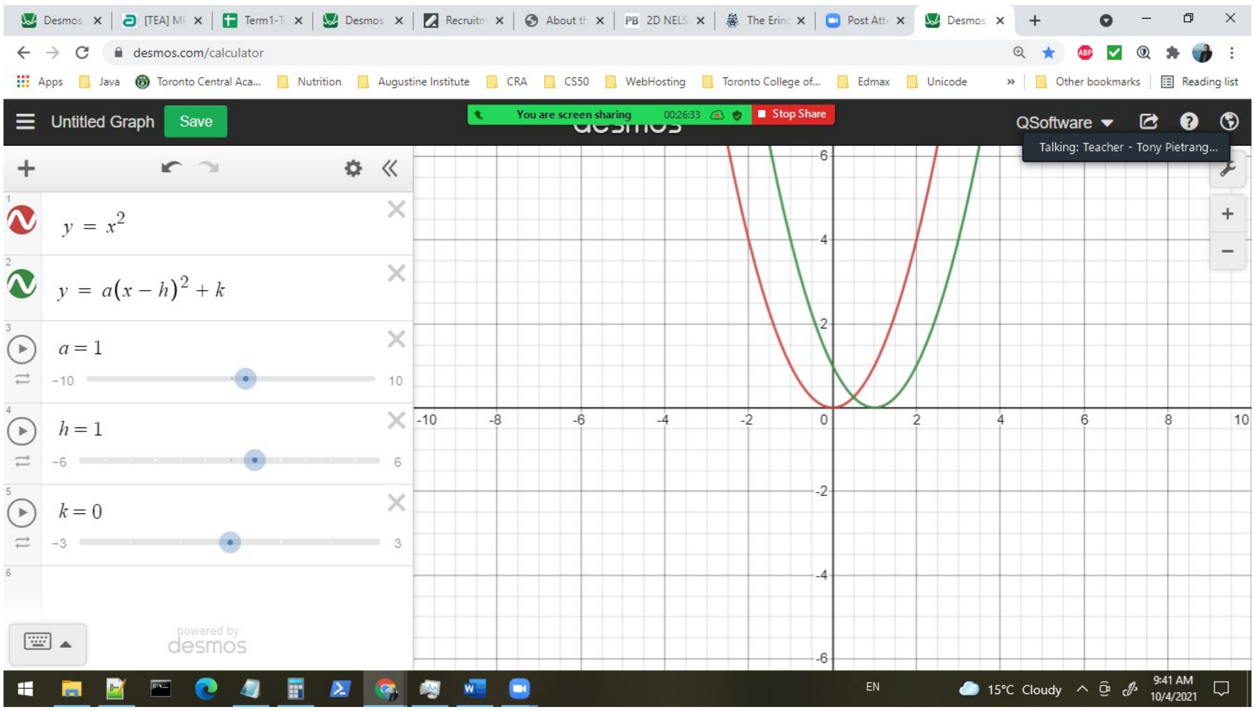
If k < 0, the transformation is downwards.

Step 2: Analyze the effects of variable h; set a = 1, k = 0.

$y = a(x - h)^2 + k$

$y = 1(x - h)^2 + 0;$
$y = (x - h)^2$

The effects of h, where h is a set of values: $-6 <= h <= 6$



The effect of when h is 1, it shifts the original function $y = x^2$ one position to the right. This is a horizontal transformation.

What is the x-intercept or zeros of the parabola?

The x-intercept is when y = 0
$y = (x - 1)^2$
$0 = (x - 1)^2$
$\sqrt{0} = \sqrt{(x - 1)^2}$

$0 = x - 1$
$-x = -x + x - 1$
$x = 1$

∴ x = 1, y = 0; at point (x, y) = (1, 0)

$(x - 1)(x - 1) = (x - 1)^2$

Later, we will analyze the parabolas of the form $y = a(x - r)(x - s)$

Conclusion: h does a horizontal transformation.

If $h > 0$, it shifts to the right.

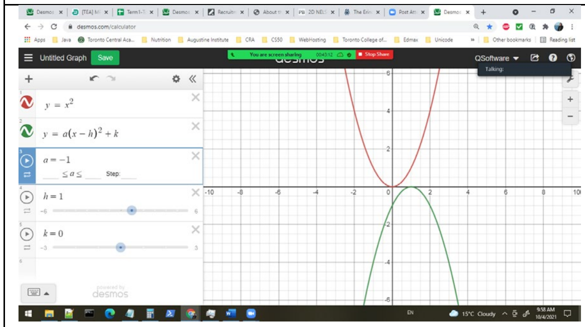
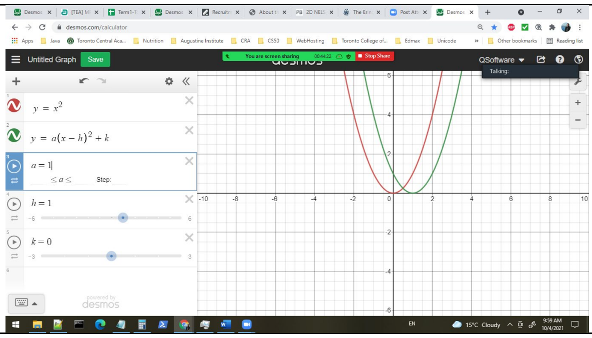
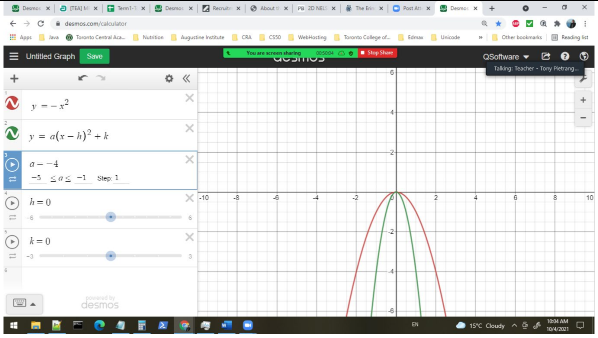
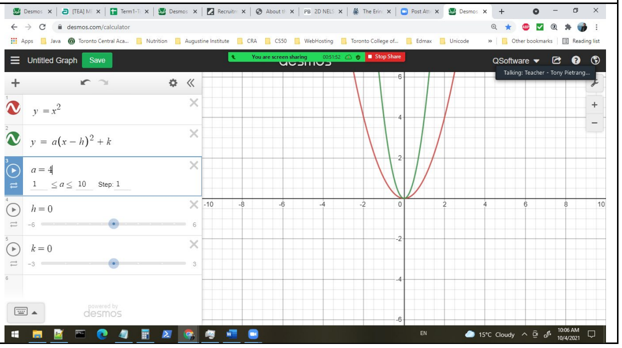
if $h < 0$, it shifts to the left.

Step 3: Analyze the effects of variable a; set h = 0, and k = 0.

Note: $a \neq 0$

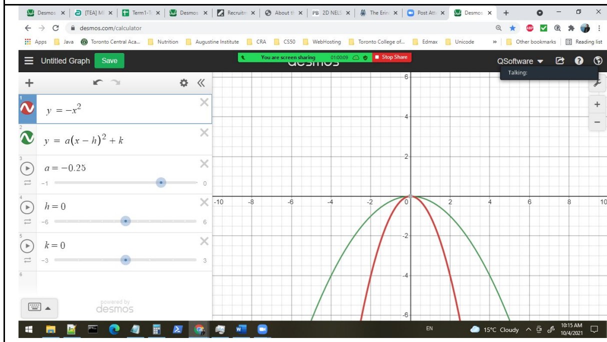
Four types of effects for a.
If $a > 0$, the parabola opens upwards. This means the graph has a minimum.

If $a < 0$, the parabola opens downwards. This means the graph as a maximum.

$a < 0$; the graph opens downwards. We have a maximum	$a > 0$; the graph opens upwards We have a minimum
	
$a < -1$; the graph opens downwards. We have a maximum It stretches the graph negatively, or y values are decreasing at much faster than $y = x^2$	$a > 1$; the graph opens upwards We have a minimum. It stretches the graph positively, or y values are increasing at much faster than $y = x^2$
	

<table>
<tr>
<td>-1 < a < 0; the graph opens downwards.
We have a maximum

It compresses or flattens the graph compared to $y = x^2$</td>
<td>0 > a > 1 ; the graph opens upwards
We have a minimum.

It compresses or flattens the graph compared to $y = x^2$</td>
</tr>
<tr>
<td>
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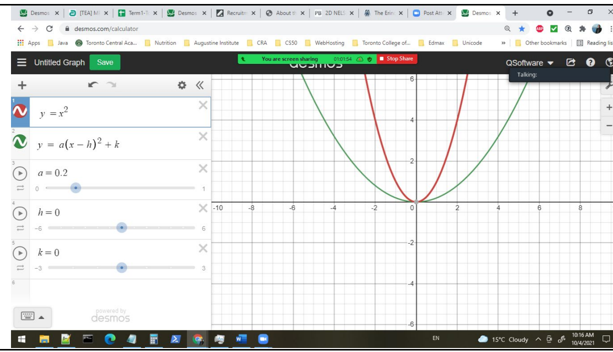
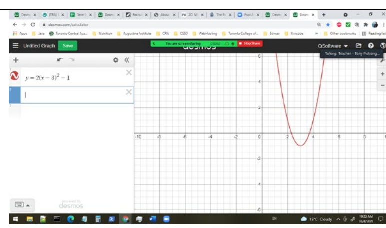
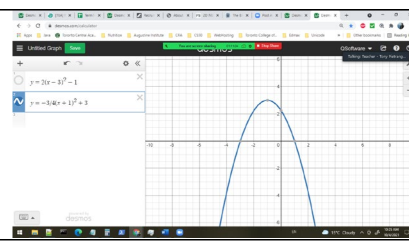
<td>
</td>
</tr>
</table>

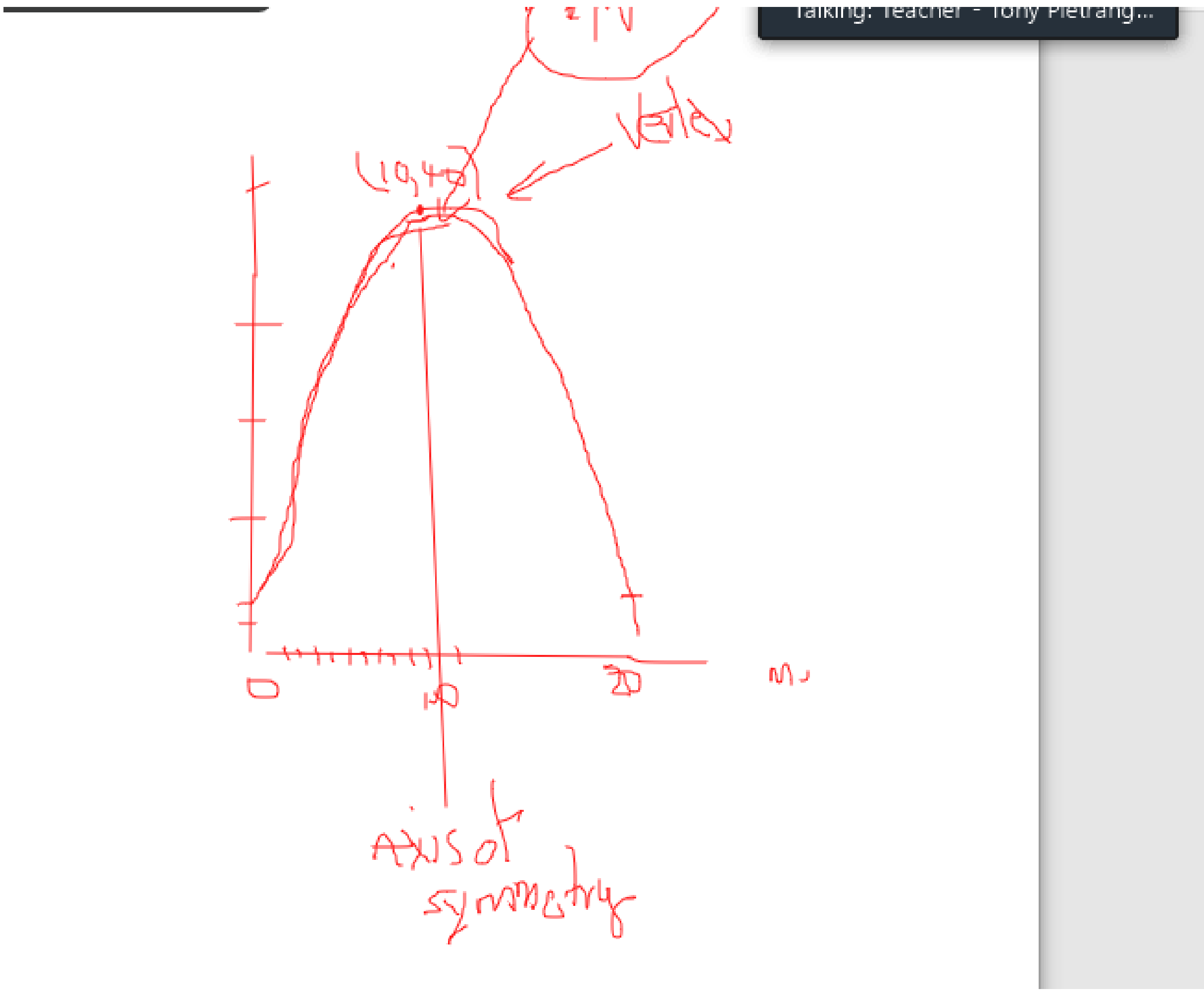
Key Concepts:

Property	$y = a(x-h)^2 + k$	$y = 2(x-3)^2 - 1$	$y = -\frac{3}{4}(x+1)^2 + 3$
Vertex	(h, k)	(3, -1)	(-1, 3)
Axis of Symmetry	x = h	x = 3	X = -1
Stretch or compression factor relative to $y = x^2$	a	2	$-\frac{3}{4}$
Direction of opening	If a > 0, the parabola has a minimum. It opens upwards and the vertex is the minimum. If a < 0, it opens downwards. it has a maximum and the vertex is the maximum	Opens upwards, and the vertex is at (3, -1). | The minimum is at the vertex, which is (3, -1).	Opens downwards, and the vertex is at (-1, 3) The maximum is at the vertex, which is (-1, 3).
Graph	General equation of a parabola		
Values x may take (Domain)	Any real numbers	Set of real numbers.	Set of real numbers.
Values y may take (Range)	If a > 0, then $y \geq k$ If a < 0, then $y \leq k$	$y \geq -1$	$y \leq 3$

Activity 4: Parabolas:

1. Describe the properties of the parabola with the equation of: $y = 2(x - 4)^2 - 3$.
2. Word Problem:
   At a fireworks display, a firework was launched from a height of 2 meters above the ground and reaches a maximum height of 40 meters at a horizontal distance of 10 meters.
   a) Determine an equation to model the flight path of the firework.
   b) The firework continues to travel an additional 1-meter horizontality, after it reaches it maximum height, before it explodes. What is its height when it explodes?
   c) At what other horizontal distance is the firework at the same height as in part b).

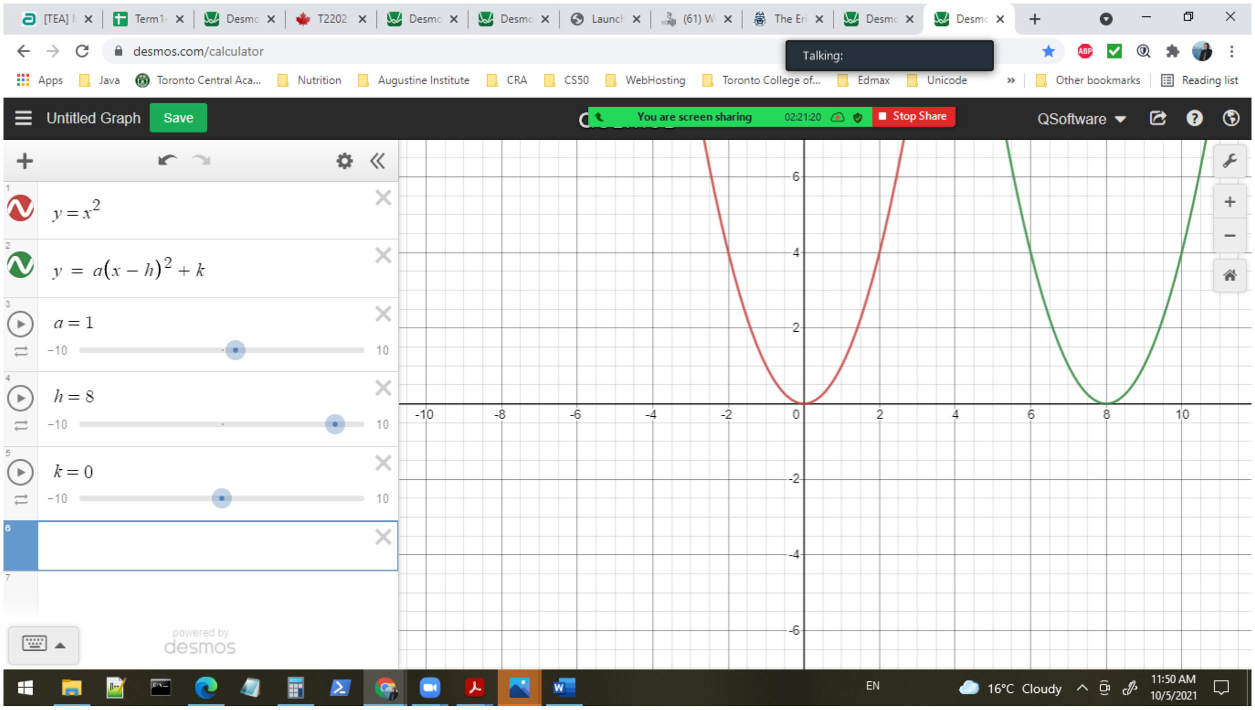
See sketch below for a rough drawing:



Date: Tuesday, October 5th, 2021

Course: MPM2D – Principles of Mathematics

Review:



Question 1: What is the vertex of the parabola in green?

Question 2: Is the vertex a maximum or minimum?

Question 3: What is the equation of the parabola in green?

Answer These questions:

Question 4: How to find the vertex of a parabola in the form of: $y = a(x - h)^2 + k$

Question 5: What is the axis of symmetry for a parabola of the form: $y = a(x - h)^2 + k$

Goal:

Analyze the form of parabola:

$y = a(x - r)(x - s)$

Activity 5/Home work:

Examples:

$y = 2(x - 5)(x + 3)$ where x is a set of integers: -6 to +6.

X value	Y value y=2(x – 5)(x + 3)	Point (x, y)
-6	y=2(-6 – 5)(-6 + 3) y=2(-11)(-3) = 66	P_1 (-6, 66)
-5	y=2(-5 – 5)(-5 + 3) y=2(-10)(-2) = 40	P_2 (-5, 40)
-4	y=2(-4 – 5)(-4 + 3)	
-3		
-2		
-1		
0		
1		
2		
3		
4		
5		
6		

Create a table of points for the parabola below.

$y = -(\frac{1}{2})(x + 2)(x - 6)$

1. Create a table of points for both parabolas
2. Plot all the points on a graph using Demos Graphing Software.
3. Add the equation of the line.
4. State the vertex of each equation.

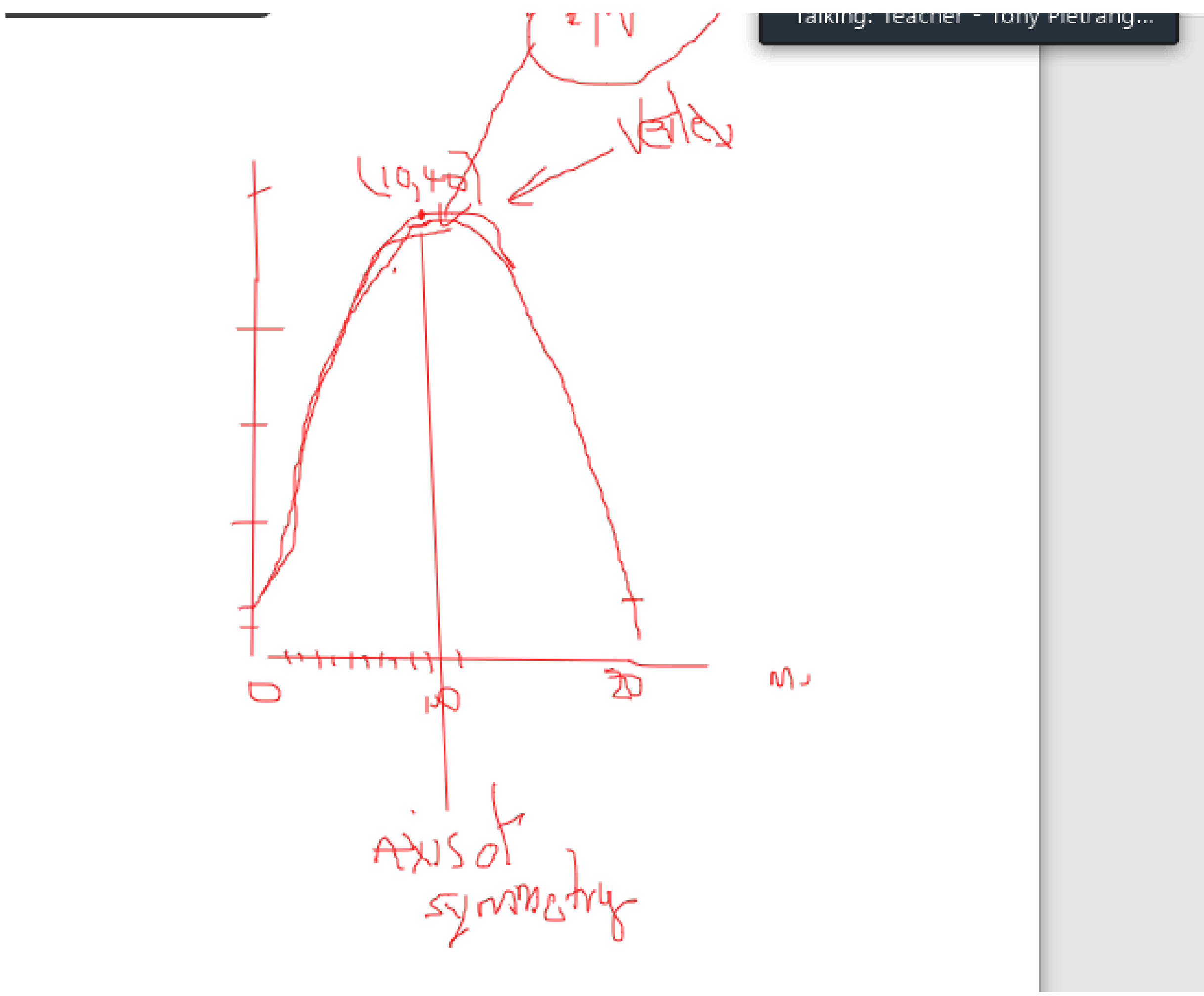
Date: Wednesday, October 6th, 2021
Course: MPM2D – Principles of Mathematics.

Solve Activity 4:

Activity 4: Parabolas:

3. Describe the properties of the parabola with the equation of: $y = 2(x - 4)^2 - 3$.
4. Word Problem:
   At a fireworks display, a firework was launched from a height of 2 meters above the ground and reaches a maximum height of 40 meters at a horizontal distance of 10 meters.
   a) Determine an equation to model the flight path of the firework.
   b) The firework continues to travel an additional 1-meter horizontality, after it reaches it maximum height, before it explodes. What is its height when it explodes?
   c) At what other horizontal distance is the firework at the same height as in part b).

See sketch below for a rough drawing:



Vertex (x, y) = (10, 40)

$y = a(x - h)^2 + k$

h is the horizontal distance in meters the fireworks will travel horizontally.
k is the vertical distance in meters the fireworks will travel vertically.

given: k = 40 meters.
h = 10 meters

The launch pad, which is the platform, where the fireworks will be propelled from. It is at a height of 2 meters, and the horizontal position is zero.

Given:
Launch pad position (x, y) = (0, 2)

Substitute the point for the vertex into the general form of the equation.

Vertex in the general form is: vertex (x, y) = point (h, k), and it opens downwards, so we know a is negative.

$y = a(x - 10)^2 + 40$

substitute the position of the launch pad (0, 2)

$y = a(x - 10)^2 + 40$
$2 = a(0 - 10)^2 + 40$

Solve for a:

$2 = a(0 - 10)^2 + 40$
$2 = 100\,a + 40$

$a = \frac{2 - 40}{100} = \frac{-38}{100} = -0.38$

substitute a = -038 into the original equation.

$y = -0.38(x - 10)^2 + 40$

a) Conclusion: The model equation for the formula of the flight path of the firework is:

$y = -0.38(x - 10)^2 + 40$

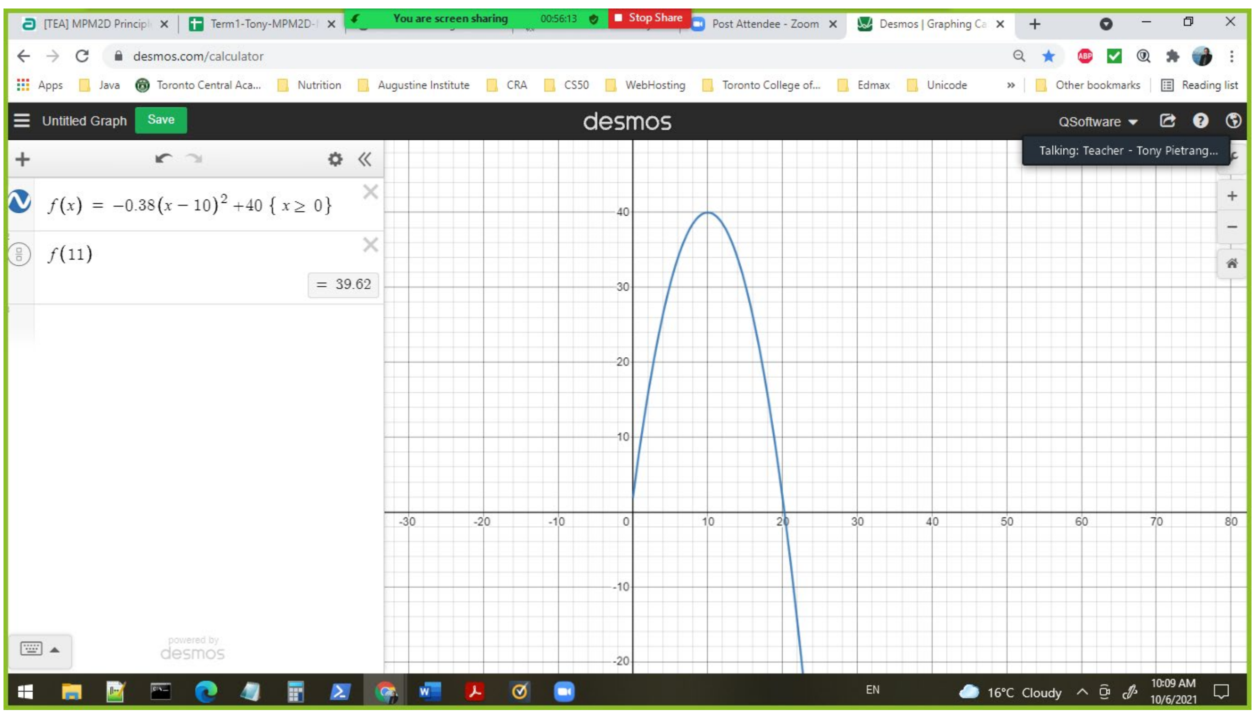
b) The firework explodes one more meter to the right of the maximum position. The fireworks explodes at x position of $x = 11$.

Substitute $x = 11$ into the above equation to determine the height of where it explodes.

$y = -0.38(x - 10)^2 + 40$
$y = -0.38(11 - 10)^2 + 40$
$y = -0.38(1)^2 + 40$
$y = 39.62$ in meters.

∴ the height of the explosion is 39.62 in meters.

f(x) = f(11) using Desmos graphing software:



f(11) = 39.62 meters.

Therefore, the height of the fireworks will explode at a height of 39.62 meters.

Part c:

At what other horizontal distance is the firework at the same height as in part b), before the maximum height.

Due to symmetry, the fireworks explodes at 11 meters horizontally. Before the fireworks reaches at the maximum height of 40 meters, the works should be at one meter less horizontally than the horizontal position of 10 meters, which is 10 – 1, 9 meters.

Examples of the form
y = a(x – r)(x – s)

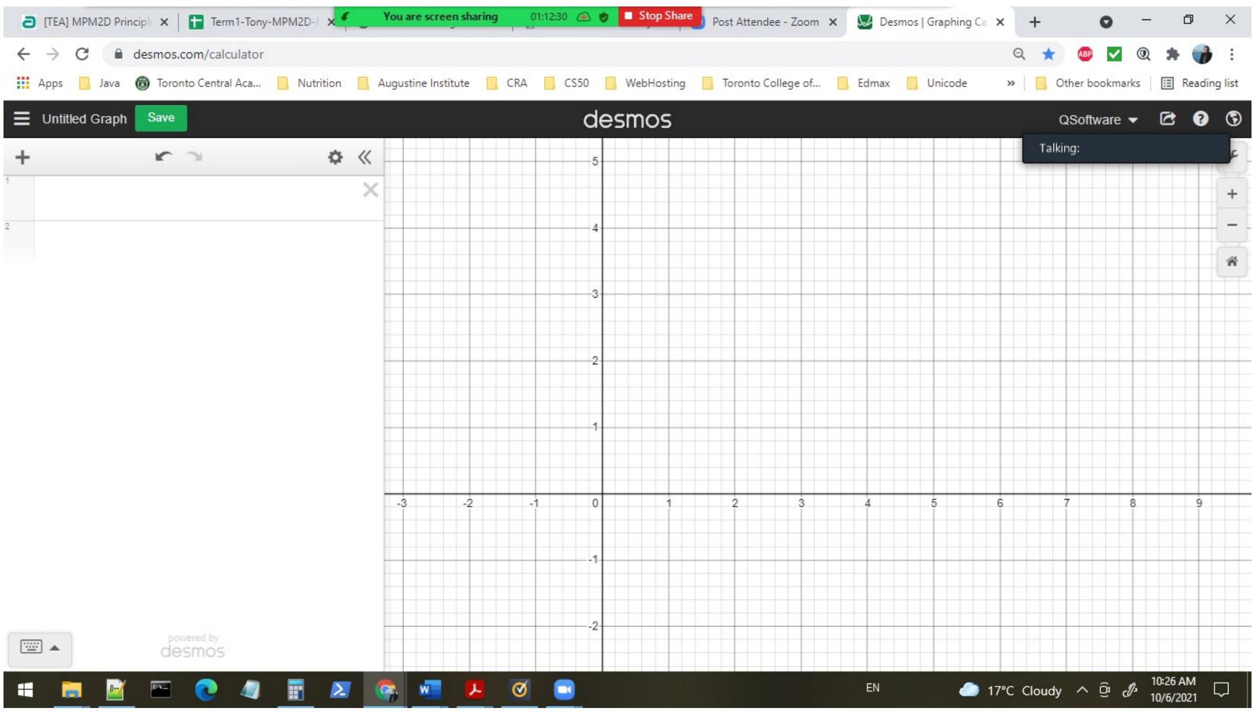
The r, s are the x-intercepts.

Example 1:

Describe the graph of the quadratic relation y = 2(x + 1)(x – 7).
Sketch the graph and label the x-intercepts, vertex, and the axis of symmetry.

Compare y = 2(x + 1)(x – 7) to y = a(x – r)(x – s)

The x-intercepts are r = -1, and s = 7



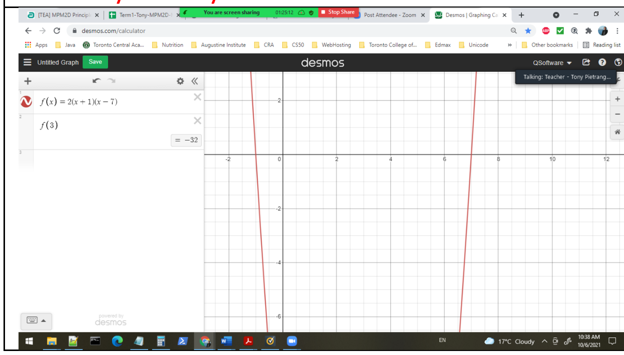
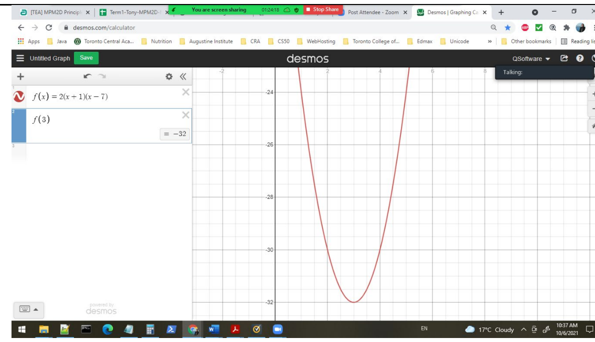
The axis of symmetry is between the two x-intercepts, which are at -1 and 7.

The axis of symmetry, which x = $\frac{-1+7}{2} = \frac{6}{2}$ = 3

To solve for the vertex (x, y) = (3, y)

Substitute x = 3 into the equation above to find the y value for the vertex.

y = 2(x + 1)(x – 7).
y = -2(3 + 1)(3 – 7)
y = 2(4)(-4) = -32
∴ vertex is at position(x, y) = (3, -32)

x-intercepts y = 2(x + 1)(x – 7). x = -1, x = 7 axis of symmetry is 3.	Vertex position (3, -32)
	

Example 2: Dufferin Gate.

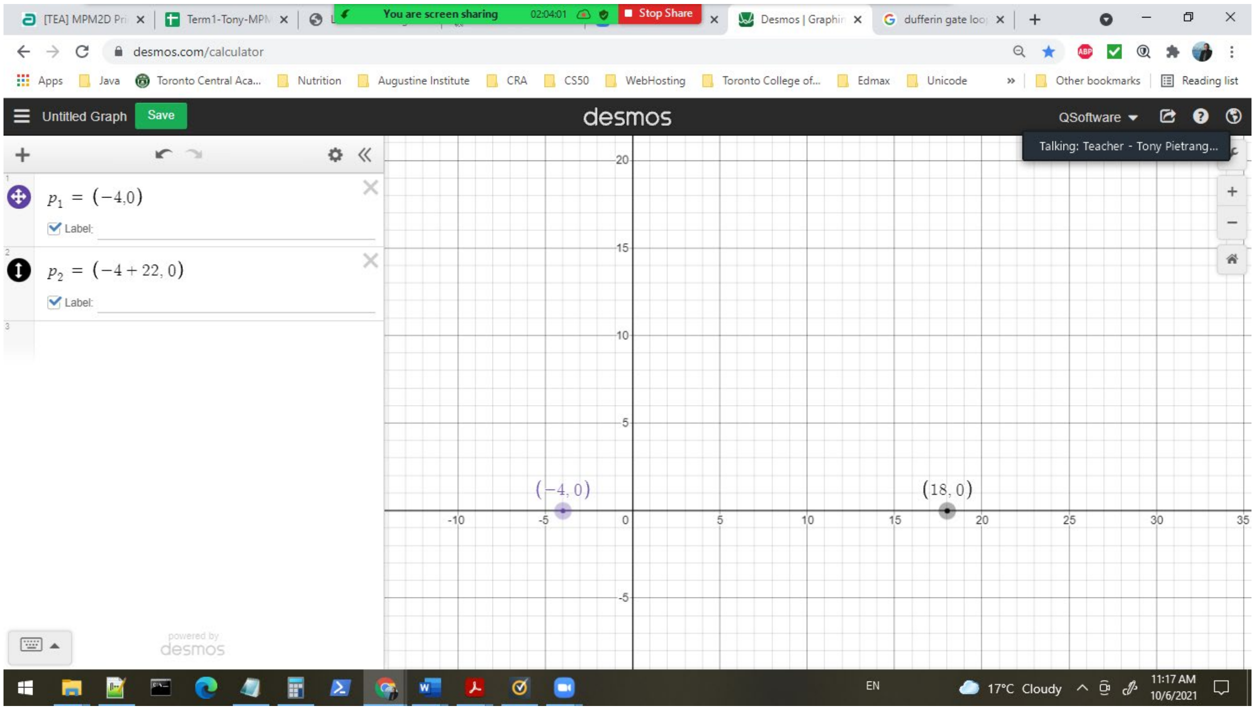
Below is a picture of dufferin gate, which has a arch that is visible to people. It is at the end of Duffering street, which is the entrace to the CNE – The Canadian National Exhibition.

The Duffering Gate is a parabolic arch that is approximately 20 meters tall and approximately 22 meters wide.

a) Sketch a graph of the arch with the left base located 4 units to the left of the y-axis.
b) Determine the equation to model the arch.

$y = a(x - r)(x - s)$

We have 1, x-intercept which is at -4. Where is the other x-intercept? If the left x-intercept is at -4, the right must be -4 + 22 metres wide, which is 18 meters.



When we have the two x-intercepts, we can calculate the vertex by knowing a parabola is symetrical.

Vertex (x, y) x position is $\frac{-4+18}{2} = \frac{14}{2} = 7$

The vertex is at x = 7, which is also the axis of symmetry.
The height is given, which is 20.

∴ vertex = (7, 20)

Using the parabola x-intercept form of: y = a(x-r)(x – s)

We have:

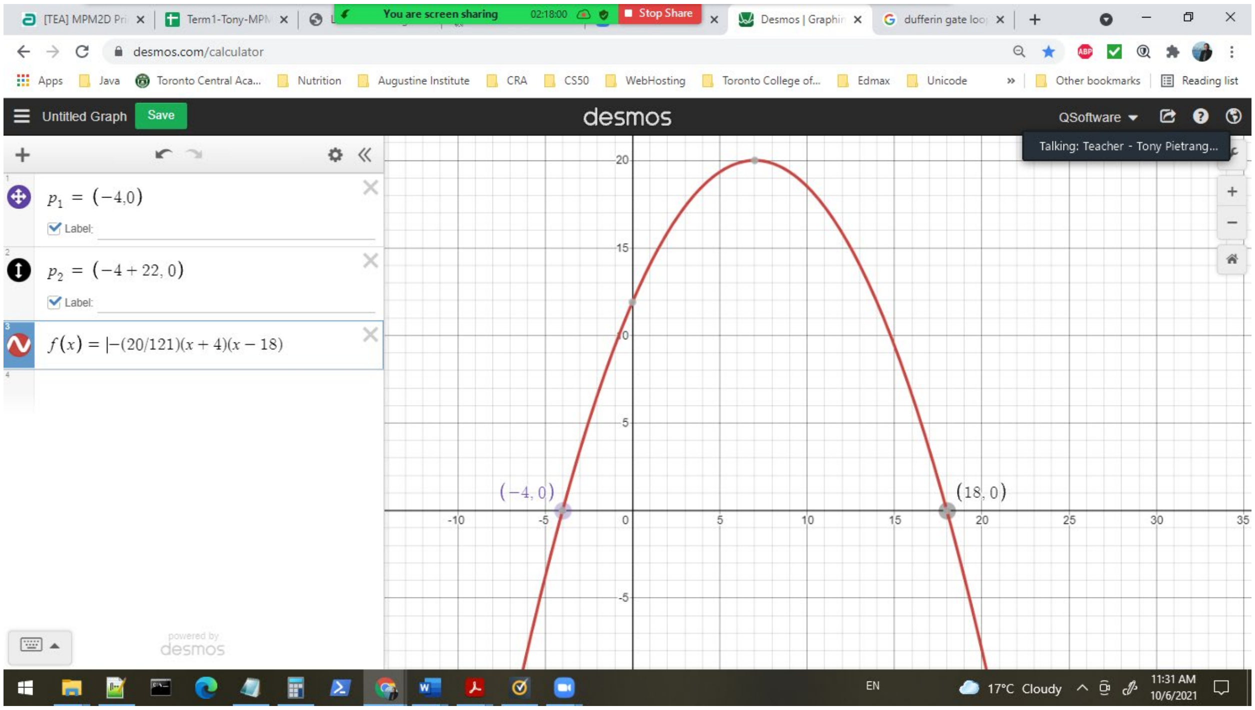
y = a(x – (-4))(x – 18)
y = a(x + 4)(x – 18)

Substitute the point of vertex = (7, 20) into the above equation.

y = a(x + 4)(x – 18)
20 = a(7 + 4)(7 – 18)
20 = a(11)(-11)
a = $-(\frac{20}{121})$
y = $-(\frac{20}{121})$(x + 4)(x – 18)

∴ the equation to represent the arch is $y = -(\frac{20}{121})(x + 4)(x - 18)$

y is the height in meters, and x is the horizontal distance in meters.



Activity 6/Home work:

Question 1: Sketch all 3 relations on the same graphs.

a) y = (x + 3)(x – 1)
b) y = 2(x + 3)(x – 1)
c) y = -2(x + 3)(x – 1)

Question 2: Sketch all three relations on the same set of axes.

Find the x-intercepts, axis of symmetry, for the following:

a) y = (x – 4)(x – 8)

b) y = $\frac{1}{2}$ (x – 4)(x – 8)
c) y = $\frac{1}{4}$ (x – 4)(x – 8)

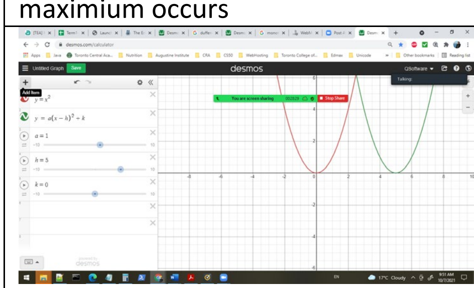
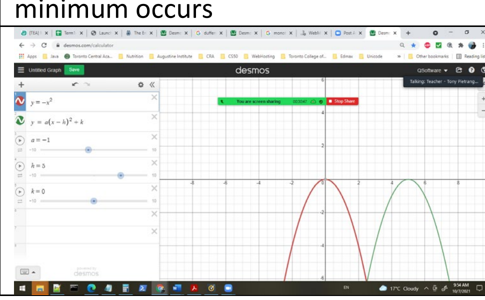
Question 3: Sketch all four relations on the same set of axes and find the x-intercepts, axis of symmetry, for the following:

a) y = (x – 6)(x – 2)
b) y = -(x + 3)(x + 7)
c) y = 2(x - 3)(x + 2)
d) y = 2(x - 4)(x + 2)

Date: Thursday, October 7th, 2021
Course: MPM2D – Principles of Mathematics.

Review:

1. Parabolas
    a) Three forms
        i) $y = ax^2 + bx + c$, where $a \neq 0$, and b, c are real numbers. The standard form.
        ii) $y = a(x – h)^2 + k$, $a \neq 0$, h and k are real numbers
            (1) h shifts the parabola horizontally.
                (a) If $h > 0$ parabola shifts to the right,
                (b) If $h < 0$ parabola shifts to the left
            (2) K shifts the parabola vertically
                (a) If $k > 0$ the shift is vertically upward.
                (b) If $k < 0$ the shift is downwards.
            (3) a determines if the parabola, is compressed or stretched.
                (a) If $a > 0$,
                    (i) the parabola opens upwards
                    (ii) the parabola has a minimum
                (b) If $a < 0$,
                    (i) the parabola opens downwards.
                    (ii) the parabola has a maximum
                (c) if $a > 1$
                    (i) the parabola stretches more quickly upwards
                    (ii) factored by a
                (d) if $a < -1$
                    (i) the parabola stretches more quickly downwards
                    (ii) factored by a
                (e) if a is a positive fraction
                    (i) the parabola is compressed or flatted
                (f) if a is a negative fraction
                    (i) the parabola is compressed or flatted
            (4) Vertex (x, y) = (h, k)

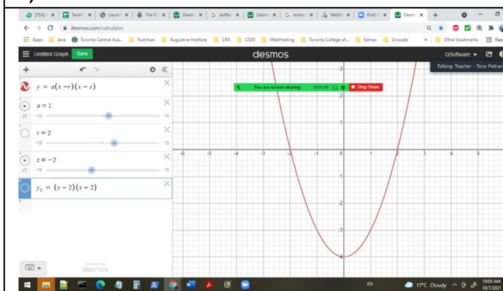
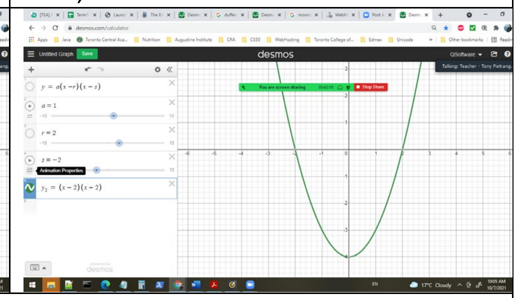
Vertex (x, y) = (h, k) $a > 0$ maximium occurs	Vertex (x, y) = (h, k) $a < 0$ minimum occurs
	

            (5) Axis of symmetry occurs at h.

iii) y = a(x – r)(x – s) – The x-intercepts or zeros form.

(1) r, s are the x values where the parabola crosses the x-axis.

Exampes of using r, s
Left graph as column as variables,
Right graph as constants:

r, s are variables	r = 2, s = -2
	

axis of symmetry is x = $\frac{r+s}{2}$

example above: r = -2, s = 2,
axis of symmetry x = $\frac{r+s}{2}$ = $\frac{-2+2}{2}$ = $\frac{0}{2}$ = 0

To determine the variable a, in form:
y = a(x – r)(x – s)

substitute any point (x, y) into equation to get the value a, which determines how the parabola opens upwards or downwards, stretches or (compresses / flattens)

use: point (x, y) = (0, -4)
r = 2
s = -2

determine a:
y = a(x – r)(x – s)
-4 = a(x – 2)(x +2)
-4 = a(0 – 2)(0 + 2)
-4 = a(-2)(+2)
-4 = -4a
a = $\frac{-4}{4}$ = -1

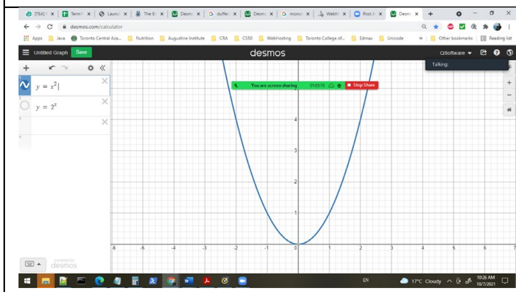
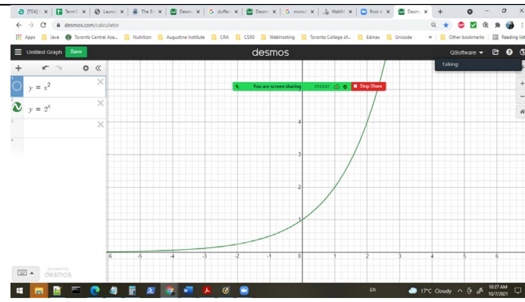
End of summary of Parabolas:

Goal:

1. Review of Negatives and zero exponents.

Question: What is an exponent?

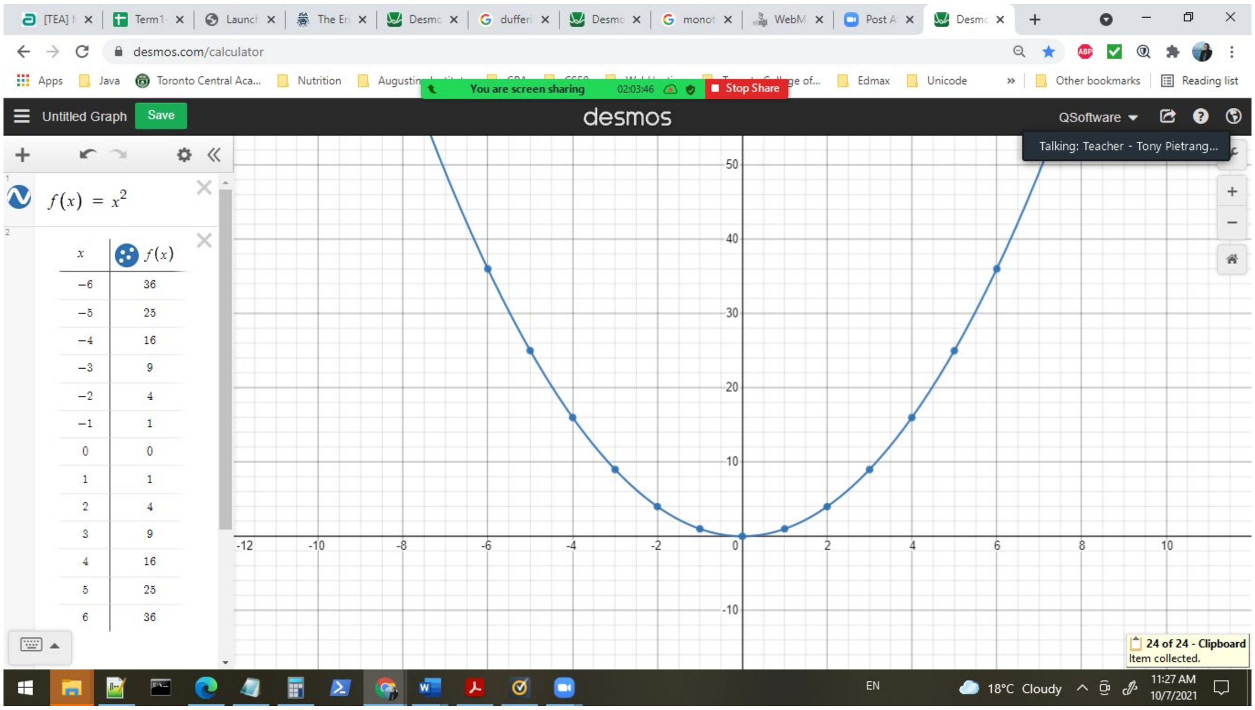
Answer: raising of variable, constant or expression to a power, i.e., a^n

$y = x^2$	$y = 2^x$
	

$y = x^2$		
Value of x	Value of $y = x^2$	Point(x, y)
-6	$y = (-6)^2 = 36$	$P_1 = (-6, 36)$
-5	$y = (-5)^2 = 25$	$P_2 = (-5, 25)$
-4	$y = (-4)^2 = 16$	$P_3 = (-4, 16)$
-3	$y = (-3)^2 = 9$	$P_4 = (-3, 9)$
-2	$y = (-2)^2 = 4$	$P_5 = (-2, 4)$
-1	$y = (-1)^2 = 1$	$P_6 = (-1, 1)$
0	$y = (0)^2 = 0$	$P_7 = (0, 0)$
1	$y = (1)^2 = 1$	$P_8 = (1, 1)$
2	$y = (2)^2 = 4$	$P_9 = (2, 4)$
3	$y = (3)^2 = 9$	$P_{10} = (3, 9)$
4	$y = (4)^2 = 16$	$P_{11} = (4, 16)$
5	$y = (5)^2 = 25$	$P_{12} = (5, 25)$
6	$y = (6)^2 = 36$	$P_{13} = (6, 36)$

Results of plotting of points above.

Instructor used: automatic point generation of Demos graphing software using a table of points and functions of the form f(x) = x^2

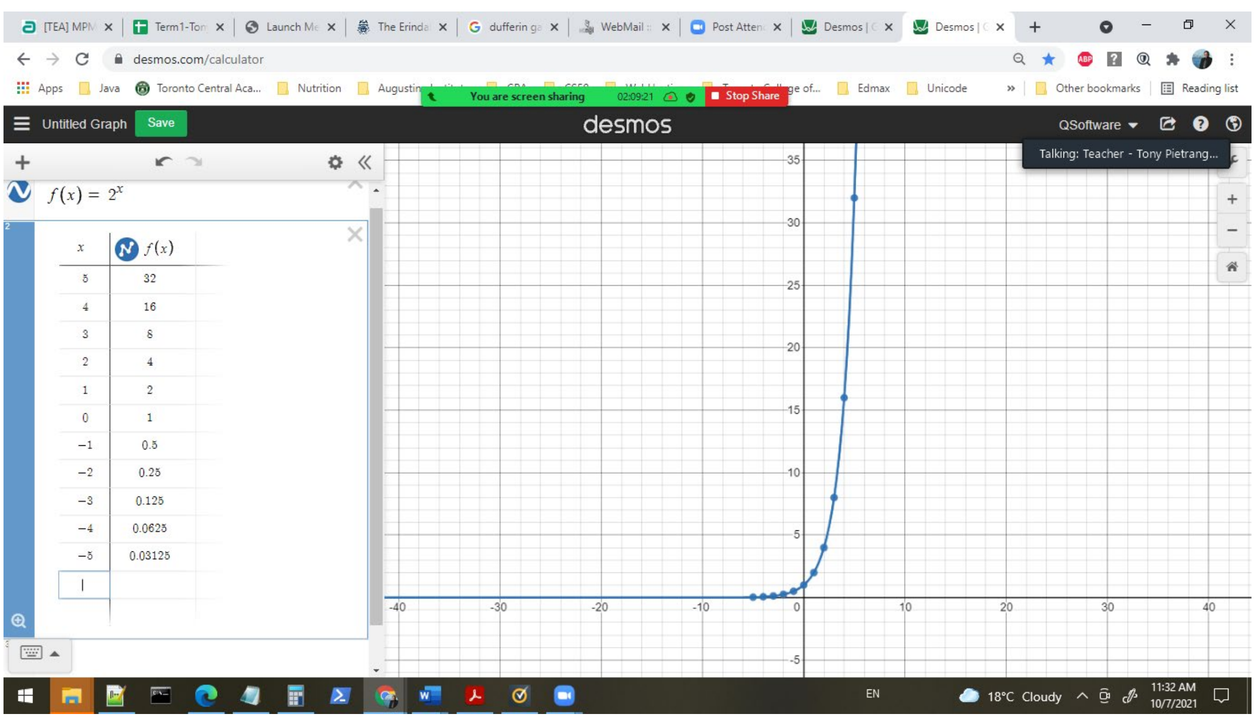


Remember $a^{-x} = 1 / (a^x)$

Create points for $y = 2^x$

$y = 2^x$		
Value of x	Value of $y = 2^x$	Point(x, y)
6	$y = 2^6 = 64$	$P_1 = (6, 64)$
5	$y = 2^5 = 32$	$P_2 = (5, 32)$
4	$y = 2^4 = 16$	$P_3 = (4, 16)$
3	$y = 2^3 = 8$	$P_4 = (3, 8)$
2	$y = 2^2 = 4$	$P_5 = (2, 4)$
1	$y = 2^1 = 2$	$P_6 = (1, 2)$
0	$y = 2^0 = 1$	$P_7 = (0, 1)$
-1	$y = 2^{-1} = 1/ (2^1) = \frac{1}{2} = 0.5$	$P_8 = (-1, 0.5)$
-2	$y = 2^{-2} = 1/ (2^2) = \frac{1}{4} = 0.25$	$P_9 = (-2, 0.25)$
-3	$y = 2^{-3} = 1/ (2^3) = \frac{1}{8} = 0.125$	$P_{10} = (-3, 0.125)$
-4	$y = 2^{-4} = 1/ (2^4) = \frac{1}{16} = 0.0625$	$P_{11} = (-4, 0.0625)$
-5	$y = 2^{-5} = 1/ (2^5) = \frac{1}{32} = 0.03125$	$P_{12} = (-5, 0.03125)$
-6	$y = 2^{-6} = 1/ (2^6) = \frac{1}{64} = 0.016625$	$P_{13} = (-6, 0.016625)$

Plot points of 2^x on a graph



Activity 7 / Home Work:

Question 1: Review each power with a positive exponent. a) 3^{-2} b) 5^{-1} c) 10^{-4} d) 7^{-3} e) $(-2)^{-4}$ f) $(-7)^{-1}$	**Question 2:** Evaluate a) 6^{-2} b) 9^{0} c) 7^{-1} d) 10^{-3} e) $(-9)^{-1}$ f) $(-12)^{-2}$ g) $(-3)^{0}$ h) 89^{0}
Question 3: Evaluate 1. $(\frac{1}{3})^{-2}$ 2. 0^{-5} 3. $-(\frac{1}{4})^{-1}$ 4. $(\frac{5}{6})^{-2}$ 5. $(-(\frac{3}{8}))^{-4}$ 3 6. $((\frac{9}{4}))^{-4}$	**Question 4:** Evaluate a) $6^{0} + 6^{-2}$ b) $8 - 8^{-1}$ c) $(4 + 3)^{0}$ d) $4^{0} + 3^{0}$