Chapter 7: Trigonometry of Right Angles
Chapter 8: Trigonometry of Acute Angles

Key formulas:

1. Pythagorean Thorem:
   $r^2 = x^2 + y^2$ ← $r = \sqrt{x^2 + y^2}$ ← $c^2 = a^2 + b^2$ ← used for right angles only
   $c^2 = \sqrt{a^2 + b^2}$ ← $c = \sqrt{a^2 + b^2}$
2. Trigonometric identities:

Standard Trig. Functions	Abbreviation		
SOH CAH TOA			
sine	$\sin(\theta)$	$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{opp}{hyp} = \frac{y}{r}$	
cosine	$\cos(\theta)$	$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{adj}{hyp} = \frac{x}{r}$	
tangent	$\tan(\theta)$	$\tan(\theta) = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{y}{x}$	
Reciprocal Functions			
sine ← cosecant	$\csc(\theta) = \frac{1}{\sin(\theta)}$	$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{hyp}{opp} = \frac{r}{y}$	
cosine ← secant	$\sec(\theta) = \frac{1}{\cos(\theta)}$	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{hyp}{adj} = \frac{r}{x}$	
tangent ← cotangent	$\cot(\theta) = \frac{1}{\tan(\theta)}$	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{adj}{opp} = \frac{x}{y}$	

3. Sine Law ← Acute Angles
   $\frac{a}{Sine(A)} = \frac{b}{Sine(B)} = \frac{c}{Sine(C)}$

4. Cosine Law ← Acute Angles
   $a^2 = b^2 + c^2 - bc(\cos(A))$
   $b^2 = a^2 + c^2 - ac(\cos(B))$
   $c^2 = a^2 + b^2 - ab(\cos(C))$

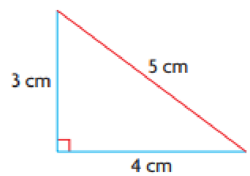
7 Getting Started

WORDS YOU NEED to Know

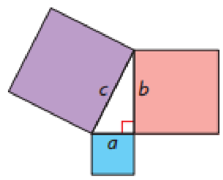
1. Match each term with the example or diagram that best represents it.

a) ratio **b)** proportion **c)** congruent triangles **d)** Pythagorean theorem **e)** hypotenuse **f)** acute angle

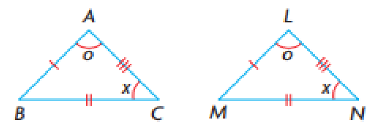
i)



iii)



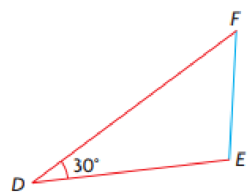
v)



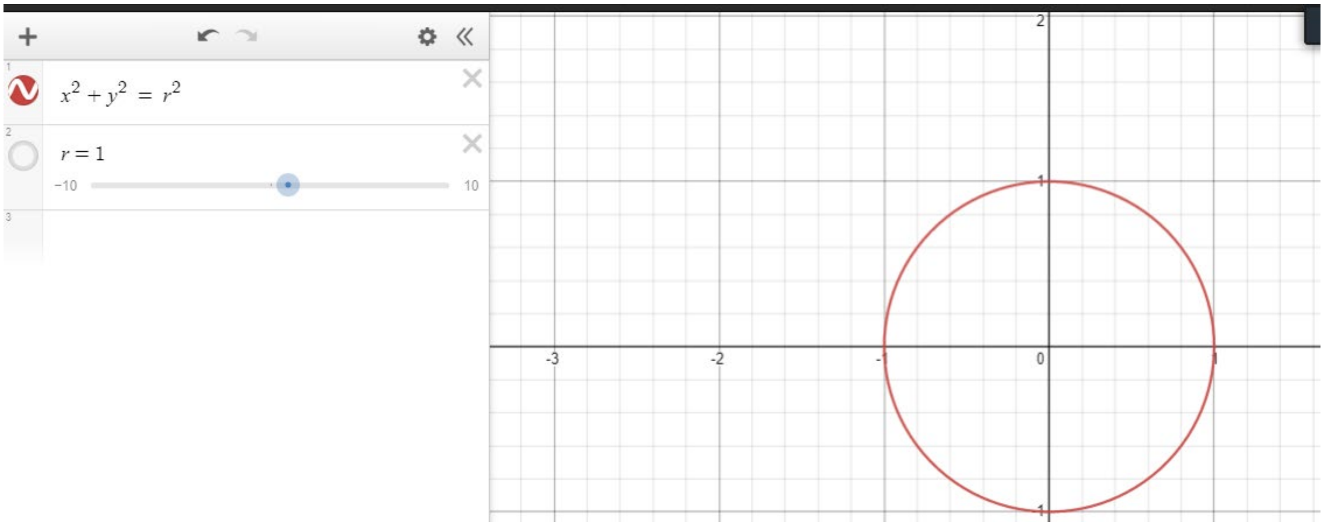
ii) $\frac{3}{5}$ or 3:5 or 3 to 5

iv) $\frac{x}{5} = \frac{3}{15}$

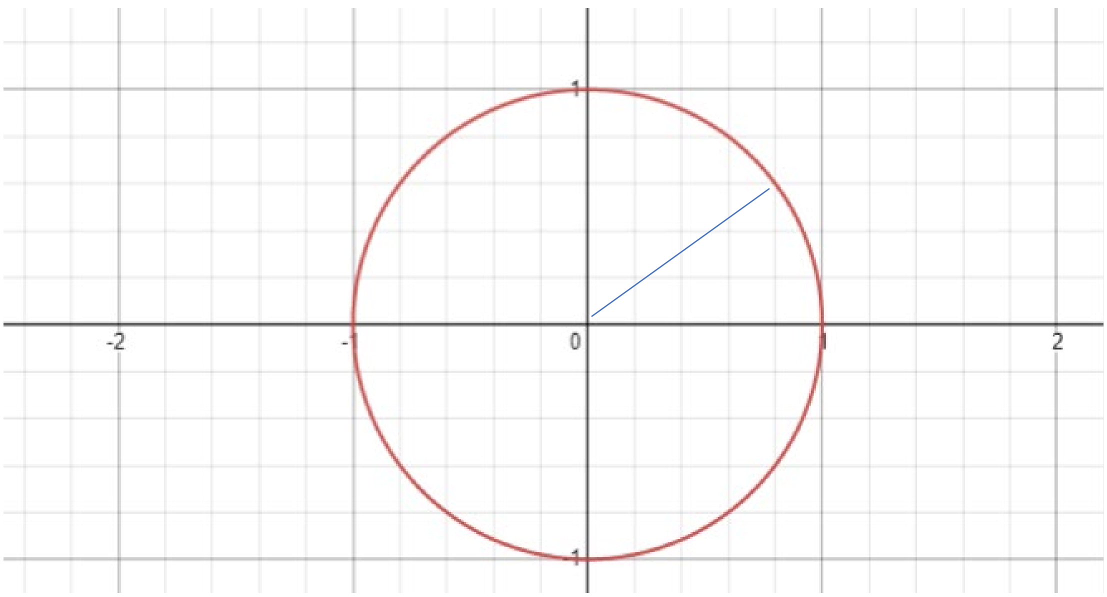
vi)



Terms	Diagram
ratio	ii)
proportion	iv)
Congruent triangles	v)
Pythagorean theorem	iii)
hypotenuse	i)
Acute angle	vi)

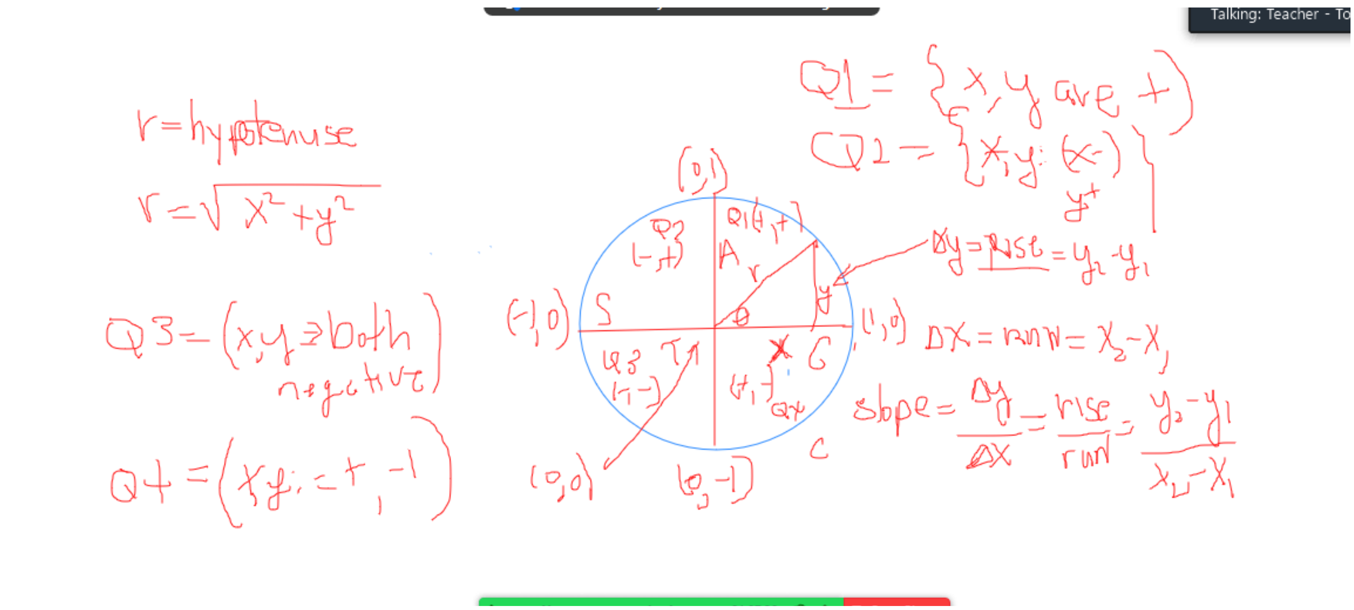


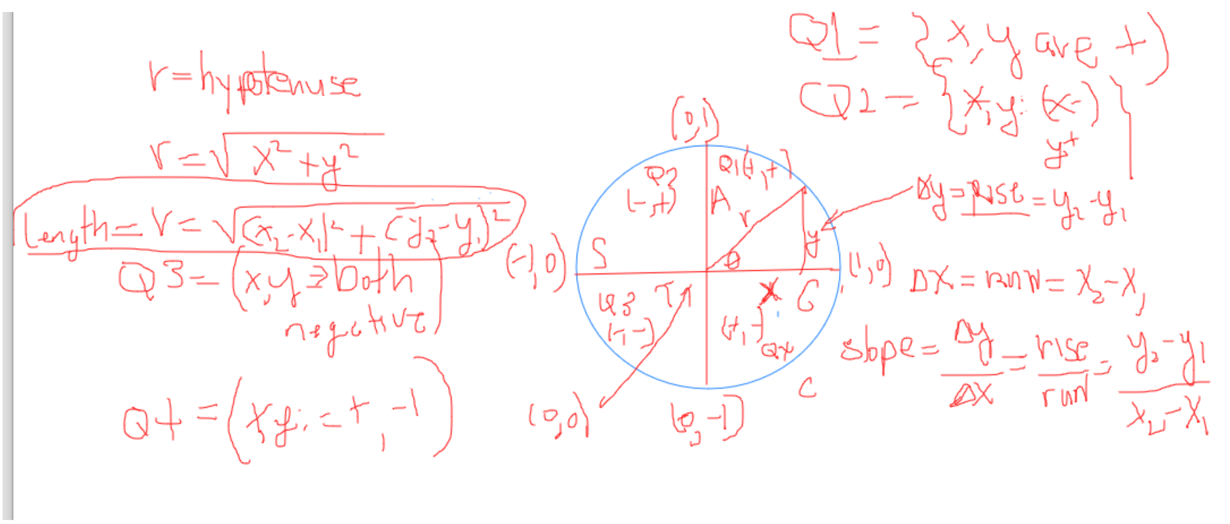
$x^2 + y^2 = r^2$
$r = 1$
−10
10
2
1
-3
-2
-1
0
1
-1



1
-2
-1
0
1
2
-1

Analysis of a Unit Circle



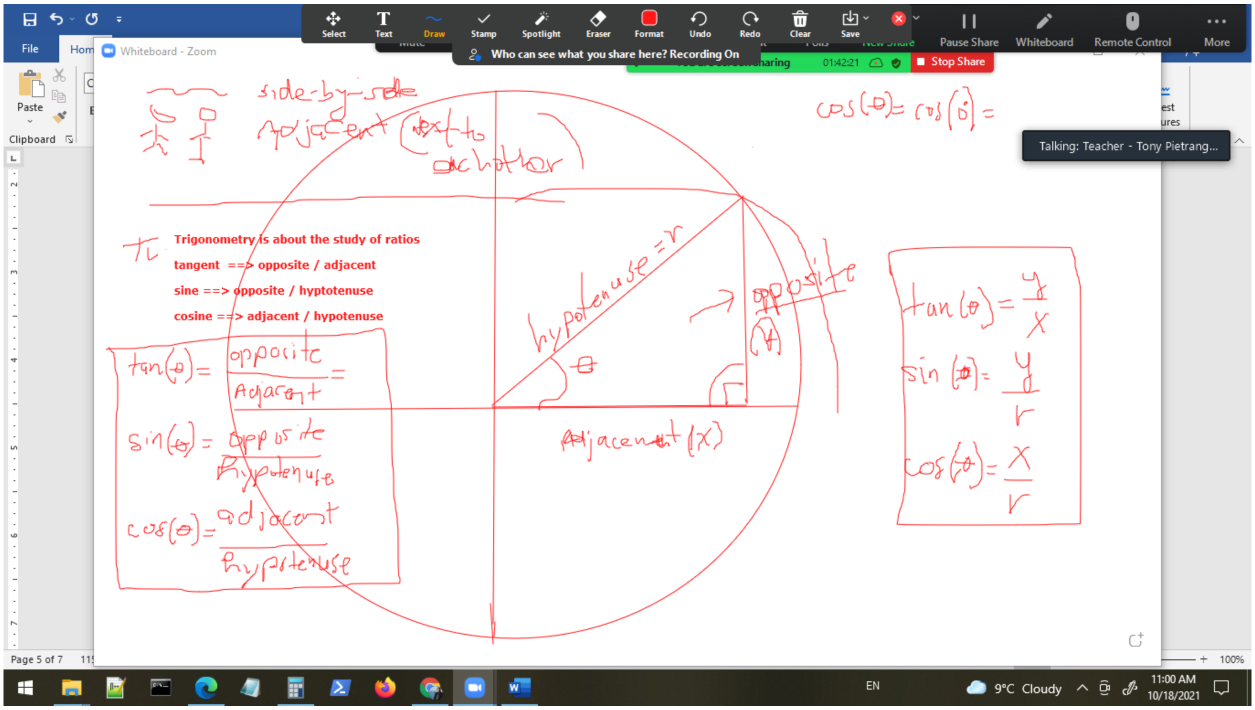


Formula(s):

1. Δy = rise = $y_2 - y_1$
2. Δx = run = $x_x - x_1$
3. Slope = m = $\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = \frac{y2 - y1}{x2 - x1}$
4. r = $\sqrt{x^2 + y^2}$
5. length = L = r = $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

By Definition:

Trigonometry



θ, angle pronounced theta

Memory Technique for Trigonometric identities:
SOH CAH TOA

$$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{opp}{hyp} = \frac{y}{r}$$

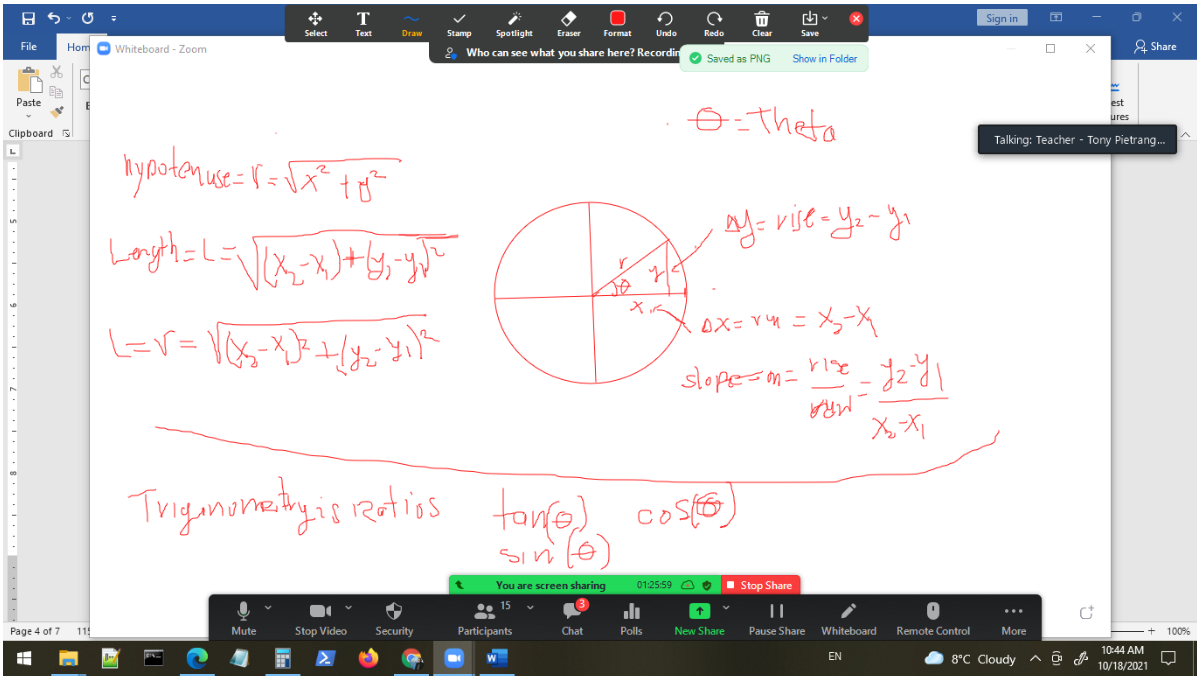
$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{adj}{hyp} = \frac{x}{r}$$

$$\tan(\theta) = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{y}{x}$$

$$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{hyp}{opp} = \frac{r}{y}$$

$$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{hyp}{adj} = \frac{r}{x}$$

$$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{adj}{opp} = \frac{x}{y}$$



$$\tan(\theta) = \frac{opposite}{adjacent} = \frac{y}{x}$$

$$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{x}{r}$$

$$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{y}{r}$$

By definition:

$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{y}{r}$

$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{x}{r}$

$\tan(\theta) = \frac{opposite}{adjacent} = \frac{y}{x}$

What happens when (θ = 0°) 🡸 r = 1, x = 1, y = 0

$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{x}{r}$

$\cos(\theta) = \frac{1}{1} = 1$

If we go to the unit circle, at 0° or point(x, y) = (1, 0), the cos(0°)= 1

$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{y}{r}$

if we go to the unit circle, at 0° or point(x, y) = (1, 0), the sin(0°) = $\frac{0}{1}$ = 0

$\tan(\theta) = \frac{opposite}{adjacent} = \frac{y}{x} = \frac{0}{1} = 0$

What happens when (θ = 90°) 🡰 r = 1, x = 0, y = 1

If we go to the unit circle, at 90° or point(x, y) = (0, 1), the cos(90°)= 0

cos(θ) = $\frac{adjacent}{hypotenuse}$ = $\frac{x}{r}$

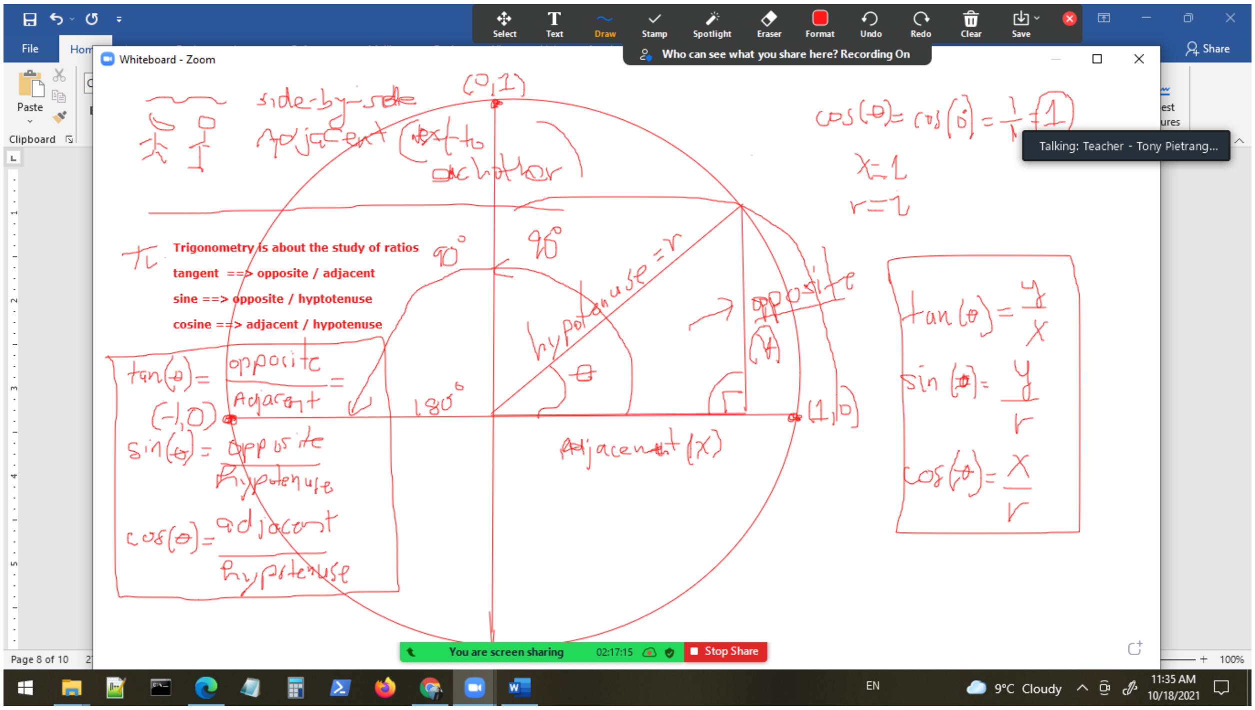
cos(90°) = $\frac{0}{1}$ = 0

at 90°

sin(90°)= $\frac{opposite}{hypotenuse}$ = $\frac{y}{r}$ = $\frac{1}{1}$ = 1

if we go to the unit circle, at 0° or point(x, y) = (1, 0), the tan(90°)

tan(90°) = $\frac{opposite}{adjacent}$ = $\frac{y}{x}$ = $\frac{1}{0}$ = ∞ = unknown



What happens when (θ = 180°) 🡐 r = 1, x = -1, y = 0
Point on circle(x, y) = (-1, 0)

If we go to the unit circle, at 180° or point(x, y) = (-1, 0)

$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{x}{r}$

$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{y}{r}$

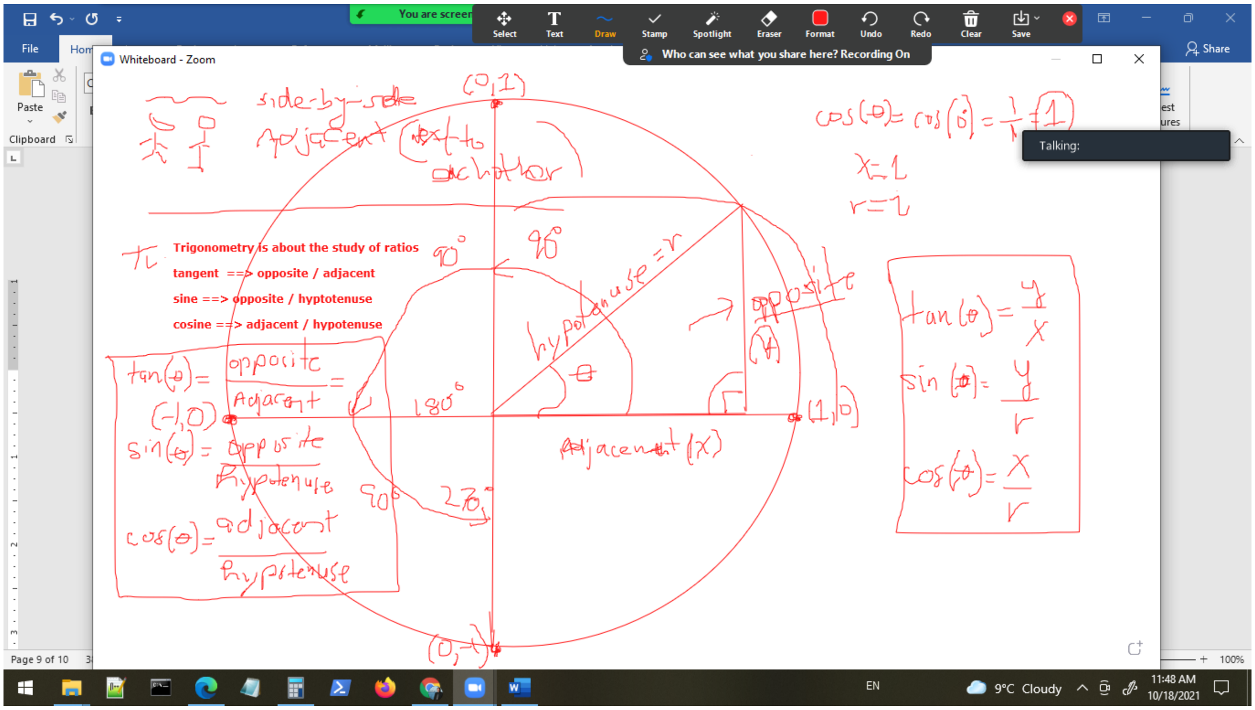
$\tan(\theta) = \frac{opposite}{adjacent} = \frac{y}{x}$

At (180°) the trigonometric values are

$\cos((180°) = \frac{adjacent}{hypotenuse} = \frac{x}{r} = \frac{-1}{1} = -1$

$\sin(180°) = \frac{opposite}{hypotenuse} = \frac{y}{r} = \frac{0}{1} = 0$

$\tan(180°) = \frac{opposite}{adjacent} = \frac{y}{x} = \frac{0}{-1} = 0$



What happens when (θ = 270°) 🡐 r = 1, x = 0, y = -1

Point on circle(x, y) = (0, -1)

cos(θ) = $\frac{adjacent}{hypotenuse}$ = $\frac{x}{r}$

sin(θ)= $\frac{opposite}{hypotenuse}$ = $\frac{y}{r}$

tan(θ) = $\frac{opposite}{adjacent}$ = $\frac{y}{x}$

At (270°) the trigonometric values are

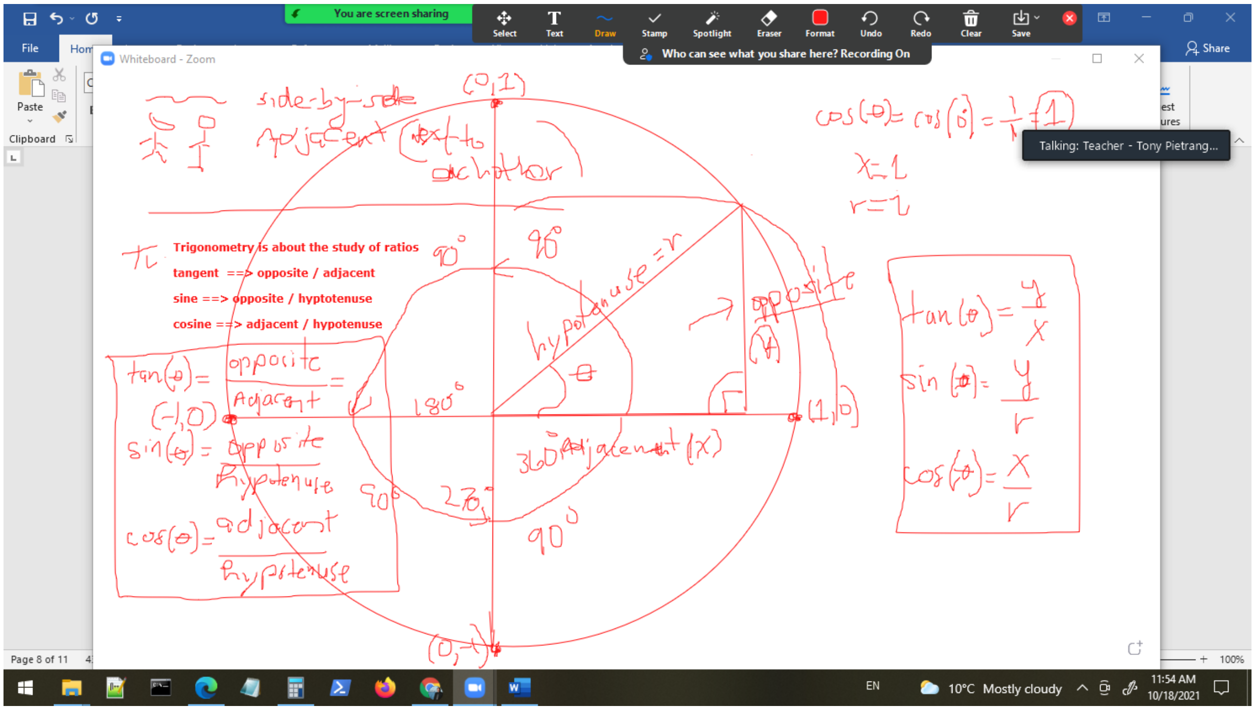
cos((270°) = $\frac{adjacent}{hypotenuse}$ = $\frac{x}{r}$ = $\frac{-0}{1}$ = 0

sin(270°) = $\frac{opposite}{hypotenuse}$ = $\frac{y}{r}$ = $\frac{-1}{1}$ = -1

tan(270°) = $\frac{opposite}{adjacent}$ = $\frac{-1}{0}$ = ∞ = unknown

At (θ = 360°), we have traversed the whole circle and are back at point(x, y) = (1, 0), which is the same

As (θ = 0°)



What happens when (θ = 360°) same as (θ = 0°) 🡐 r = 1, x = 1, y = 0
Point on circle(x, y) = (1, 0)

$\cos(\theta) = \dfrac{adjacent}{hypotenuse} = \dfrac{x}{r}$

$\sin(\theta) = \dfrac{opposite}{hypotenuse} = \dfrac{y}{r}$

$\tan(\theta) = \dfrac{opposite}{adjacent} = \dfrac{y}{x}$

At (360°) same as 360°, the trigonometric values are

$\cos((360°) = \dfrac{adjacent}{hypotenuse} = \dfrac{x}{r} = \; = \dfrac{1}{1} = 1$

$\sin(360°) = \dfrac{opposite}{hypotenuse} = \dfrac{y}{r} = \dfrac{0}{1} = 0$

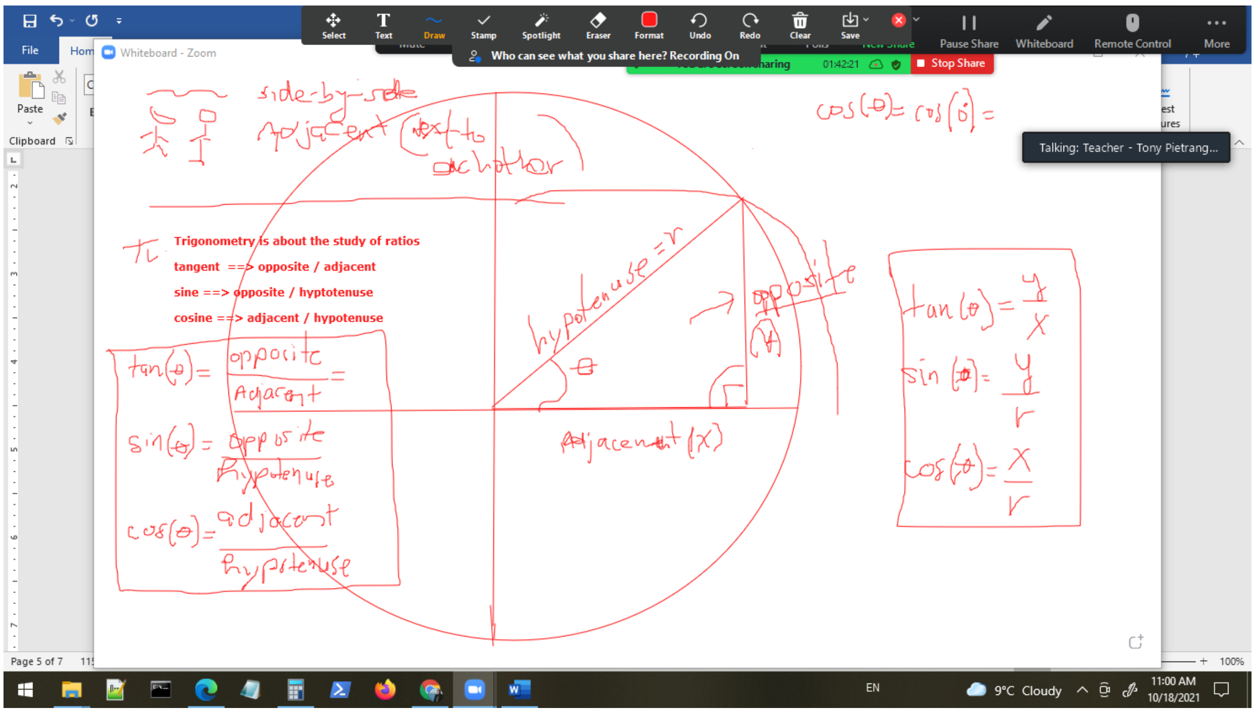
$\tan(360°) = \dfrac{opposite}{adjacent} = \dfrac{y}{x} = \dfrac{0}{1} = 0$

Date Created: Tuesday, October 19th, 2021

Review of Identities for Trigonometric Functions:

By Definition:

Trigonometry



θ, angle pronounced theta

Memory Technique for Trigonometric identities:
SOH CAH TOA

$\sin(\theta) = \frac{opposite}{hypotenuse} = \frac{opp}{hyp} = \frac{y}{r}$

$\cos(\theta) = \frac{adjacent}{hypotenuse} = \frac{adj}{hyp} = \frac{x}{r}$

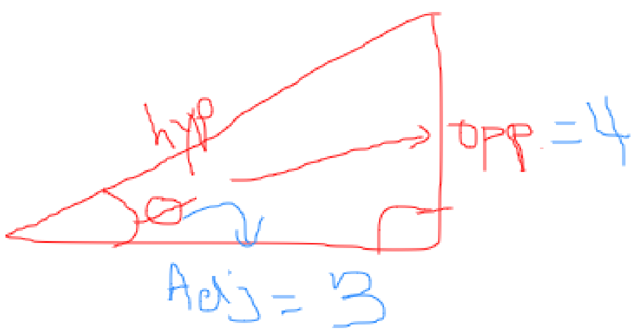
$\tan(\theta) = \frac{opposite}{adjacent} = \frac{opp}{adj} = \frac{y}{x}$

$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{hyp}{opp} = \frac{r}{y}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{hyp}{adj} = \frac{r}{x}$

$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{adj}{opp} = \frac{x}{y}$

Example 1:



Question 1:
Find the length of the missing side.

Question 2:
Find the values of all 6 trigonmetric identities

Question 1:

Solution: to utilize the Pythagorean Theorem.

$c^2 = a^2 + b^2$

c = $\sqrt{(3)^2 + (4)^2}$ =

c = $\sqrt{9 + 16}$

c = $\sqrt{25}$ = 5

opp = 4
Adj = 3
Hyp = 5

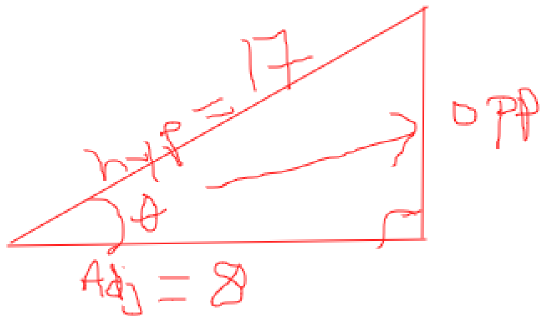
Question 2:
Solution: Find all the values of the trigonometric identities.

opp = 4
Adj = 3
Hyp = 5

SOH CAH TOA

sine	sin(θ)	sin(θ)= $\frac{opposite}{hypotenuse}$ = $\frac{opp}{hyp}$ = $\frac{4}{5}$
cosine	cos(θ)	cos(θ) = $\frac{adjacent}{hypotenuse}$ = $\frac{adj}{hyp}$ = $\frac{3}{5}$
tangent	tan(θ)	tan(θ) = $\frac{opposite}{adjacent}$ = $\frac{opp}{adj}$= $\frac{4}{3}$
cosecant	csc(θ) = $\frac{1}{\sin(\theta)}$	csc(θ) = $\frac{1}{\sin(\theta)}$ = $\frac{hyp}{opp}$ = $\frac{5}{4}$
secant	sec(θ) = $\frac{1}{\cos(\theta)}$	sec(θ) = $\frac{1}{\cos(\theta)}$ = $\frac{hyp}{adj}$ = $\frac{5}{3}$
cotangent	cot(θ) = $\frac{1}{\tan(\theta)}$	cot(θ) = $\frac{1}{\tan(\theta)}$ = $\frac{adj}{opp}$= $\frac{3}{4}$

Example 2:



Question 1: Find the length of the opposite side the angle.

Question 2: Find all the trigonometric values of all the 6 identities.

Question 1: Find the length of the opposite side of the angle Theta (θ)

Use Pythagorean Theorem:

$c^2 = a^2 + b^2$

$c^2 = a^2 + b^2$

b = $\sqrt{(c)^2 - (a)^2}$

b = $\sqrt{(17)^2 - (8)^2}$

b = $\sqrt{289 - 64}$ = $\sqrt{225}$

b = 15 ← This is length of opposite side.

opp = 15
Adj = 8
Hyp = 17

Once we have all the lengths of the three sides, we can calculate the ratios of the trigonometric functions.

Example 2:

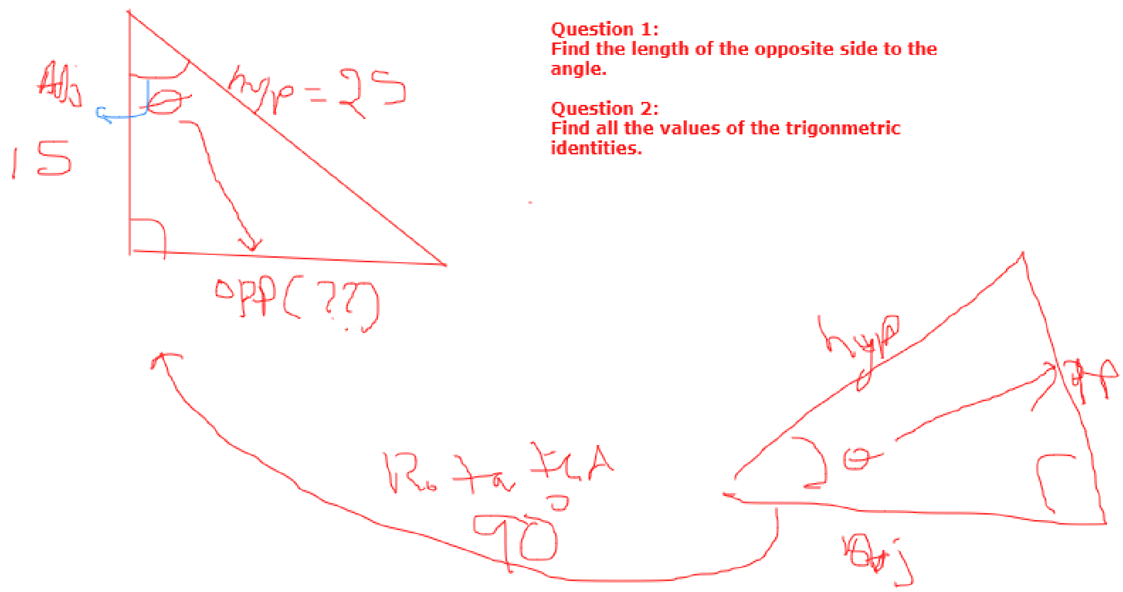
Question 2: Find all the values of the trigonometric identities.

opp = 15
Adj = 8
Hyp = 17

SOH CAH TOA

sine	sin(θ)	sin(θ)= $\frac{opposite}{hypotenuse}$ = $\frac{opp}{hyp}$ = $\frac{15}{17}$
cosine	cos(θ)	cos(θ) = $\frac{adjacent}{hypotenuse}$ = $\frac{adj}{hyp}$ = $\frac{8}{17}$
tangent	tan(θ)	tan(θ) = $\frac{opposite}{adjacent}$ = $\frac{opp}{adj}$= $\frac{15}{8}$
cosecant	csc(θ) = $\frac{1}{\sin(\theta)}$	csc(θ) = $\frac{1}{\sin(\theta)}$ = $\frac{hyp}{opp}$ = $\frac{17}{15}$
secant	sec(θ) = $\frac{1}{\cos(\theta)}$	sec(θ) = $\frac{1}{\cos(\theta)}$ = $\frac{hyp}{adj}$ = $\frac{17}{8}$
cotangent	cot(θ) = $\frac{1}{\tan(\theta)}$	cot(θ) = $\frac{1}{\tan(\theta)}$ = $\frac{adj}{opp}$= $\frac{8}{15}$

Example 3:



Question 1:
Find the length of the opposite side to the angle.

Question 2:
Find all the values of the trigonometric identities.

Question 1: Find the length of the opposite side of the angle Theta (θ)

Use Pythagorean Theorem:

$c^2 = a^2 + b^2$

$c^2 = a^2 + b^2$

b = $\sqrt{(25)^2 - (15)^2}$

b = $\sqrt{625 - 225}$

b = $\sqrt{400}$

b = 20 ← This is length of opposite side.

opp = 20
Adj = 15
Hyp = 25

Example 3:

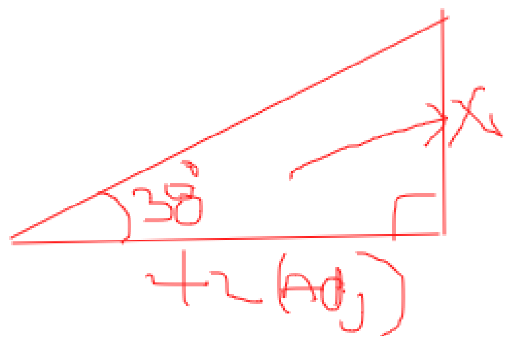
Question 2: Find all the values of the trigonometric identities.

opp = 20
Adj = 15
Hyp = 25

SOH CAH TOA

sine	sin(θ)	sin(θ)= $\frac{opposite}{hypotenuse}$ = $\frac{opp}{hyp}$ = $\frac{20}{25}$ = $\frac{4}{5}$
cosine	cos(θ)	cos(θ) = $\frac{adjacent}{hypotenuse}$ = $\frac{adj}{hyp}$ = $\frac{15}{25}$ = $\frac{3}{5}$
tangent	tan(θ)	tan(θ) = $\frac{opposite}{adjacent}$ = $\frac{opp}{adj}$ = $\frac{20}{15}$ = $\frac{4}{3}$
cosecant	csc(θ) = $\frac{1}{\sin(\theta)}$	csc(θ) = $\frac{1}{\sin(\theta)}$ = $\frac{hyp}{opp}$ = $\frac{25}{20}$ = $\frac{5}{4}$
secant	sec(θ) = $\frac{1}{\cos(\theta)}$	sec(θ) = $\frac{1}{\cos(\theta)}$ = $\frac{hyp}{adj}$ = $\frac{25}{15}$ = $\frac{5}{3}$
cotangent	cot(θ) = $\frac{1}{\tan(\theta)}$	cot(θ) = $\frac{1}{\tan(\theta)}$ = $\frac{adj}{opp}$ = $\frac{15}{20}$ = $\frac{3}{4}$

Example 4:



Question 1: Find the value or length of x, which is opposite side of the angle.

Logic/Reasoning:
What trigonmetric function should we use to find the value of x.

We have 6 trigonometric identities.
Which one should we use to find the value of x?

θ = 38°
adj = 42

tan(θ) = $\frac{opposite}{adjacent}$ = $\frac{opp}{42}$= $\frac{x}{42}$

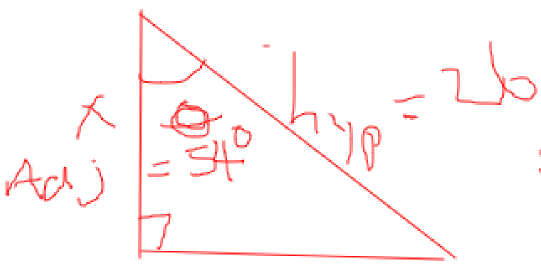
tan(38°) = $\frac{x}{42}$ ← isolate and solve for x; Multiply both sides by 42.

(42) tan(38°) = $\frac{x}{42}$ (42)

x = (42) tan(38°)
x = 42 (0.781285)

x = 32.81

Example 5:



Question 1:
What trigonometric identity do we use to find the value of x.

SOH CAH TOA

The trigonometric identity that we use that has hypotenuse, and adjacent angle is the cosine identity.

θ = 54°
hyp = 26
solve for x.

cos(54°) = $\frac{adj}{hyp} = \frac{x}{26}$ ← isolate and solve for x; Multiply both sides by 26.

(26) cos(54°) = $\frac{x}{26}$ (26)

x = (26) cos(54°)

x = (26) (0.587785)

x = 15.28

Tomorrow, we will at least do four (4) more examples for these trigonometric identities.

Activity 13:

Identify all the 6 trigonometric identities.

1.

Standard Trig. Functions	Abbreviation		
SOH CAH TOA			
Reciprocal Functions			

Date Created: Wednesday, October 20th, 2021

Review: now

SOH CAH TOA

S = $\frac{O}{H}$ = sine = $\frac{Opposite}{Hypotenuse}$

C = $\frac{A}{H} = cosine = \frac{Adjacent}{Hypotenuse}$

T = $\frac{O}{A}$ = $tangent = \frac{Opposite}{Adjecent}$

Goal: To further understand and expose student to reverse functions of the standard trigonometric functions.

SOH CAH TOA

ACTIVITY 14: -

Complete the table sin(θ), cos(θ), tan(θ) of the angles in table below for angles provided.
Complete the table $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$ for the ratios below.

Trigonometric Function	Abbreviation	Specific Angles θ	Value (Ratio)	Inverse Function		
sine	sin(θ)	θ = 0° θ = 30° θ = 45° θ = 60° θ = 90° add 90° to above θ = 120° θ = 135° θ = 150° θ = 180° add 90° to above θ = 210° θ = 225° θ = 240° θ = 270°	0 0.5 0.7071 0.8660 1 0.8660 0.7071 0.50 0 -0.50 -0.7071 -0.8660 -1	acrsine	$\sin^{-1}(0)$ $\sin^{-1}(0.5)$ $\sin^{-1}(0.7071)$ $\sin^{-1}(0.8660)$ $\sin^{-1}(1)$	0° 30° 45° 59.997 1

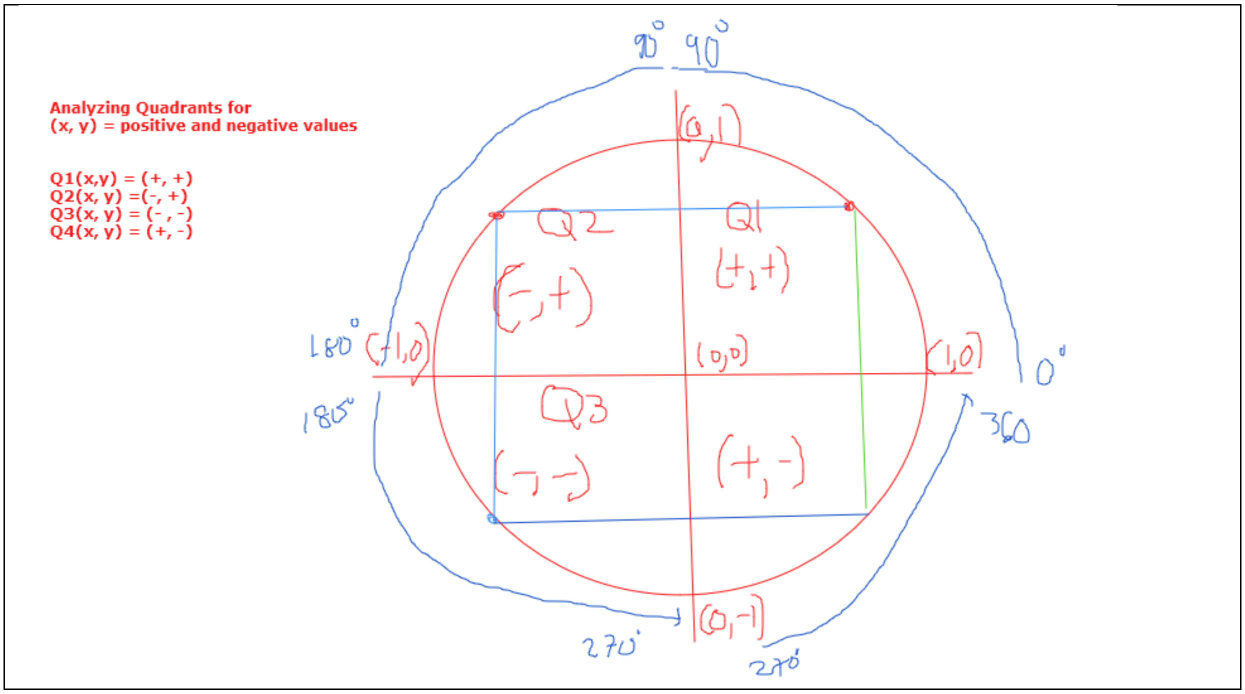
		add 90° to above θ = 300° θ = 315° θ = 330° θ = 360°	 -0.8660 -0.7071 -0.50 0			

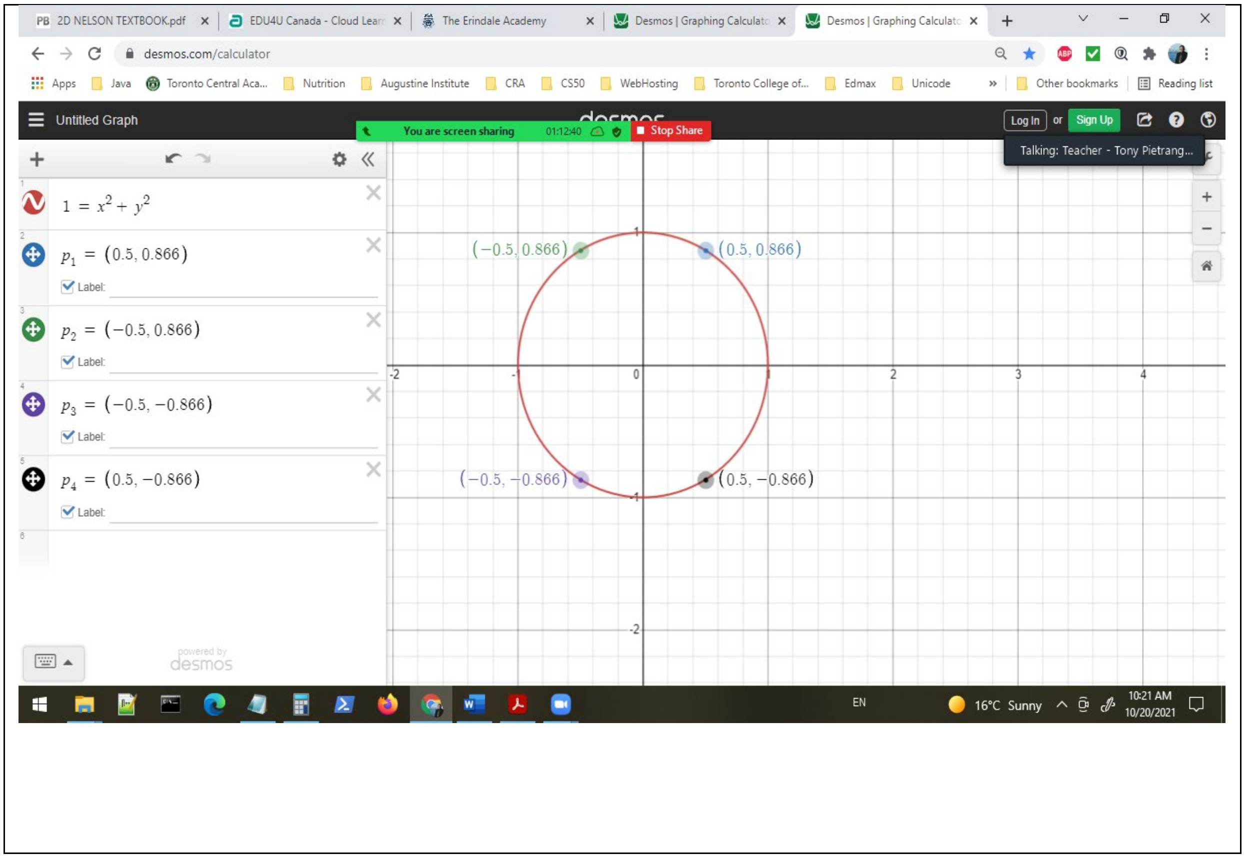
Trigonometric Function	Abbreviation	Specific Angles θ	Value (Ratio)	Inverse Function		
cosine	cos(θ)	θ = 0°	1	acrcosine	$\cos^{-1}(1)$	0°
		θ = 30°	0.8660			
		θ = 45°	0.7071			
		θ = 60°	0.50			
		θ = 90°	0			
		add 90° to above				
		θ = 120°	-0.50			
		θ = 135°	-0.7071			
		θ = 150°	-0.8666			
		θ = 180°	-1.0			
		add 90° to above				
		θ = 210°				
		θ = 225°				
		θ = 240°	0			
		θ = 270°				
		add 90° to above				
		θ = 300°				
		θ = 315°	1.0			
		θ = 330°				
		θ = 360°				

Trigonometric Function	Abbreviation	Specific Angles θ	Value (Ratio)	Inverse Function		
tangent	tan(θ)	θ = 0° θ = 30° θ = 45° θ = 60° θ = 90° add 90° to above θ = 120° θ = 135° θ = 150° θ = 180° add 90° to above θ = 210° θ = 225° θ = 240° θ = 270° add 90° to above θ = 300° θ = 315° θ = 330° θ = 360°	0 0.57735 1 1.73205 unknown	acrtan	$\tan^{-1}(0)$	0°

<u>**Analysis of Standard unit Circle:**</u>

1. Positive, Negative values of coordinates for each Quadrant
2. Reflection of values of points in other quadrants changes sign of values
3. In a unit circle Point(x, y) = Point(cos(θ), sin(θ)): The value of x = cos(θ), the vale of y = sin(θ)

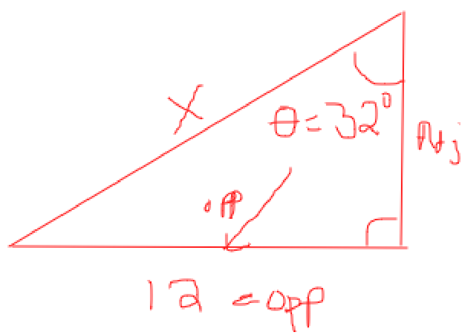




2D NELSON TEXTBOOK.pdf
EDU4U Canada - Cloud Learn
The Erindale Academy
Desmos | Graphing Calculator
Desmos | Graphing Calculator
desmos.com/calculator
Untitled Graph
You are screen sharing
Stop Share
Log In
or
Sign Up
Talking: Teacher - Tony Pietrang...
$1 = x^2 + y^2$
$p_1 = (0.5, 0.866)$
Label:
$p_2 = (-0.5, 0.866)$
Label:
$p_3 = (-0.5, -0.866)$
Label:
$p_4 = (0.5, -0.866)$
Label:
$(-0.5, 0.866)$
$(0.5, 0.866)$
$(-0.5, -0.866)$
$(0.5, -0.866)$
powered by desmos
16°C Sunny
10:21 AM
10/20/2021

Example 6:

Question:
What trigonometric function should we use to calculate then length or value of x?



SOH CAH TOA

$\sin(\theta) = \frac{Opposite}{Hypotenuse}$

$\sin(32°) = \frac{12}{x}$

solve for x.

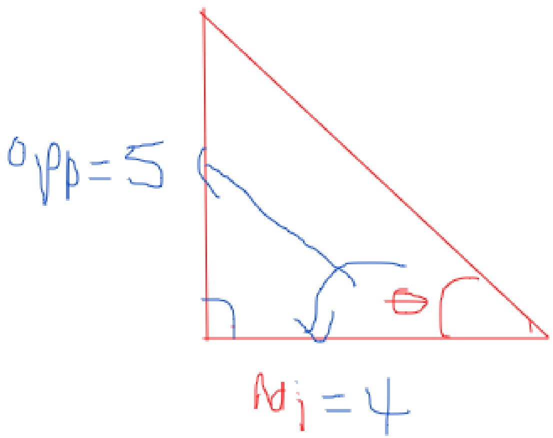
$\sin(32°) = \frac{12}{x}$

(x) $\sin(32°)= 12$

$x = \frac{12}{\sin(32°)}$

$x = \frac{12}{0.52992} \cong 22.64$

Example 7:



Question:
What trigomometric function should we to find the size of the angle theta.

SOH CAH TOA

T = $\frac{O}{A}$ = $tangent = \frac{Opposite}{Adjecent}$

tan(θ) = $\frac{Opposite}{Adjecent}$ = $\frac{5}{4}$ = 1.25

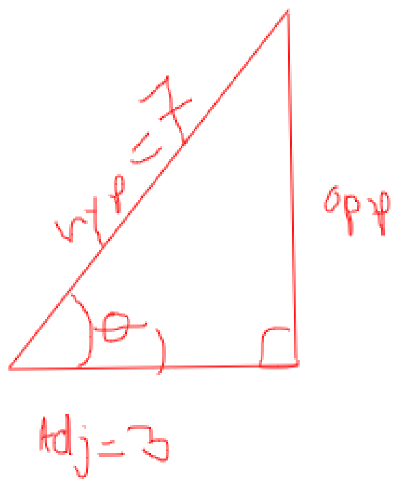
tan(θ) = 1.25

$\tan^{-1}(\tan(\theta)) = \tan^{-1}(1.25)$

$\theta = \tan^{-1}(1.25)$

$\theta \cong$ 51.34°

Example 8:



Question:
Find the size of the angle theta based soley on the facts given.

SOH CAH TOA

C = $\frac{A}{H}$ = $cosine = \frac{Adjecent}{Hypotenuse}$

cos(θ) = $\frac{Adjecent}{Hypotenuse}$ = $\frac{3}{7}$

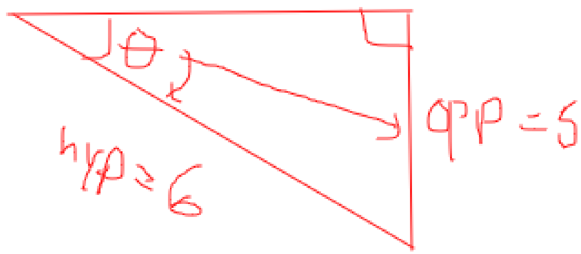
$\cos^{-1}(\cos(\theta)) = \cos^{-1}(\frac{3}{7})$

$\theta = \cos^{-1}(\frac{3}{7})$

$\theta \cong 64.62\,°$

Example 9:

Question:
Based on the given facts in the diagram, what trigonometric function can we use to calculate the angle?



SOH CAH TOA

S = $\frac{O}{H}$ = $sine = \frac{Opposite}{Hypotenuse}$

sin(θ) = $\frac{Opposite}{Hypotenuse}$ = $\frac{5}{6}$

$\sin^{-1}(\sin(\theta)) = \sin^{-1}(\frac{5}{6})$

$\theta = \sin^{-1}(\frac{5}{6})$

$\theta \cong$ 56.44°

Date Created: Thursday, Oct. 21st, 2021

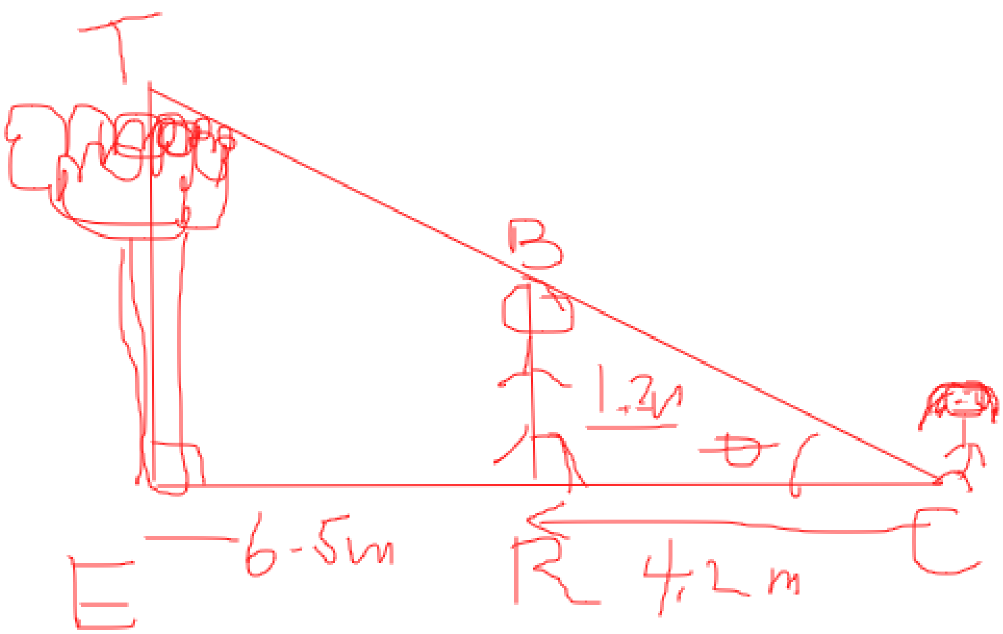
Word Problem: Similar Triangles – All the angles are the same size, and the length of the sides are a ratio to the other triangle.

Example 1:

Objective: To measure the height of a tree

To measure the height of a tree, Cynthia has her little brother, BR, stand so that the tip of his shadow coincides with the tip of the tree's shadow, at Point C.

Cynthia's brother, who is 1.2 m tall, is 4.2 m from Cynthia, who is standing at C, and 6.5 m from the base of the tree. Find the height of the tree TE. Solve to the nearest tenth of a metre.



Objective: Find TE

$\frac{TE}{EC} = \frac{BR}{RC}$ 🡐 Isolate TE

TE = $\frac{BR}{RC}$ x EC

TE = $\frac{1.2}{4.2}$ x $10.7 = 3.057$

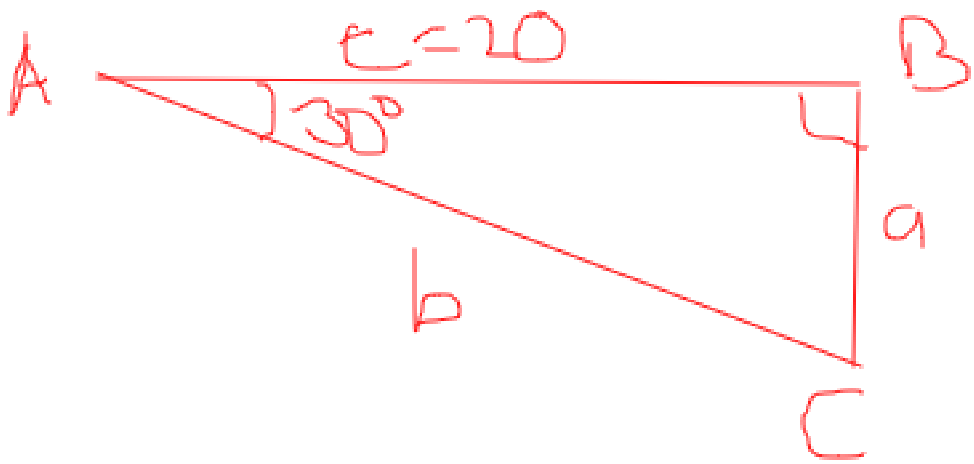
TE ≅ 3.1 metres.

Literacy:

To solve a triangle - means to find all the six (6) measurements of a triangle:

1. 3 sides
2. 3 angles.

Example: 2 – Solve a triangle.



Solution:

Label all the sides according to the labels required.

Find Angle <C.

<C = 180° - 30° - 90°

<C = 180° - 120°

<C = 60°

Find side b, which is the hypotenuse:

SOH CAH TOA

C = $\frac{A}{H}$ = cosine $= \frac{Adjecent}{Hypotenuse}$

$\cos(\theta) = \frac{Adjecent}{Hypotenuse}$

$\cos(\theta) = \frac{Adjecent}{Hypotenuse} = \frac{Adjecent}{b}$

$\cos(30°) = \frac{Adjecent}{Hypotenuse} = \frac{20}{b}$

$b \cong \frac{20}{\cos(30°)} = \frac{20}{\cos(30°)} = 23.094$

Find side a:

TOA

T = $\frac{O}{A}$ = tangent $= \frac{Opposite}{Adjacent}$

$\tan(\theta) = \frac{Opposite}{Adjacent}$

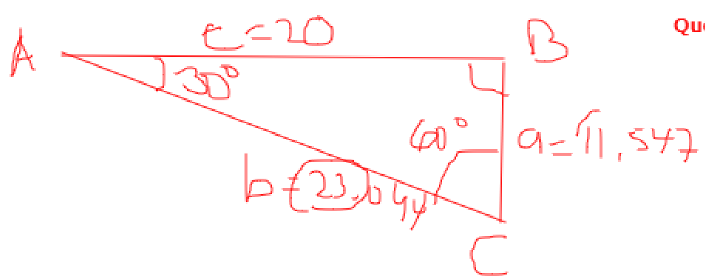
$\tan(\theta) = \frac{Opposite}{Adjacent} = \frac{Oppisite}{20}$

$\tan(30°) = \frac{Opposite}{Adjacent} = \frac{c}{20}$

c = (20) tan(30°) = (20) (0.57735)

$c \cong 11.547$

3 Angles	3 sides
a = 11.547	$<A = 30°$
b = 23.094	$<B = 90°$
c = 20	$<C = 60°$



Question: Solve the triangle.

Vocabulary:

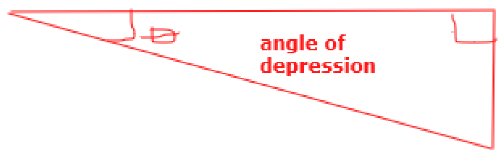
Angle of Depression: -

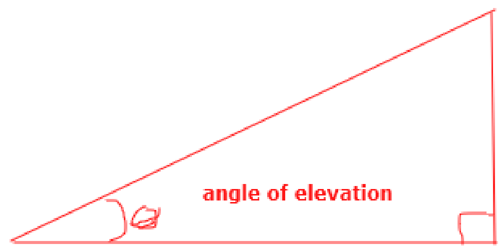
- is the angle below the horizontal

- also called the angle of **declination**.

Angle of Elevation: -

- angle measurement above the horizontal.
- also called the angle of inclination.





Example 3:

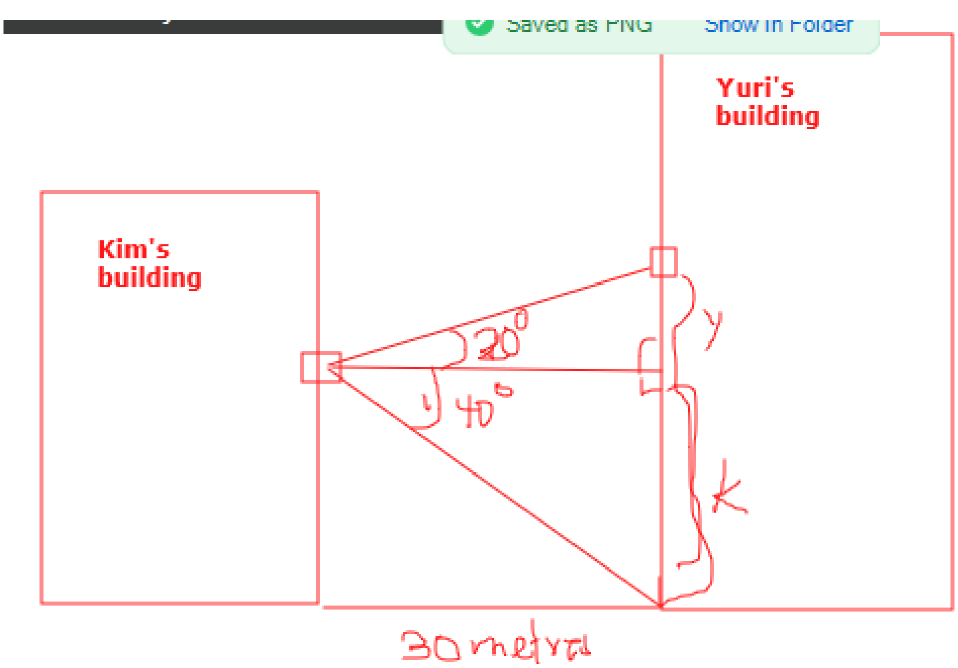
Kim and Yuri live in apartment buildings next to each other that are 30 meters apart as shown in diagram.

The **angle of depression** from Kim's balcony to where Yuri's building meets the ground at a 40° angle.

The angle of elevation from Kim's balcony to Yuri's balcony is at a 20°

Objective:

1. How high is Kim's balcony above the ground to the nearest meter? Here we are determining the vertical height of Kim's apartment and label k in the diagram.
2. How high is Yuri's balcony above the ground to the nearest meter?



Let k be the height of Kim's apartment above the ground.
We have one known side of the triangle, which is 30 meters, which is the adjacent side to the both angles 40°, and 20°, which are the angles of depression and elevation respectively.

TOA

$\tan(\theta) = \frac{Opposite}{Adjacent}$

$\tan(40°) = \frac{Opposite}{Adjacent} = \frac{k}{30}$

solve for k

k = (30) $\tan(40°)$

k = (30) (0.839099) = 25.1730
k $\cong 25$ metres.

Let y be the height of Yuri's apartment from Kim's apartment.

solve for y

SOH CAH TOA

TOA

$\tan(\theta) = \frac{Opposite}{Adjacent}$

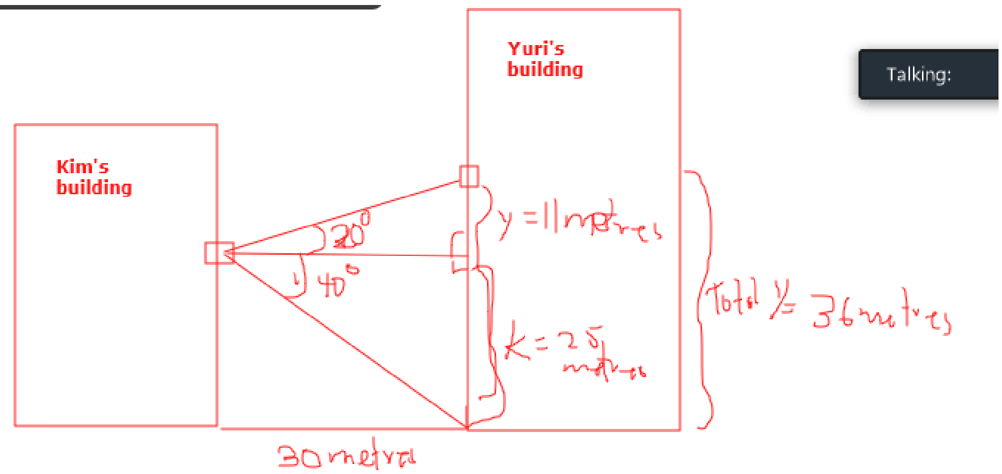
$\tan(20°) = \frac{Opposite}{Adjacent} = \frac{y}{30}$

y = (30) $\tan(20°)$
y = (30) (0.363970) = 10.919
y $\cong 11$ metres.

Therefore, the height of Yuri's apartment relative to Kim's is 11 meters.

The total height of Yuri's apartment relative to the ground is 11 + 25 = 36 meters.

The vertical height for Kim's apartment relative to the round is 25 meters.



Activity 15: Find the distance between two people from the based of a flag pole.

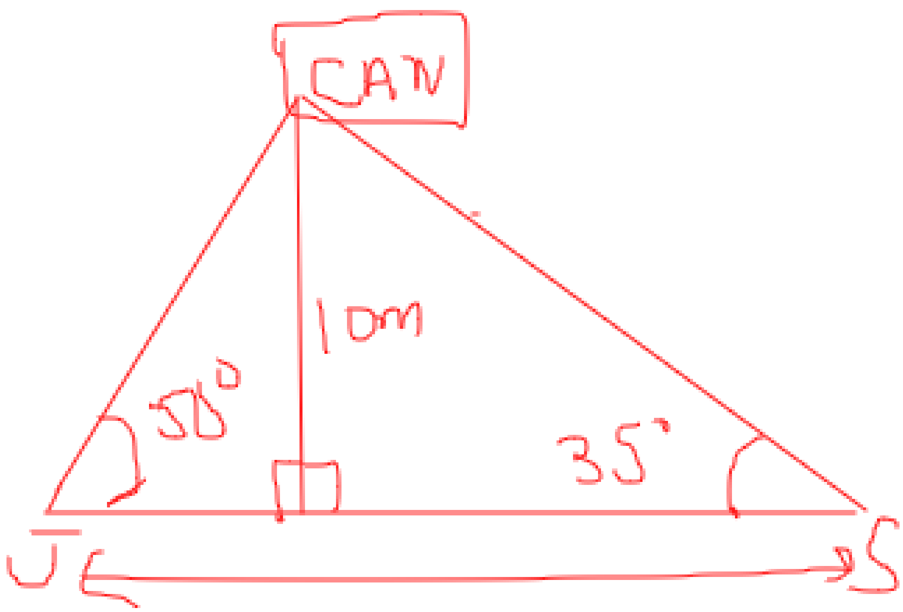
Word Problem:

Jack and Sangita are facing each other on the opposite sides of a 10-metre flagpole.

From Jack's point of view, the top of the flagpole is at an angle of elevation of 40°.

From Sangita's point of view, the top of the flagpole is at an angle of elevation of 35°.

Question: How far apart are the Jack and Sangita?



Find the distance between Jack and Sangita?

Topic: Trigonometry of Acute Angles.

Vocabulary: -

sine law

Cosine law

Trigonometry comes from two Greek words, that together means “measurement of triangles”.

Triangles are special!! Any polygon can be split into two or more triangles. If you can solve triangles, you can solve many more complex polygons.

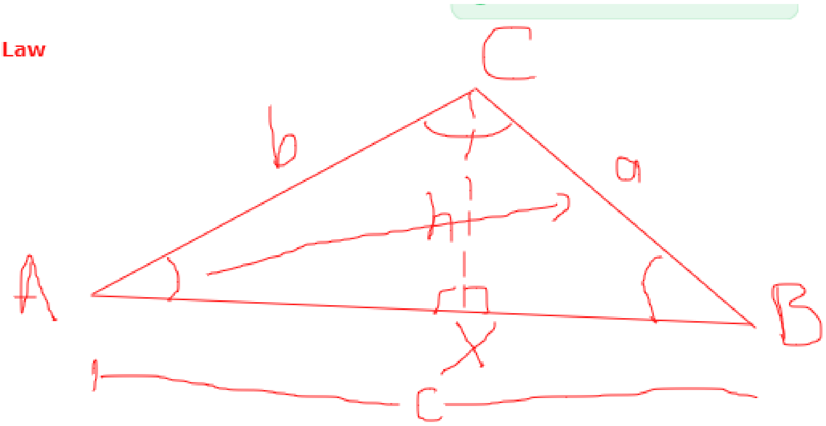
1. **The Sine law:**

   Sine Law ← Acute Angles

$$\frac{a}{Sine(A)} = \frac{b}{Sine(B)} = \frac{c}{Sine(C)}$$

The Sine Law is the relationship between, the sides and their opposite angles in any acute triangle Δ ABC

Develop the Sine Law

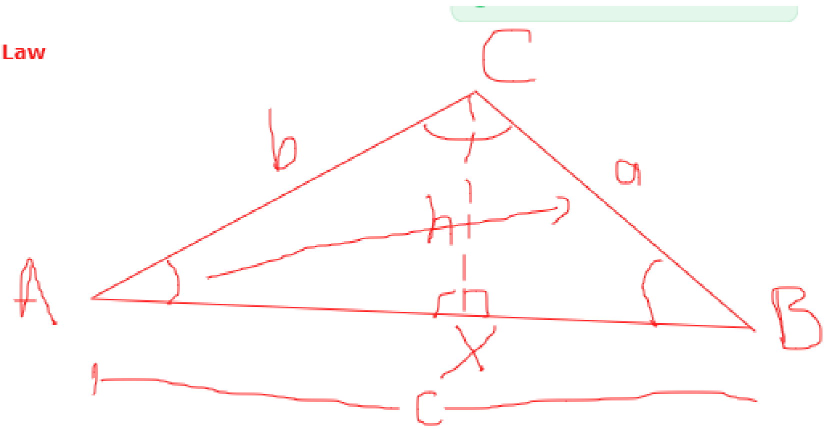


Step 1: Focus on ΔAXC

Find the height h in terms of other sides.

sin C = S = $\frac{O}{H}$ = $sine\ C = \frac{Opposite}{Hypotenuse}$

Develop the Sine Law



Solve for h.

SOH CAH TOA

sin A = $sine\ A = \frac{Opposite}{Hypotenuse}$

sin A = $\frac{h}{b}$

h = b · sin A 🡐 focusing on triangle **ΔAXC**

Step 2: Focus on ΔBXC

Find the height h in terms of other sides.

sin B = $sine\ B = \frac{Opposite}{Hypotenuse}$

sin B = $\frac{h}{a}$

h = a · sin B 🡐 focusing on triangle **ΔBXC**

Step 3: Equate both h equation of triangles: ΔAXC and ΔBXC

h = b · sin A; h = a · sin B

b · sin A = a · sin B 🡐 divide both sides by sin B

$\frac{\text{b} \cdot \sin \text{A}}{\sin \text{B}} = \frac{\text{a} \cdot \sin \text{B}}{\sin \text{B}}$

$\frac{\text{b} \cdot \sin \text{A}}{\sin \text{B}} = \text{a}$ 🡐 divide both sides by sin A

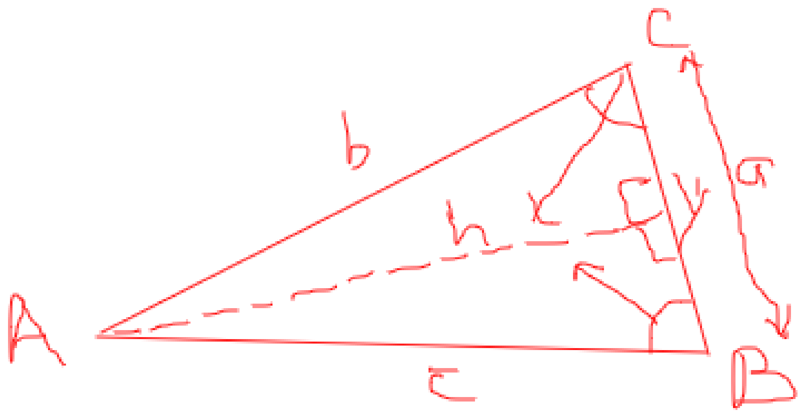
$\frac{\text{a}}{\sin \text{A}} = \frac{\text{b}}{\sin \text{B}}$ 🡐 Similarity we can do the same for the other triangle.

Date Created: Friday, October 21st, 2021
Date Modified: Friday, February 23rd, 2023

Completion of derivation of Sine Law for $\frac{c}{\sin C}$

The above process can be repeated using a different altitude. See diagram below:

Complete Derivation of Sine Law:



Step 4: Focus on ΔCYA

SOH CAH TOA

sin C = $sine\ C = \frac{Opposite}{Hypotenuse}$

sin C = $\frac{h}{b}$

h = b · sin C

Step 5: Focus on ΔBYA

sin B = $sine\ B = \frac{Opposite}{Hypotenuse}$

sin B = $\frac{h}{c}$

h = c · sin B

Step 6: Equate both h equation of triangles: ΔCYA and ΔBYA

h = b · sin C

h = c · sin B

b · sin *C* = c · sin B 🡐 divide both sides by sin B

$$\frac{b \cdot \sin C}{\sin B} = \frac{c \cdot \sin B}{\sin B}$$

$\frac{b \cdot \sin C}{\sin B}$ = c 🡐 divide both sides by sin C

$$\frac{b \cdot \sin C}{\sin B \cdot \sin C} = \frac{c}{\sin C}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Include $\frac{a}{\sin A} = \frac{b}{\sin B}$

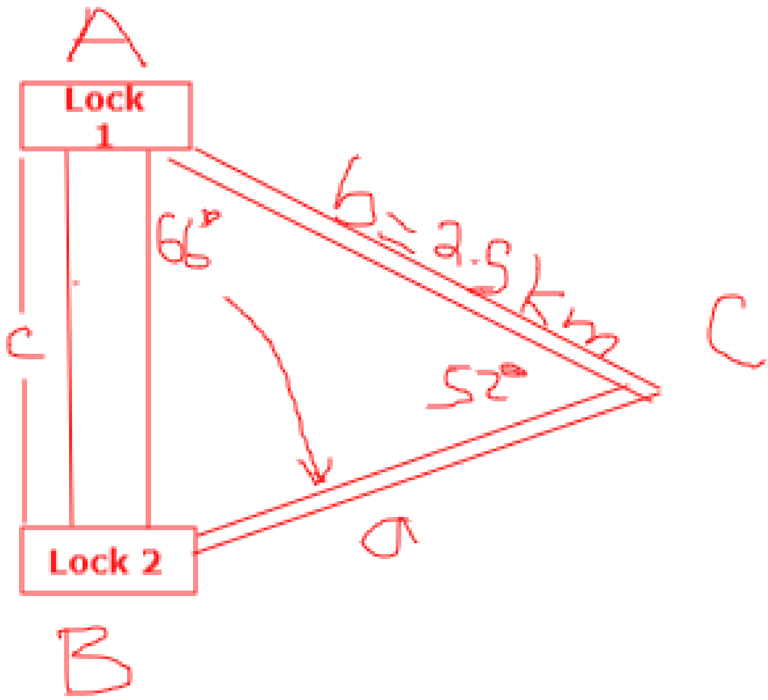
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ 🡐 Sine Law developed from diagram above.

Example 1: Find a side length using the Sine Law:

A bicycle path forms a 66° angle with one lock of a canal.

At a distance of 2.5 km along the bicycle path, the angle separating this lock from the next lock is 52°.

How far apart are the two locks, to the nearest tenth of a km.



Using the Sine Law:

$\frac{b}{\sin B} = \frac{c}{\sin C}$

<B = 180° – 66° – 52°
<B = 62°

$\frac{b}{\sin 62°} = \frac{c}{\sin 52°}$

$c = \frac{b \cdot \sin 52°}{\sin 62°}$

$c = \frac{(2.5) \cdot (0.78801)}{(0.88294)}$

c = 2.2311 km

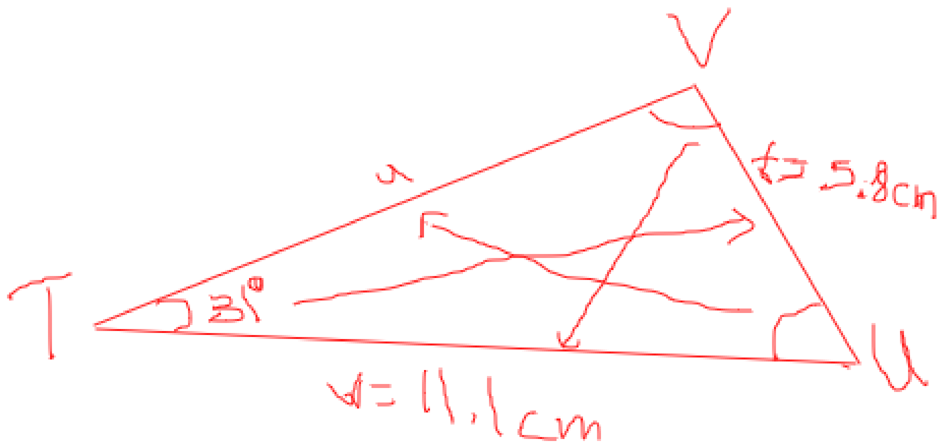
c = 2.2 km

By using the Sine Law, Lock 1 and Lock 2 are 2.2 km apart.

Example 2: Find an angle using the Sine Law:

In an acute ΔTUV,

given:
TU = 11.1 cm
UV = 5.8 cm
$<T = 31°$



Find the measurement of the $<V$, to the nearest degree.

Using the Sine Law:

$$\frac{t}{\sin T} = \frac{v}{\sin V} = \frac{u}{\sin U} \quad \text{or}$$

$$\frac{\sin T}{t} = \frac{\sin V}{v} = \frac{\sin U}{u}$$

$$\frac{\sin T}{t} = \frac{\sin V}{v}$$

$$\sin V = \frac{v \cdot \sin T}{t}$$

$$\sin V = \frac{(11.1) \cdot \sin 31°}{5.8}$$

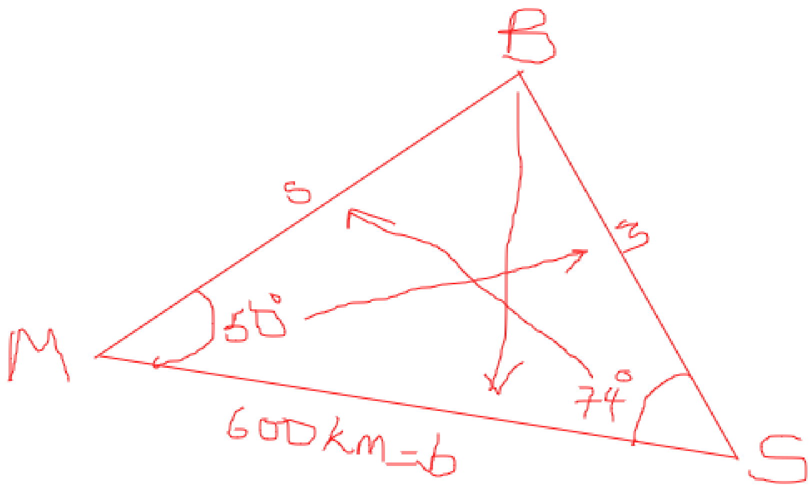
$$\sin V = \frac{(11.1) \cdot (0.515038)}{5.8}$$

$\sin V = 0.985676$
$\sin^{-1}(\sin V) = \sin^{-1}(0.985676)$
$<V \cong 80.29°$

$<V \cong 80°$ is the nearest degree.

<u>Activity 16: Using the Sine Law – Find the Perimeter Length of the Bermuda Triangle:</u>

Use the information given on the diagram to determine the perimeter of the Bermuda Triangle, to the nearest kilometer.



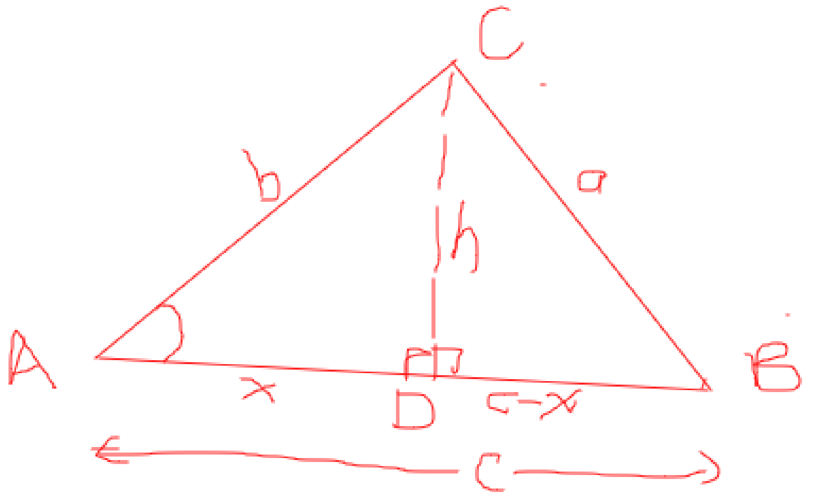
M is the label for the city of Miami.
B is the label for Bermuda.
S is the label for the city of San Juan of Portico Rico.

Let s is the length of distance between Miami and Bermuda.
Let m is the length in km distance between San Juan and Bermuda.

Find the Total length (Perimeter) to the nearest km.

<u>Hint: Use the Sine Law.</u>

The Cosine Law: - is the relationship between the cosine of an angle and its lengths of the three sides in any acute Δ ABC.



The Cosine Law relates to the cosine of any angle to the three side lengths of an acute triangle.

To derive the Cosine Law, draw a triangle and add the altitude, h, from on of the vertices.

The altitude splits ΔABC into two right angle triangles ΔADC and ΔBDC.

Let AD = x.

Let DB = c – x.

Step 1: Focus on ΔADC

From the Pythagorean Theorem:
$b^2 = x^2 + h^2$

SOH CAH TOA

Also, cosine ration gives:
$\cos A = \frac{Adjacent}{Hypotenuse} = \frac{x}{b}$

$x = b \cdot \cos A$

Step 2: Focus on ΔBDC

From the Pythagorean Theorem:
$a^2 = h^2 + (c - x)^2$ ← expand term

$a^2 = h^2 + c^2 - 2cx + x^2$ ← rearrange formula

$a^2 = x^2 + h^2 + c^2 - 2cx$

$a^2 = (x^2 + h^2) + c^2 - 2cx$

$a^2 = (x^2 + h^2) + c^2 - 2cx$

From step 1, previous triangle ΔADC

$b^2 = x^2 + h^2$ 🡐 substitute into equation below.

$a^2 = (x^2 + h^2) + c^2 - 2cx$

$a^2 = b^2 + c^2 - 2cx$

From step 1, previous triangle ΔADC

$x = b \cdot \cos A$ 🡐 substitute into equation below.

$a^2 = b^2 + c^2 - 2cx$
$a^2 = b^2 + c^2 - 2c(b \cdot \cos A)$
$a^2 = b^2 + c^2 - 2bc(\cos A)$

∴ formula below, lets you calculate the length of a, if you know the lengths of b, and c, and the <A.

$a^2 = b^2 + c^2 - 2bc(\cos A)$

Similarly, you can derive the other two formulas for the other sides the same way.

$b^2 = a^2 + c^2 - 2ac(\cos B)$

$c^2 = a^2 + b^2 - 2ab(\cos C)$

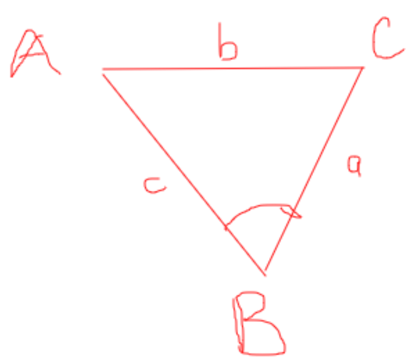
Literacy Connection:

A **contained angle** is the interior angle that is formed at the vertex of the two adjacent sides in a triangle.

If we have a contained angle, we can solve for the side opposite side (b) to the <B.

Example of a contained Angle

The sides a and c contains <B



Example 1: Find length of side b using the Cosine Law:

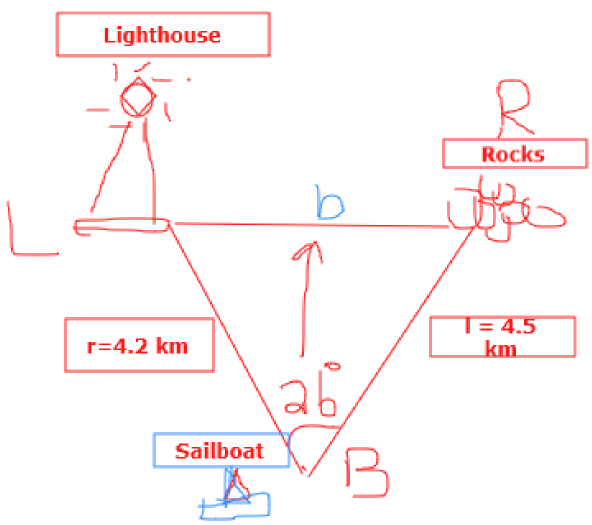
Occasionally, when a ship's navigational systems are not working, a ship's captain can use trigonometry to find important distances and directions.

It is very important for ships to avoid hazards such as rocks that protrude through shallow waters. In the example below, the boat is sailing North through a narrow straight marked by distance d.

A lighthouse (L) marks the Western Shoreline, while a buoy indicates a rock hazard (R) directly east of the lighthouse, as shown in diagram below.

The $<$B is 26°, r = 4.2 km, l = 4.5 km

What width is the section of the straight (mark b) to the nearest tenth of a kilometer?



Question: Can we use the Sine Law to solve this problem?
Response: NO! This problem can not be solved with Sine Law since we do not have one complete ratio.

$$\frac{\sin L}{l} = \frac{\sin R}{r} = \frac{\sin B}{b}$$

Substitute Given facts:

$$\frac{\sin L}{4.5} = \frac{\sin R}{4.2} = \frac{\sin 26°}{b}$$

Solve the Problem above with the Cosine Law:

$b^2 = r^2 + l^2 - 2rl(\cos B)$

Substitute values:

$<B$ is 26°
r = 4.2 km
l = 4.5 km

$b^2 = (4.2)^2 + (4.5)^2 - 2(4.2)(4.5)(\cos 26°)$

$b^2 = 17.64 + 20.25 - (37.8)(0.898794)$

$b^2 = 37.89 - 33.9744$

$b^2 = 3.9155$
$b = \sqrt{3.9155} = 1.9787$

$b \cong 2.0$ km.

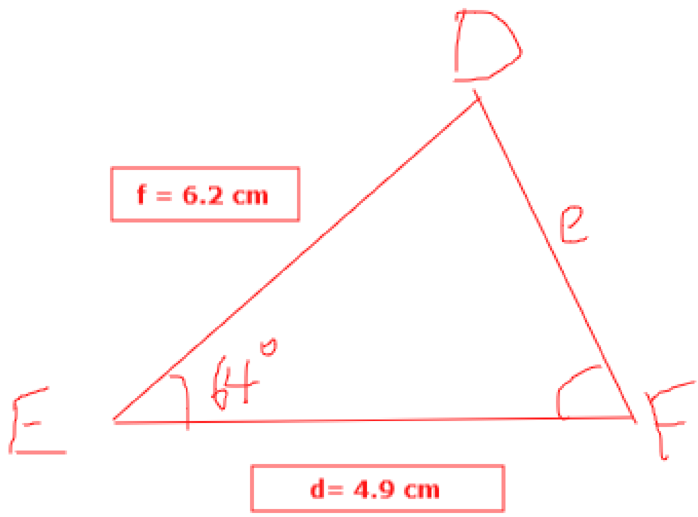
$\therefore$ The width of the straight is approximately 2.0 km

Example 2: Solve the Triangle:

In acute triangle ΔDEF, d = 4.9 cm, f = 6.2 cm and <E = 64°. Solve ΔDEF.

To solve a triangle means to find the values for the 3 sides and 3 angles.

Round the measurements to the nearest degree or the tenth of a centimeter.



Note: <E is a contained interior angle of sides f, e of known values.

Since we know the measurements of <E, and values sides f, e, we can determine the length of side e.

Using the Cosine Law for the triangle above ΔDEF:

$e^2 = d^2 + f^2 – 2df(\cos E)$

d = 4.9
f = 6.2
<E = 64°

$e^2 = (4.9)^2 + (6.2)^2 – 2(4.9)(6.2)(\cos 64°)$

$e^2 = 24.01 + 38.44 – 2(4.9)(6.2) (\cos 64°)$

$e^2 = 62.45 – 60.76(0.4383711)$
$e^2 = 62.45 – 26.6354$
$e^2 = 35.81$
$\sqrt{e^2} = \sqrt{35.81}$

$e \cong 5.98$ cm
$e \cong 6.0$ cm

Using the Sine Law:

$$\frac{\sin F}{f} = \frac{\sin E}{e} = \frac{\sin D}{d}$$

$$\frac{\sin D}{d} = \frac{\sin E}{e}$$

$$\sin D = \frac{d \cdot \sin E}{e}$$

d = 4.9
e = 6.0 (Just calculated)
<E = 64°

$$\sin D = \frac{(4.9) \cdot \sin 64°}{6.0}$$

$$\sin D = \frac{(4.9) \cdot (0.898794)}{6.0}$$

sin D = 0.734015

$\sin^{-1}(\sin D) = \sin^{-1}(0.734015)$

<D ≅ 47.22°

<D ≅ 47° is the nearest degree.

Total interior angles of a Triangle is 180°.

<F = 180° - 64° - 47°

<F = 180° - 111°

<F ≅ 69°

Solving triangle ΔDEF is a following:

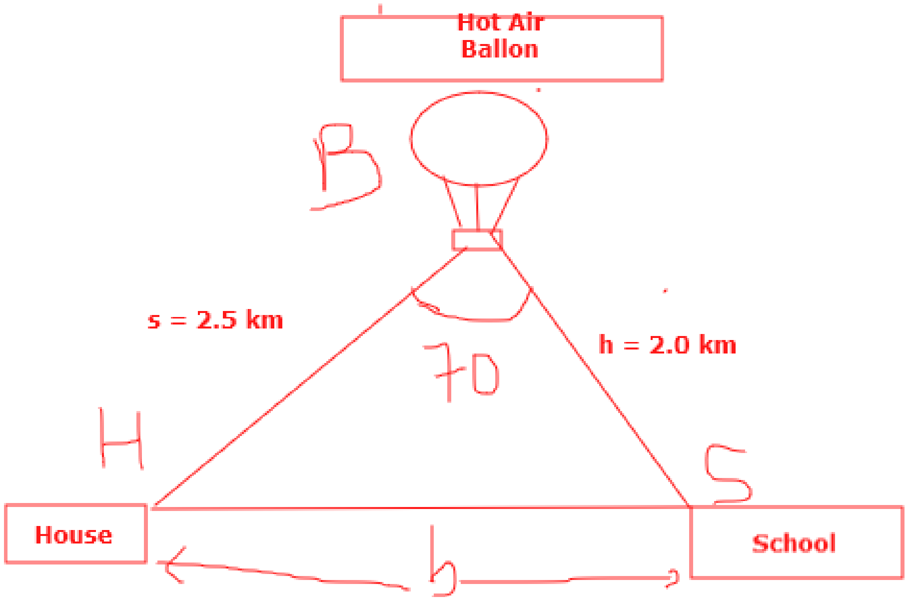
Sides:

d = 4.9
e = 6.0 (Just calculated)
f = 6.2

Angles:

<D ≅ 47° (Just calculated)
<E = 64°
<F ≅ 69° (Just calculated)

Activity 17: Use the Cosine Law to find the distance between two objects:



Chandra is riding in a hot-air balloon and spots her house and her school. She estimates how far away they are from her, and the angle separating their lines of sight, as shown in above.

1. How far apart is Chandra's house and school, that is, solve for b.