

<b>COURSE NAME: MPM2D – Principles of Mathematics (Final Exam – Accumulative)</b>	
<b>MPM2D: Final Exam</b>  <b>Teacher: Antonio Pietrangelo</b>  <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <b>Time: (3 + 1) Hours – with ESL accommodation</b> </div>  <div style="border: 1px solid black; padding: 5px;"> <b>Pages: 24</b> </div>	<b>Student's Name: Instructor</b> <b>Student#: Answer Key</b>  <div style="border: 1px solid black; padding: 5px; margin: 5px 0;"> <b>Due Date: Thursday, February 29<sup>th</sup>, 2024 @ 01:00 pm (EST)</b> </div>  <div style="border: 1px solid black; padding: 5px;"> <b>Mark: ✓ /100 ✓</b> </div>

Categories	Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application
Symbol	K/U	T/I	C	A
Weight	✓ 25 % ✓	✓ 25 % ✓	✓ 25 % ✓	✓ 25 % ✓
Level	✓   ✓	✓   ✓	✓   ✓	✓   ✓

### Overall Expectations:

Expectations as listed in the Ontario Curriculum course outline for your specific course.

<p><b>Specific Expectations</b></p> <p><b>Unit 1 - Systems of Linear Equations</b></p> <p>1.1 Representing Linear Relations          1.2 Solving Linear Equations          1.3 Graphically Solving Linear Systems          1.4 Solving Linear Systems: Substitution          1.5 Equivalent Linear Systems          1.6 Solving Linear Systems: Elimination          1.7 Exploring Linear Systems</p> <p><b>Chapter 2: Analytic Geometry: Line Segments and Circles</b></p> <p>2.1 Midpoint of a Line Segment          2.2 Length of a Line Segment          2.3 Equation of a Circle          2.4 Classifying Figures on a Coordinate Grid          2.5 Verifying Properties of Geometric Figures          2.6 Exploring Properties of Geometric Figures          2.7 Using Coordinates to Solve Problems</p> <p><b>Chapter 3: Graphs of Quadratic</b></p>
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- 3.1 Exploring Quadratic Relations
- 3.2 Properties of Graphs of Quadratic Relations
- 3.3 Factored Form of a Quadratic Relation
- 3.4 Expanding Quadratic Expressions
- 3.5 Quadratic Models Using Factored Form
- 3.6 Exploring Quadratic and Exponential Graphs

#### **Chapter 4: Factoring Algebraic**

- 4.1 Common Factors in Polynomials
- 4.2 Exploring the Factorization of Trinomials
- 4.3 Factoring Quadratics:  $ax^2 + bx + c$ , where  $a = 1$ , and  $b, c$  are integers
- 4.4 Factoring Quadratics:  $ax^2 + bx + c$ , where  $a \neq 1$  and  $a, b, c$ , are integers
- 4.5 Factoring Quadratics: Special Cases
- 4.6 Reasoning about Factoring Polynomials

#### **Chapter 5: Applying Quadratic**

- 5.1 Stretching/Reflecting Quadratic Relations
- 5.2 Exploring Translations of Quadratic Relations
- 5.3 Graphing Quadratics in Vertex Form
- 5.4 Quadratic Models Using Vertex Form
- 5.5 Solving Problems Using Quadratic Relations
- 5.6 Connecting Standard and Vertex Forms

#### **Chapter 6: Quadratic Equations**

- 6.1 Solving Quadratic Equations
- 6.2 Exploring the Creation of Perfect Squares
- 6.3 Completing the Square
- 6.4 The Quadratic Formula
- 6.5 Interpreting Quadratic Equation Roots
- 6.6 Solving Problems Using Quadratic Models

#### **Chapter 7: Similar Triangles and Trigonometry**




- 7.1 Congruence and Similarity in Triangles
- 7.2 Solving Similar Triangle Problems
- 7.3 Exploring Similar Right Triangles
- 7.4 The Primary Trigonometric Ratios
- 7.5 Solving Right Triangles
- 7.6 Solving Right Triangle Problems




#### **Chapter 8: Acute Triangle Trigonometry**

- 8.1 Exploring the Sine Law
- 8.2 Applying the Sine Law
- 8.3 Exploring the Cosine Law
- 8.4 Applying the Cosine Law
- 8.5 Solving Acute Triangle Problems

**Rubrics:**

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Knowledge – Understanding of:  <b>(Final Exam:            Topics: 1.1 to            8.5)</b>	demonstrates insufficient understanding	demonstrates limited understanding	demonstrates some understanding	demonstrates considerable understanding	demonstrates thorough understanding	✓
					<b>Individual:            Assigned:</b>	✓ _____ -

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Thinking and Inquiry (What if scenarios) of <b>(Final Exam: Topics: 1.1 to 8.5)</b>	demonstrates insufficient ability to apply different scenarios	demonstrates limited ability to apply different scenarios	demonstrates some ability to apply different scenarios	demonstrates considerable ability to apply different scenarios	demonstrates through ability to apply different scenarios 	
					<b>Individual:</b>	 _____

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Communication  <b>Communicates effectively with the use of</b>  <b>(Final Exam: Topics: 1.1 to 8.5)</b>	demonstrates <b>insufficient</b> ability to communicate effectively	demonstrates <b>limited</b> ability to communicate effectively	demonstrates <b>some</b> ability to communicate effectively	demonstrates <b>considerable</b> ability to communicate effectively	demonstrates <b>through</b> ability to communicate effectively  	
					<b>Individual:</b>	 _____

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
<p><b><u>Application:</u></b></p> <p>Demonstrates the ability to apply mathematical principles to real world situations.</p> <p><b>(Final Exam: Topics: 1.1 to 8.5)</b></p>	demonstrates <b>insufficient ability</b>	demonstrates <b>limited ability</b>	demonstrates <b>some ability</b>	demonstrates <b>considerable ability</b>	demonstrates <b>thorough ability</b>	✓
					Individual: ✓	✓

**PART A: KNOWLEDGE AND UNDERSTANDING (K/U) – 25%**

**2 Marks Per Question**

**Instructions:**

**Question 1: The Centroid of a triangle is a single point outside the triangle? (True or False)**

**False**

**Question 2: A Centroid splits the medians of a triangle by a ratio of 2:1? (True or false)**

**True**

**Question 3: The  $\cos 90^\circ$  is equal to 1? (True or False)**

**False**

**Question 4: The tangent of angle is a ratio two sides of a right-angle triangle: Adjacent / Opposite? (True or False)**

**False**

**Question 5: A scalene triangle has all the sides the same? (True or false)**

**False**

**Question 6: When two triangles are the identical, they are said to be congruent? (True or false)**

**True**

**Question 7: An Obtuse triangle is a triangle that has one angle equal to 90 degrees? (True or False)**

**False**

**Question 8:** A square is not a parallelogram? (True or False)

**False**

**Question 9:** A parallelogram has two sides that are parallel and the other two sides are not parallel? (True or False)

**False**

**Question 10:** (True or False) The equation for the midpoint of a line segment is  $M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

**True**

**Question 11:** A trapezoid is a quadrilateral with exactly one pair of parallel lines? (True or false)

**True**

**Question 12:** A Rhombus is a parallelogram with all sides equal? (True or False)

**True**

**Question 13:** A rectangle is a parallelogram with 4 right angles, but sides has two pair of sides with equal lengths? (True or false)

**True**

**Question 14:** A square has 4 equal sides with only two angles that are 90 degrees? (True or False)

**False**



**Question 15:** The right bisector of an isosceles triangle splits a triangle into two equal parts? (True or False)

**True**

**Question 16:** The equation of  $y=ax^2$ , if  $a$  is negative the parabola has a maximum? (True or False)

**True**

**Question 17:** The equation of  $y=ax^2$ , if  $a$  is between  $0 < a < 1$ , the parabola is widened or compressed? (True or False)

**True**

**Question 18:** The equation of  $y=a(x-h)^2$ , when  $h > 0$  the quadratic is shifted or transformed upwards or downwards by the  $h$  value? (True or False)

**False**

**Question 19:** For the equation  $y=x^2$ , there is no minimum for  $y$ ? (True or False)

**False**

**Question 20:** For the equation  $y = -x^2$  maximum value for  $y$  is zero? (True or False)

**True**

**Question 21:** The line of symmetry for a parabola is at  $x$  value of the vertex( $x, y$ )? (True or False)

**True**

**Question 22:** A minimum or a maximum is on the vertex of a quadratic equations? (True or False)

**True**

**Question 23:** The y-coordinate of the vertex(x, y) is the point where the line of symmetry is for a quadratic equation? (True or False)

**False**

**Question 24:** The equation of  $y=a(x-r)(x-s)$ , the r, and s values are where the zeros occur? (True or False)

**True**

**Question 25:** The equation of  $y=x^2 + k$ , the k transforms the quadratic relation left or right? (True or False)

**False**

**Question 26:** (True or False) equation for midpoint is  $M(x,y) = (\frac{run}{2}, \frac{rise}{2})$ ?

**False**

$$\text{Midpoint } M(x,y) = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$$

Note: rise =  $\Delta y = y_2 - y_1$ ; run =  $\Delta x = x_2 - x_1$

**Question 27:** (True or False) An Isosceles triangle has two sides that are the same?

**True**

**Question 28:** (True or False) An Isosceles triangle as two angles that are the same?

**True**

**Question 29:** (True or False) An equilateral triangle has no sides the same?

**False**

**Question 30:** The  $\sin 90^\circ$  has a value of 1? (True or False)

**True**

**PART B: THINKING AND INQUIRY (T/I) – 25 %**

**5 Marks Per Question**

**Show your work: If you do not you will get zero.**

**Question 1: Find the equation of line that is perpendicular to the points A(0,10) and B(8, 6)?**

$$\text{Slope } M_{AB} = \frac{\Delta y}{\Delta x} = \frac{6-10}{8-0} = \frac{-4}{8} = \frac{-1}{2}$$

$$\text{Mid Point of } M_{AB} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right) = \left( \frac{0+8}{2}, \frac{10+6}{2} \right) = \left( \frac{8}{2}, \frac{16}{2} \right) = (4, 8)$$

Lines that are perpendicular, the product of their slopes is **-1**.

$$M_2 \cdot M_1 = -1.$$

$$M_2 = \frac{-1}{M_1} = \frac{-1}{\left(\frac{-1}{2}\right)} = \frac{\left(\frac{-1}{1}\right)}{\left(\frac{-1}{2}\right)} \cdot \frac{\left(\frac{2}{1}\right)}{\left(\frac{2}{1}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-2}{2}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-2}{2}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-1}{1}\right)} = 2$$

$y = mx + b$  ← substitute slope and a point to find y-intercept.

$$8 = (2)(4) + b$$

$$8 = 8 + b$$

$$8 - 8 = b$$

$$0 = b$$

Equation of a line that is perpendicular to points A(0,10), B(8, 6) is  **$y = 2x$** .

**Question 2: Identify two points that are on the circumference of the circle  $x^2 + y^2 = 5^2$**

Two points that are on the circumference of the circle are  
for example: (0, 5), (0, -5), (5, 0), (-5, 0).

**Question 3:** Evaluate the algebraic expression when  $a = 4$ ,  $b = 2$ ,  $c = -3$ ,  $d = -5$

$$4a^3 + 6b + 2c - 8d =$$

$$\begin{aligned} 4a^3 + 6b + 2c - 8d &= 4(4)^3 + 6(2) + 2(-3) - 8(-5) \\ &= 256 + 12 - 6 + 40 \\ &= \mathbf{302} \end{aligned}$$

**Question 4:** Find the vertex for equation  $y = (x - 3)^2 + 6$

$$\text{Vertex}(x, y) = (h, k) = \mathbf{(3, 6)}$$

**Question 5:** Find the line of symmetry for  $y = 2(x-8)(x+4)$

$$\text{Axis of Symmetry } x = \frac{(r + s)}{2} = \frac{(8 + -4)}{2} = \frac{4}{2} = \mathbf{2}$$

**Question 6:** Find the  $\sin A$ ,  $\cos A$ ,  $\tan A$ , when  $A$  is an angle of  $45^\circ$

$$\sin 60^\circ \cong \mathbf{0.8660}$$

$$\cos 60^\circ \cong \mathbf{0.5}$$

$$\tan 60^\circ = \mathbf{1.7321}$$

**Question 7:** Find the slope between the two Points P(4, 8) and Q(10, 16)?

$$\text{Slope} = M_{pq} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{16 - 8}{10 - 4} = \frac{8}{6} = \frac{4}{3}$$

**Question 8:** Find the MidPoint between the two points A(2, -3) and B(6, 9)

$$\text{Midpoint } M_{AB}(x,y) = \left( \frac{2 + 6}{2}, \frac{-3 + 9}{2} \right) = \left( \frac{8}{2}, \frac{6}{2} \right) = (4, 3)$$

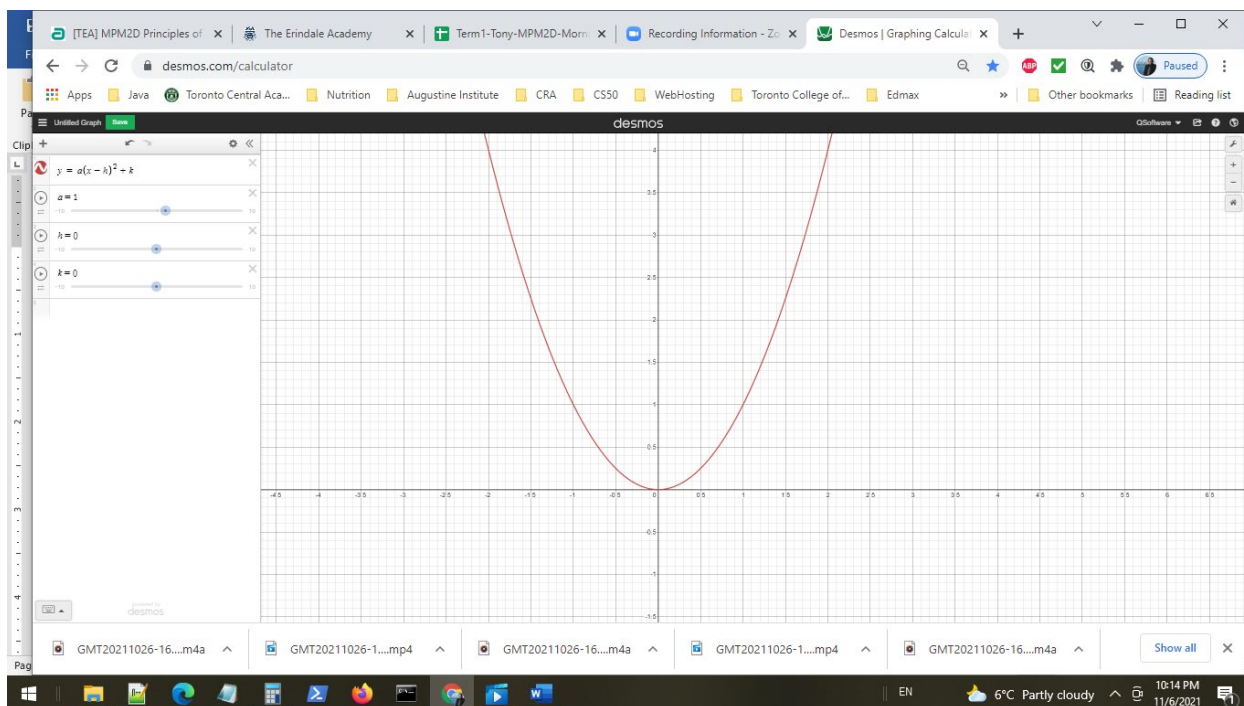
**PART C: COMMUNICATION (C) – 25%**

**10 Marks Per Question**

**Question 1: Quadratic Equation form  $y = a(x - h)^2 + k$**

**Please explain how does variables  $a$ ,  $h$ , and  $k$  affect the transformations of a parabola relative to  $y = x^2$ ?**

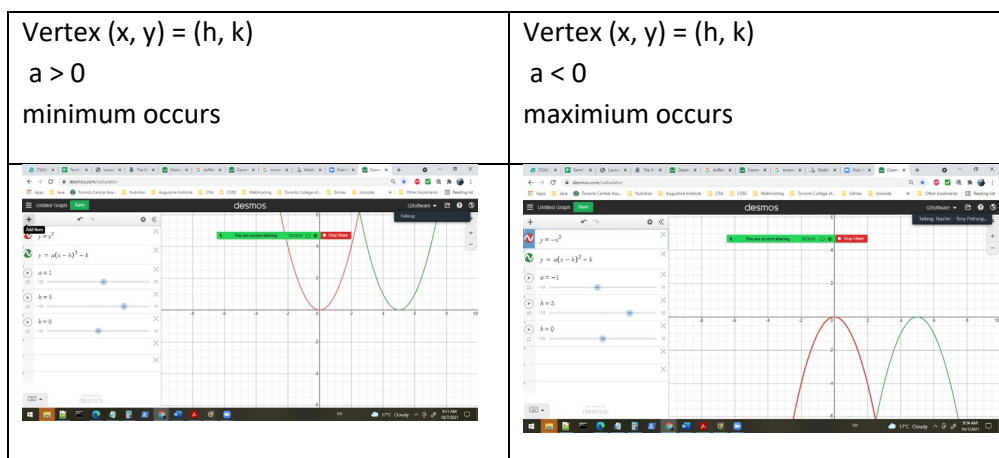
**Example the affects of the three variables:**



$y = a(x - h)^2 + k$ ,  $a \neq 0$ ,  $h$  and  $k$  are real numbers

- (1)  $h$  shifts the parabola horizontally.
  - (a) If  $h > 0$  parabola shifts to the right,
  - (b) If  $h < 0$  parabola shifts to the left
- (2)  $k$  shifts the parabola vertically
  - (a) If  $k > 0$  the shift is vertically upward.
  - (b) If  $k < 0$  the shift is downwards.
- (3)  $a$  determines if the parabola, is compressed or stretched.
  - (a) If  $a > 0$ ,
    - (i) the parabola opens upwards
    - (ii) the parabola has a minimum

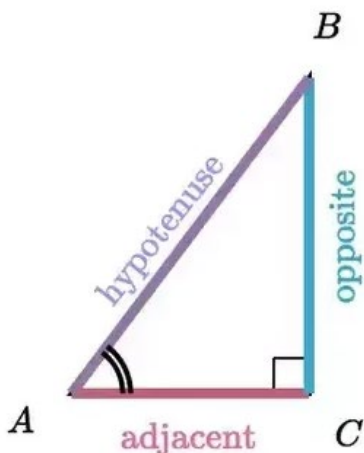
- (b) If  $a < 0$ ,
    - (i) the parabola opens downwards.
    - (ii) the parabola has a maximum
  - (c) if  $a > 1$ 
    - (i) the parabola stretches more quickly upwards
    - (ii) factored by  $a$
  - (d) if  $a < -1$ 
    - (i) the parabola stretches more quickly downwards
    - (ii) factored by  $a$
  - (e) if  $a$  is a positive fraction
    - (i) the parabola is compressed or flatted
  - (f) if  $a$  is a negative fraction
    - (i) the parabola is compressed or flatted
- (4) Vertex  $(x, y) = (h, k)$



- (5) Axis of symmetry occurs at  $h$ .

**Question 2:** In trigonometry explain the acronym SOH CAH TOA in terms of the three primary trigonometric functions.

**Example the acronym:**



$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}}$$

Based on the definitions of the three primary trigonometric identities as stated in the above diagram, by using the first character of each identity you will get SOH CAH TOA.

For example:

$$\sin(A) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \text{SOH}$$

$$\cos(A) = \frac{\text{Adjacent}}{\text{Hypotenuse}} = \text{CAH}$$

$$\tan(A) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \text{TOA}$$



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**PART D: APPLICATION (A) – 25%**

**10 Marks Per Question**

Use your own graph paper?

**Question 1:** Draw quadrilateral and label the points A(-2, 1), B(-1, -3), C(4, -1), and D(3, 3),

The midpoint to line segment AB call it E.

The midpoint to line segment BC call it F.

The midpoint to line segment DC call it G.

The midpoint to line segment AD call it H.

What type of parallelograms are ABCD, and EFGH?

Points: A(-2, 1), B(-1,-3), C(4, -1), and D(3, 3)

$\text{Mid}_{AB} = \left( \frac{-2 + -1}{2}, \frac{1 - 3}{2} \right)$	$E(x, y) = \left( \frac{-3}{2}, \frac{-2}{2} \right) = \left( \frac{-3}{2}, -1 \right)$	$E\left(\frac{-3}{2}, -1\right)$
$\text{Mid}_{BC} = \left( \frac{-1 + 4}{2}, \frac{-3 + (-1)}{2} \right)$	$F(x, y) = \left( \frac{3}{2}, \frac{-4}{2} \right) = \left( \frac{3}{2}, -2 \right)$	$F\left(\frac{3}{2}, -2\right)$
$\text{Mid}_{CD} = \left( \frac{4 + 3}{2}, \frac{-1 + 3}{2} \right)$	$G(x, y) = \left( \frac{7}{2}, \frac{2}{2} \right) = \left( \frac{7}{2}, 1 \right)$	$G\left(\frac{7}{2}, 1\right)$
$\text{Mid}_{DA} = \left( \frac{3 - 2}{2}, \frac{3 + 1}{2} \right)$	$H(x, y) = \left( \frac{1}{2}, \frac{4}{2} \right)$	$H\left(\frac{1}{2}, 2\right)$

Formula for Slope of Line Segment:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Original Points	Midpoints:
<b>A(-2, 1),</b> <b>B(-1,-3),</b> <b>C(4, -1),</b> <b>D(3, 3)</b>	<b>E</b> $(\frac{-3}{2}, -1)$ <b>F</b> $(\frac{3}{2}, -2)$ <b>G</b> $(\frac{7}{2}, 1)$ <b>H</b> $(\frac{1}{2}, 2)$

Slopes (M) = (AB, DC)	Slopes (M) = (AD, BC)
$M_{AB} = \frac{-3 - 1}{-1 - (-2)} = \frac{-4}{1}$	$M_{AD} = \frac{3 - 1}{3 - (-2)} = \frac{2}{5}$
$M_{DC} = \frac{3 - (-1)}{3 - 4} = \frac{4}{-1}$	$M_{BC} = \frac{-1 - (-3)}{4 - (-1)} = \frac{2}{5}$

$$M_{AB} = M_{DC} = -4$$

$$M_{AD} = M_{BC} = \frac{2}{5}$$

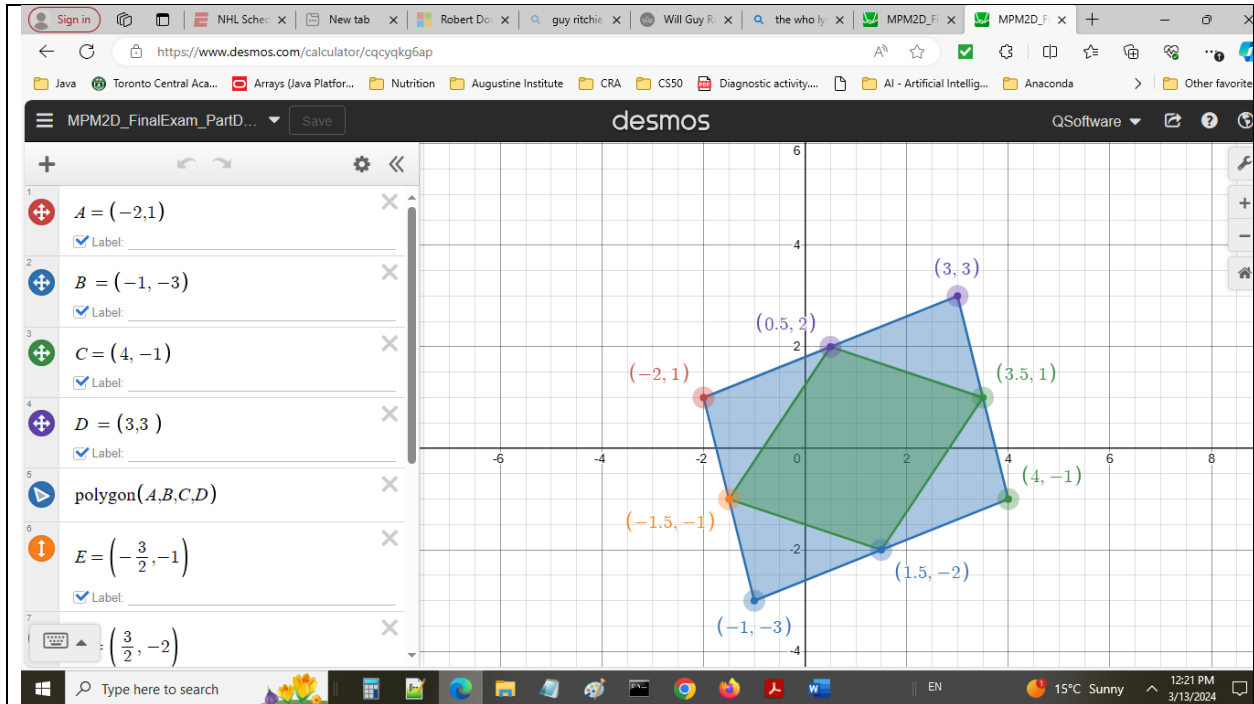
∴ The outer quadrilateral is a parallelogram since the opposite sides the slopes are the same.

Slopes (M) = (EH, FG)	Slopes (M) = (EF, HG)
$M_{EH} = \frac{2 - (-1)}{(\frac{1}{2} - (-\frac{3}{2}))} = \frac{3}{(\frac{4}{2})} = \frac{3}{2}$	$M_{EF} = \frac{-2 - (-1)}{(\frac{3}{2} - (-\frac{3}{2}))} = \frac{-1}{(\frac{6}{2})} = \frac{-1}{3}$
$M_{FG} = \frac{1 - (-2)}{(\frac{7}{2} - (\frac{3}{2}))} = \frac{3}{(\frac{4}{2})} = \frac{3}{2}$	$M_{HG} = \frac{1 - (2)}{(\frac{7}{2} - (\frac{1}{2}))} = \frac{-1}{(\frac{6}{2})} = \frac{-1}{3}$

$$M_{EH} = M_{FG} = \frac{3}{2}$$

$$M_{EF} = M_{HG} = \frac{-1}{3}$$

∴ The inner quadrilateral is a parallelogram



Both quadrilaterals are parallelograms. If student may further research the lengths and angles to if the quadrilaterals can be special type of other shapes such as: squares, rectangles, rombus, or kites.

**Question 2: Use Method of elimination to solve the equations of lines:**

1.  $8x - 2y = 4$
2.  $6x + 2y = 38$

Solve the equation of the lines algebraically as well as plotting the graphs of the two lines on the same graph.

$$\begin{aligned} \textcircled{1} \quad & 8x - 2y = 4 \\ \textcircled{2} \quad & 6x + 2y = 38 \quad \leftarrow \text{add equation } \textcircled{1} \text{ and } \textcircled{2} \end{aligned}$$

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$$14x + 0y = 42$$

$$x = \frac{42}{14} = 3$$

Substitute  $x = 3$  into equation  $\textcircled{2}$

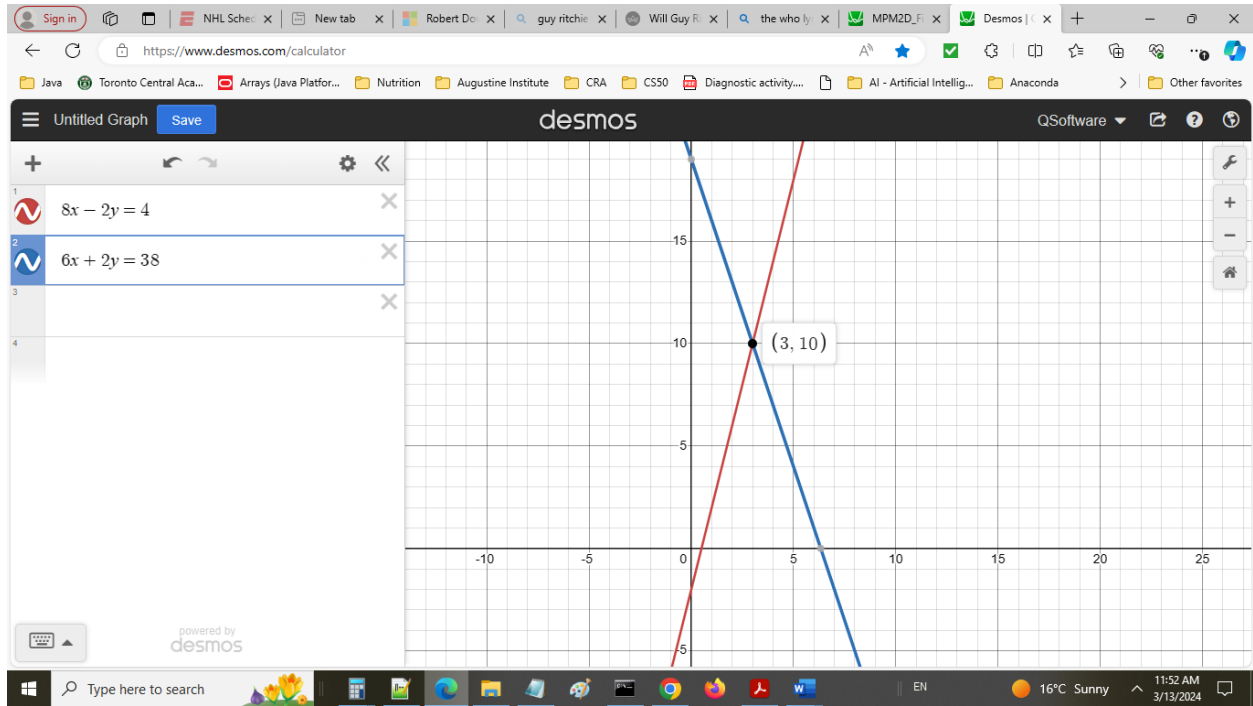
$$\begin{aligned} \textcircled{2} \quad & 6x + 2y = 38 \\ 6(3) + 2y &= 38 \\ 18 + 2y &= 38 \\ 2y &= 38 - 18 \\ 2y &= 20 \\ y &= 10 \end{aligned}$$

Point is  $P(x, y) = (3, 10)$

substitute into equation  $\textcircled{1}$  and  $\textcircled{2}$

$\textcircled{1} \quad 8x - 2y = 4$ $\begin{aligned} \text{L.S.} &= 8x - 2y \\ \text{L.S.} &= 8(3) - 2(10) \\ \text{L.S.} &= 24 - 20 \\ \text{L.S.} &= 4 \end{aligned}$ $\text{R.S.} = 4$ $\therefore \text{L.S.} = \text{R.S.}$	$\textcircled{2} \quad 6x + 2y = 38$ $\begin{aligned} \text{L.S.} &= 6x + 2y \\ \text{L.S.} &= 6(3) + 2(10) \\ \text{L.S.} &= 18 + 20 \\ \text{L.S.} &= 38 \end{aligned}$ $\text{R.S.} = 38$ $\therefore \text{L.S.} = \text{R.S.}$
<p style="color: red;">Since point <math>(x, y) = (3, 10)</math> stratifies both equations.</p> <p style="color: red;">This is the point of intersection.</p>	

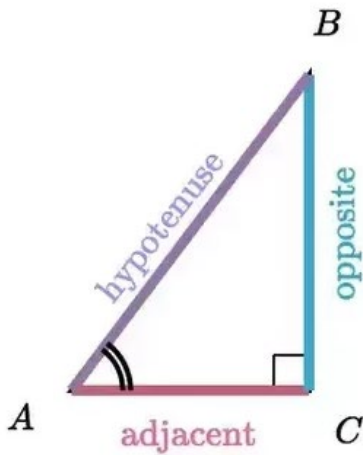
Attach graph here:



Graph shows  $P(x, y) = (3, 10)$  as point of intersection as well.

**Question 3:**

Given the following Trigonometric ratios below:

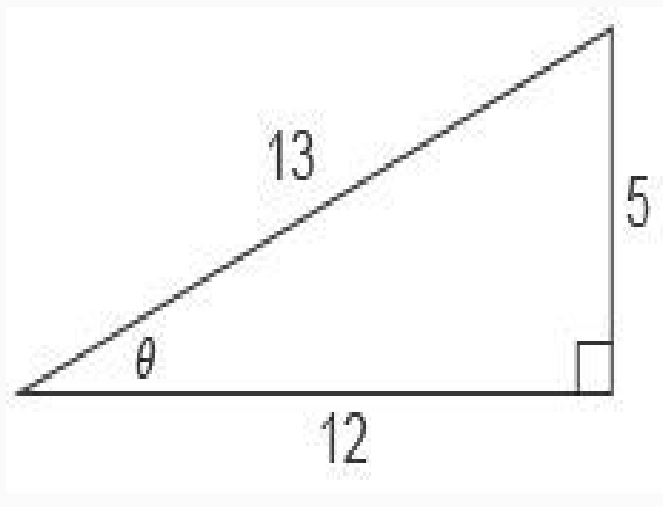


$$\sin(A) = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(A) = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(A) = \frac{\text{opposite}}{\text{adjacent}}$$

Find the values for  $\sin(A)$ ,  $\cos(A)$ ,  $\tan(A)$  for a triangle with the lengths of these sides.



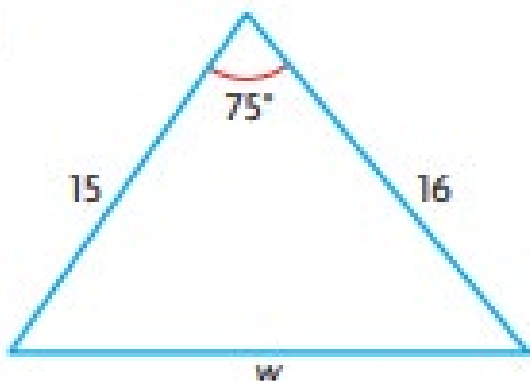
Leave answers in fractions.

$$\sin(\theta) = \frac{5}{13}$$

$$\cos(\theta) = \frac{12}{13}$$

$$\tan(\theta) = \frac{5}{12}$$

**Question 4:** Use the Cosine Law to find the length of side W.



$$w^2 = a^2 + b^2 - 2ab\cos(w)$$

$$w^2 = (15)^2 + (16)^2 - 2(15)(16)\cos(75^\circ)$$

$$w = \sqrt{481 - 480(0.25881)}$$

$$w = \sqrt{481 - 480(0.25881)}$$

$$w = 18.86$$