THE ERINDALE ACADEMY

COURSE NAME: MPM2D – Principles of Mathematics (Final Exam – Accumulative)

MPM2D: Final Exam

Teacher: Antonio Pietrangelo

Time: (3 + 1) Hours – with ESL accommodation

Pages: 20

Student's Name: Instructor
Student#: Answer Key

Due Date: Monday, October 25th, 01:00 pm (EST)

Mark: /100

Categories	Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application
Symbol	K/U	T/I	C	A
Weight	25 %	25 %	25 %	25 %
Level				

Overall Expectations:

Expectations as listed in the Ontario Curriculum course outline for your specific course.

Specific Expectations

Unit 1 - Systems of Linear Equations
1.1 Representing Linear Relations
1.2 Solving Linear Equations
1.3 Graphically Solving Linear Systems
1.4 Solving Linear Systems: Substitution
1.5 Equivalent Linear Systems
1.6 Solving Linear Systems: Elimination
1.7 Exploring Linear Systems

Chapter 2: Analytic Geometry: Line Segments and Circles
2.1 Midpoint of a Line Segment
2.2 Length of a Line Segment
2.3 Equation of a Circle
2.4 Classifying Figures on a Coordinate Grid
2.5 Verifying Properties of Geometric Figures
2.6 Exploring Properties of Geometric Figures
2.7 Using Coordinates to Solve Problems

Chapter 3: Graphs of Quadratic
3.1 Exploring Quadratic Relations

Rubrics:

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Knowledge – Understanding of: **(Final Exam: Topics: 1.1 to 8.5)**	demonstrates insufficient understanding	demonstrates limited understanding	demonstrates some understanding	demonstrates considerable understanding	demonstrates thorough understanding	
					Individual: **Assigned:**	_____

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Thinking and Inquiry (What if scenarios) of **(Final Exam: Topics: 1.1 to 8.5)**	demonstrates insufficient ability to apply different scenarios	demonstrates limited ability to apply different scenarios	demonstrates some ability to apply different scenarios	demonstrates considerable ability to apply different scenarios	demonstrates through ability to apply different scenarios	
					Individual:	_____

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Category	Level R (0 – 49%	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Communication **Communicates effectively with the use of** **(Final Exam: Topics: 1.1 to 8.5)**	demonstrates **insufficient** ability to communicate effectively	demonstrates **limited** ability to communicate effectively	demonstrates **some** ability to communicate effectively	demonstrates **considerable** ability to communicate effectively	demonstrates **through** ability to communicate effectively	
					Individual:	_____

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Category	Level R (0 – 49%	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Application: Demonstrates the ability to apply mathematical principles to real world situations. **(Final Exam: Topics: 1.1 to 8.5)**	demonstrates **insufficient ability**	demonstrates **limited ability**	demonstrates **some ability**	demonstrates **considerable ability**	demonstrates **thorough ability**	
					Individual:	_____

THE ERINDALE ACADEMY

PART A: KNOWLEDGE AND UNDERSTANDING (K/U) – 25%

2 Marks Per Question

Instructions:

Question 1: (True or False) equation for midpoint is M(x,y) = $(\frac{run}{2}, \frac{rise}{2})$?

False

Midpoint M(x,y) = $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

Note: rise = Δy = y2 – y1; run = Δx = x2 – x1

Question 2: (True or False) An Isosceles triangle has two sides that are the same?

True

Question 3: (True or False) An Isosceles triangle as two angles that are the same?

True

Question 4: (True or False) An equilateral triangle has no sides the same?

False

Question 5: The sin90° has a value of 1? (True or False)

True

Question 6: The Centroid of a triangle is a single point outside the triangle? (True or False)

False

Question 7: A Centroid splits the medians of a triangle by a ratio of 2:1? (True or false)

True

Question 8: The cos90° is equal to 1? (True or False)

False

Question 9: **The tangent of angle is a ratio two sides of a right-angle triangle: Adjacent / Opposite? (True or False)**

False

Question 10: **A scalene triangle has all the sides the same? (True or false)**

False

Question 11: **When two triangles are the identical, they are said to be congruent? (True or false)**

True

Question 12: **An Obtuse triangle is a triangle that has one angle equal to 90 degrees? (True or False)**

False

Question 13: **A square is not a parallelogram? (True or False)**

False

Question 14: **A parallelogram has two sides that are parallel and the other two sides are not parallel? (True or False)**

False

Question 15: **(True or False) The equation for the midpoint of a line segment is M(x,y) =** $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$

True

Question 16: **A trapezoid is a quadrilateral with exactly one pair of parallel lines? (True or false)**

True

Question 17: **A Rhombus is a parallelogram with all sides equal? (True or False)**

True

Question 18: **A rectangle is a parallelogram with 4 right angles, but sides has two pair of sides with equal lengths? (True or false)**

True

Question 19: **A square has 4 equal sides with only two angles that are 90 degrees? (True or False)**

False

Question 20: **The right bisector of an isosceles triangle splits a triangle into two equal parts? (True or False)**

True

Question 21: **The equation of $y=ax^2$, if a is negative the parabola has a maximum? (True or False)**

True

Question 22: **The equation of $y=ax^2$, if a is between $0 < a < 1$, the parabola is widened or compressed? (True or False)**

True

Question 23: **The equation of $y=a(x-h)^2$, when $h > 0$ the quadratic is shifted or transformed upwards or downwards by the h value? (True or False)**

False

Question 24: **For the equation $y=x^2$, there is no minimum for y? (True or False)**

False

Question 25: **For the equation $y = -x^2$ maximum value for y is zero? (True or False)**

True

Question 26: **The line of symmetry for a parabola is at x value of the vertex(x, y)? (True or False)**

True

Question 27: **A minimum or a maximum is on the vertex of a quadratic equations? (True or False)**

True

Question 28: **The y-coordinate of the vertex(x, y) is the point where the line of symmetry is for a quadratic equation? (True or False)**

False

Question 29: **The equation of y=a(x-r)(x-s), the r, and s values are where the zeros occur? (True or False)**

True

Question 30: **The equation of $y=x^2 + k$, the k transforms the quadratic relation left or right? (True or False)**

False

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PART B: THINKING AND INQUIRY (T/I) – 25 %

5 Marks Per Question

Show your work: If you do not you will get zero.

Question 1: **Find the slope between the two Points P(2,6) and Q(8, 12)?**

Slope = $\mathbf{M_{pq}} = \frac{\Delta y}{\Delta y} = \frac{y2-y1}{x2-x1} = \frac{12-6}{8-2} = \frac{6}{6} = 1$

Question 2: **Find the MidPoint between the two points A(2, -5) and B(5, 7)**

Midpoint $\mathbf{M_{AB}(x,y)} = (\frac{2+5}{2}, \frac{-5+7}{2}) = (\frac{7}{2}, \frac{2}{2}) = (\frac{7}{2}, 1)$ ✓

Question 3: **Find the equation of line that is perpendicular to circle $x^2 + y^2 = 10^2$ and perpendicular to the points A(0,10) and B(8, 6)?**

Slope M_{AB} = $\frac{\Delta y}{\Delta y} = \frac{6-10}{8-0} = \frac{-4}{8} = \frac{-1}{2}$

Mid Point of M_{AB} = $(\frac{x1+x2}{2}, \frac{y1+y2}{2}) = (\frac{0+8}{2}, \frac{10+6}{2}) = (\frac{8}{2}, \frac{16}{2}) = (4, 8)$

Lines that are perpendicular, the product of their slopes is -1.

$M_2 \cdot M_1$ = -1.

M_2 = $\frac{-1}{\mathbf{M1}} = \frac{-1}{(\frac{-1}{2})} = \frac{(\frac{-1}{1})}{(\frac{-1}{2})} \cdot \frac{(\frac{-2}{1})}{(\frac{2}{1})} = \frac{(\frac{-2}{1})}{(\frac{-2}{2})} = \frac{(\frac{-2}{1})}{(\frac{-2}{2})} = \frac{(\frac{-2}{1})}{(\frac{-1}{1})} = 2$

y = mx + b ← **substitute slope and a point to find y-intercept.**
8 = (2)(4) + b
8 = 8 + b
8 – 8 = b
0 = b

Equation of a line that is perpendicular to points A(0,10), B(8, 6) is y = 2x.

Question 4: **Identify two points that are on the circumference of the circle $x^2 + y^2 = 4^2$**

Two points that are on the circumference of the circle are
for example: **(0, 4), (0, -4), (4, 0), (-4, 0)**.

Question 5: Evaluate the algebraic expression when a = 2, b = 3, c = -2, d = -1

$3a^3 + 4b + 3c - 2d = 3(2)^3 + 4(3) + 3(-2) - 2(-1)$
$= 24 + 12 - 6 + 2$
$3a^3 + 4b + 3c - 2d = 32$

Question 6: **Find the vertex for equation $y = (x - 2)^2 + 5$**

Vertex(x, y) = (h, k) = **(2, 5)**

Question 7: Find the line of symmetry for y = 2(x-6)(x+5)

Axis of Symmetry $x = \frac{(r + s)}{2} = \frac{(6 + -5)}{2} = \frac{1}{2}$

Question 8: **Find the sinA, cosA, tanA, when A is an able of 45°**

sin45° ≅ 0.70711

cos45° ≅ 0.70711

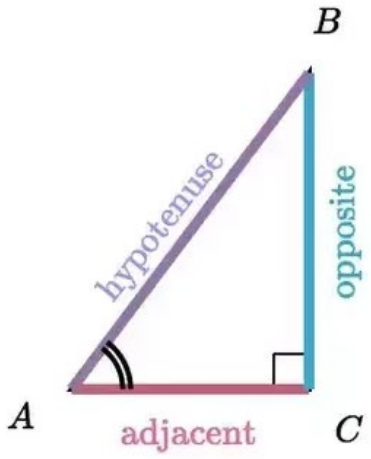
tan45° = 1

PART C: COMMUNICATION (C) – 25%

10 Marks Per Question

Question 1: In trigonometry explain the acronym SOH CAH TOA in terms of the three primary trigonometric functions.

Example the acronym:



$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}}$$

Based on the definitions of the three primary trigonometric identities as stated in the above diagram, by using the first character of each identity you will get SOH CAH TOA.

For example:

$$\mathbf{s}\text{in(A)} = \frac{Opposite}{Hypotenuse} = \text{SOH}$$

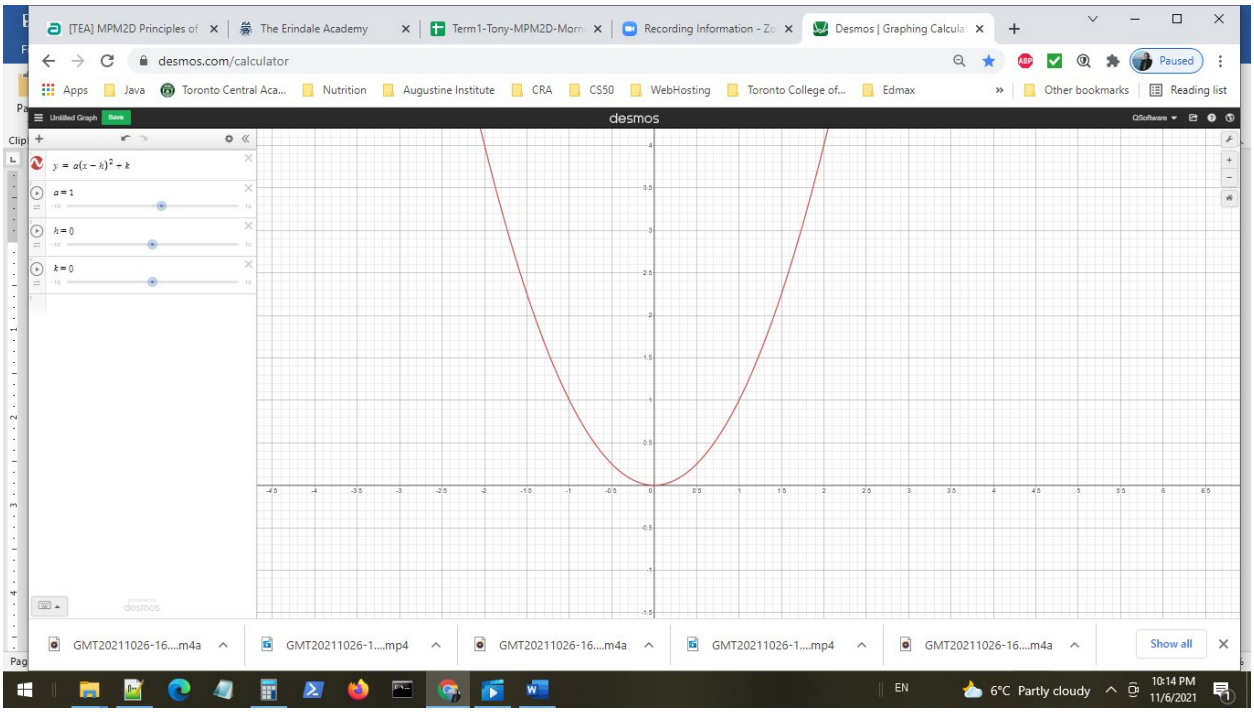
$$\mathbf{c}\text{os(A)} = \frac{\boldsymbol{Adjacent}}{\boldsymbol{Hypotenuse}} = \text{CAH}$$

$$\mathbf{t}\text{an(A)} = \frac{Opposite}{Hypotenuse} = \text{TOA}$$

Question 2: Quadratic Equation form $y = a(x - h)^2 + k$

Please explain how does variables a, h , and k affect the transformations of a parabola relative to $y = x^2$?

Example the affects of the three variables:



$y = a(x - h)^2 + k$, $a \neq 0$, h and k are real numbers

(1) h shifts the parabola horizontally.
  (a) If $h > 0$ parabola shifts to the right,
  (b) If $h < 0$ parabola shifts to the left
(2) K shifts the parabola vertically
  (a) If $k > 0$ the shift is vertically upward.
  (b) If $k < 0$ the shift is downwards.
(3) a determines if the parabola, is compressed or stretched.
  (a) If $a > 0$,
    (i) the parabola opens upwards
    (ii) the parabola has a minimum
  (b) If $a < 0$,
    (i) the parabola opens downwards.
    (ii) the parabola has a maximum
  (c) if $a > 1$
    (i) the parabola stretches more quickly upwards

(ii) factored by a

(d) if a < -1

(i) the parabola stretches more quickly downwards

(ii) factored by a

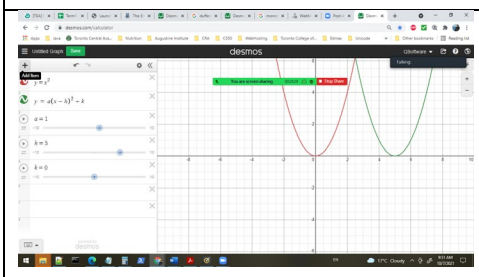
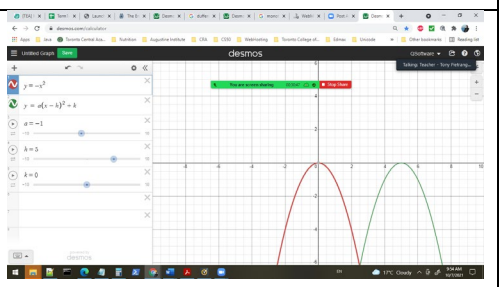
(e) if a is a positive fraction

(i) the parabola is compressed or flatted

(f) if a is a negative fraction

(i) the parabola is compressed or flatted

(4) Vertex (x, y) = (h, k)

Vertex (x, y) = (h, k) a > 0 maximium occurs	Vertex (x, y) = (h, k) a < 0 minimum occurs
	

(5) Axis of symmetry occurs at h.

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PART D: APPLICATION (A) – 25%

10 Marks Per Question

Use your own graph paper?

<u>Question 1</u>: Draw quadrilateral and label the points A(-2, 1), B(3, 3), C(4, -1), and D(-1, -3)

The midpoint to line segment AB call it E.

The midpoint to line segment BC call it F.

The midpoint to line segment DC call it G.

The midpoint to line segment AD call it H.

What type of parallelograms are ABCD, and EFGH?

Points: A(-2, 1), B(3, 3), C(4, -1), D(-1,-3)

$Mid_{AB} = (\frac{-2+3}{2}, \frac{1+3}{2})$	$E(x, y) = (\frac{1}{2}, \frac{4}{2}) = (\frac{1}{2}, 2)$	$E(\frac{1}{2}, 2)$
$MidBC = (\frac{3+4}{2}, \frac{3+(-1)}{2})$	$F(x, y) = (\frac{7}{2}, \frac{2}{2}) = (\frac{7}{2}, 1)$	$F(\frac{7}{2}, 1)$
$Mid_{CD} = (\frac{4+(-1)}{2}, \frac{-1+-3}{2})$	$G(x, y) = (\frac{3}{2}, \frac{4}{2}) = (\frac{3}{2}, -2)$	$G(\frac{3}{2}, -2)$
$Mid_{DA} = (\frac{-1+(-2)}{2}, \frac{-3+1}{2})$	$H(x, y) = (\frac{-3}{2}, \frac{-2}{2})$	$H(\frac{-3}{2}, -1)$

Formula for Slope of Line Segment:

Slope = $\frac{\Delta y}{\Delta x} = \frac{y2-y1}{x2-x1}$

Original Points	Midpoints:
A(-2, 1) B(3, 3) C(4, -1) D(-1,-3)	E($\frac{-1}{2}$, 2) F($\frac{7}{2}$, 1) G($\frac{3}{2}$, -2) H($\frac{-3}{2}$, -1)

Slopes (M) = (AB, DC)	Slopes (M) = (AD, BC)
$M_{AB} = \frac{3-1}{3-(-2)} = \frac{2}{5}$ $M_{DC} = \frac{-1-(-3)}{4-(-1)} = \frac{2}{5}$	$M_{AD} = \frac{-3-1}{-1-(-2)} = -4$ $M_{BC} = \frac{-1-3}{4-3} = -4$

$M_{AB} = M_{DC} = \frac{2}{5}$

$M_{AD} = M_{BC} = -4$

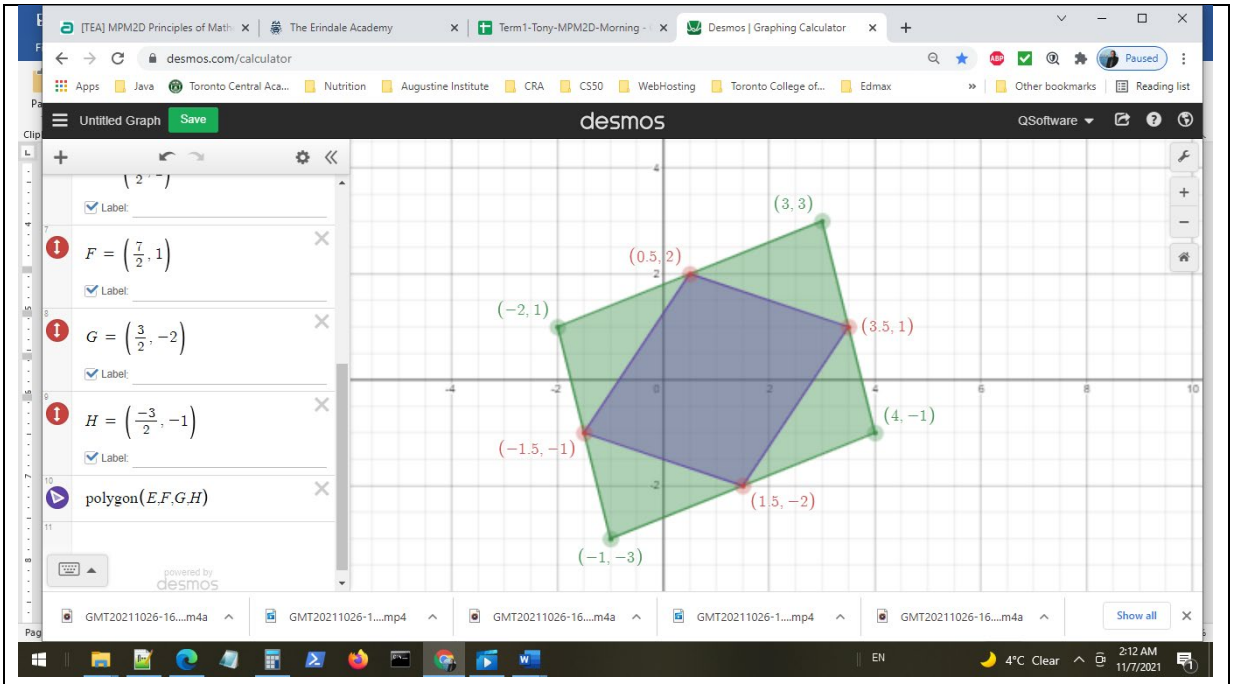
∴ The outer quadrilateral is a parallelogram since the opposite sides the slopes are the same.

Slopes (M) = (EH, FG)	Slopes (M) = (EF, HG)
$M_{EH} = \frac{-1-2}{(\frac{-3}{2}-(\frac{-1}{2}))} = \frac{-3}{-2} = \frac{3}{2}$ $M_{FG} = \frac{-2-1}{(\frac{3}{2}-(\frac{7}{2}))} = \frac{-3}{(\frac{-4}{2})} = \frac{3}{2}$	$M_{EF} = \frac{1-2}{(\frac{7}{2}-(\frac{-1}{2}))} = \frac{1-2}{(\frac{6}{2})} = \frac{-1}{3}$ $M_{HG} = \frac{-2-1}{(\frac{3}{2}-(\frac{7}{2}))} = \frac{-1}{3}$

$M_{EH} = M_{FG} = \frac{3}{2}$

$M_{EF} = M_{HG} = \frac{-1}{3}$

∴ The inner quadrilateral is a parallelogram



Both quadrilaterals are parallelograms. If student may further research the lengths and angles to if the quadrilaterals can be special type of other shapes such as: sqaures, rectangles, rombus, or kites.

Question 2: Use Method of elimination to solve the equations of lines:

1. **4x – 1y = 2**
2. **3x + y = 19**

Solve the equation of the lines algebraically as well as plotting the graphs of the two lines on the same graph.

① 4x - 1y = 2
② 3x + y = 19 ← add equation ① and ②

7x + 0y = 21

$x = \frac{21}{7} = 3$

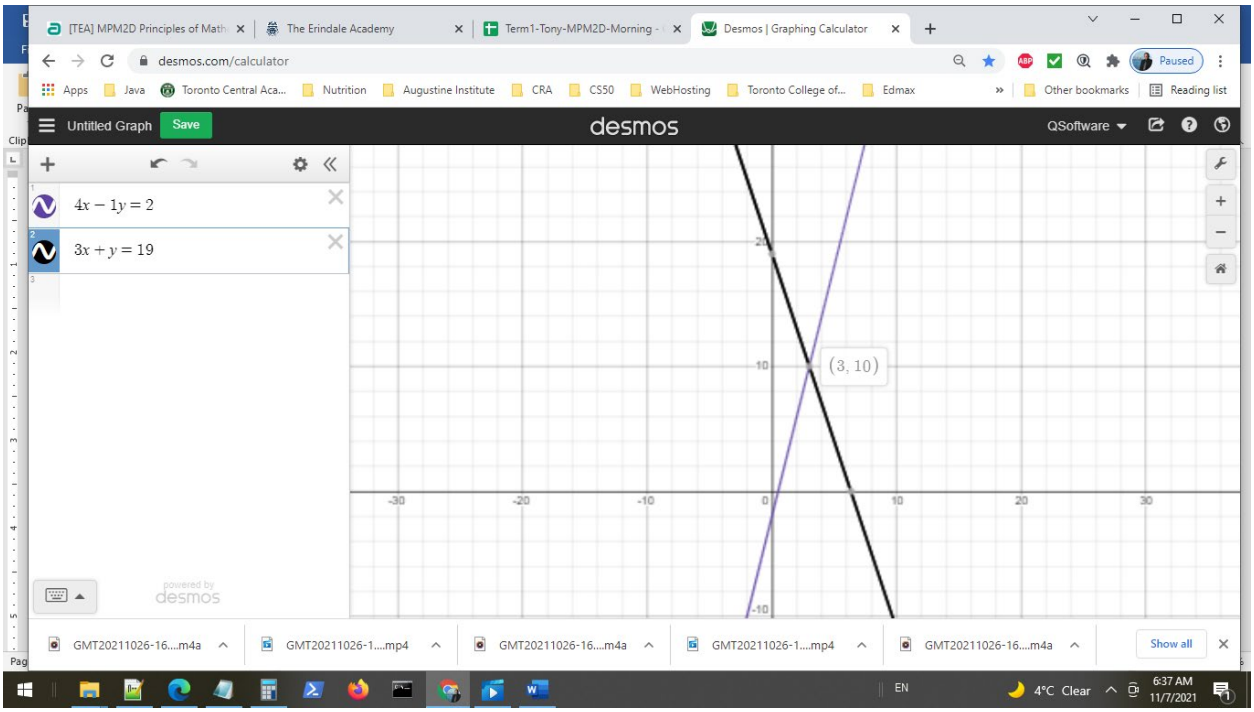
Substitute x = 3 into equation ②

② 3x + y = 19
3(3) + y = 19
9 + y = 19
y = 19 – 9
y = 10

Point is P(x, y) = (3, 10)
substitute into equation ① and ②

① 4x - 1y = 2	② 3x + y = 19
L.S. = 4x - 1y L.S. = 4(3) – 1(10) L.S. = 12 - 10 L.S. = 2 R.S = 2 ∴ L.S = R.S.	L.S. = 3x + y L.S. = 3(3) + 10 L.S. = 19 L.S. = 19 R.S = 19 ∴ L.S = R.S.
Since point (x, y) = (3, 10) stratifies both equations. This is the point of intersection.	

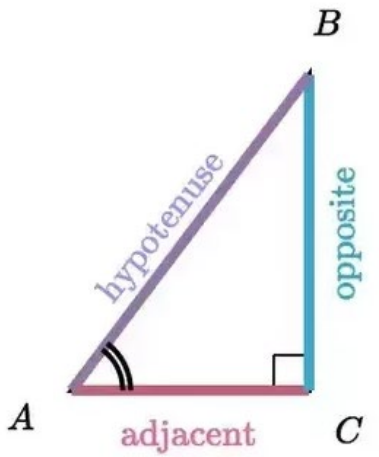
Attach graph here



Graph shows P(x, y) = (3, 10) as point of intersection as well.

Question 3:

Given the following Trigonometric ratios below:

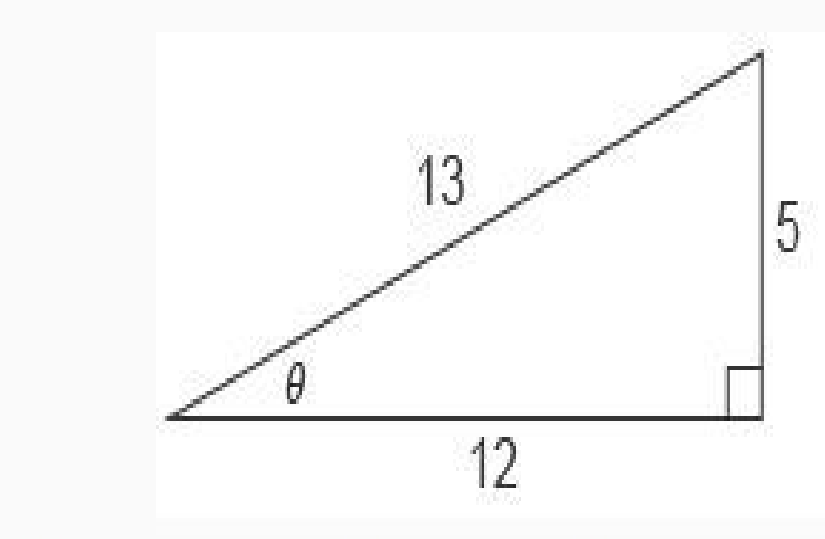


$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}}$$

Find the values for sin(A), cos(A), tan(A) for a triangle with the lengths of these sides.



Leave answers in fractions.

sin(θ) = $\frac{5}{13}$

cos(θ) = $\frac{12}{15}$

tan(θ) = $\frac{5}{12}$