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| COURSE NAME: MPM2D – Principles of Mathematics (Mid Term Exam) | |
| MPM2D: Mid Term Exam Teacher: Antonio Pietrangelo | Student's Name: Instructor Student#: Answer Key |
| Time: (3 + 1) Hours – with ESL accommodation | Due Date: Tuesday, January 30th, 2024 @ 01:30 pm (EST) |
| Pages: 20 | Mark: ✓ /100 ✓ |

| Categories | Knowledge/ Understanding | Thinking/Inquiry/ Problem Solving | Communication | Application |
|------------|-----------------------------|--------------------------------------|---------------|-------------|
| Symbol | K/U | T/I | C | A |
| Weight | 25 % | 25 % | 25 % | 25 % |
| Level | ✓✓ | ✓✓ | ✓✓ | ✓✓ |

Overall Expectations:

Expectations as listed in the Ontario Curriculum course outline for your specific course.

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| <p>Specific Expectations</p> <p>Unit 1 - Systems of Linear Equations</p> <p>1.1 Representing Linear Relations 1.2 Solving Linear Equations 1.3 Graphically Solving Linear Systems 1.4 Solving Linear Systems: Substitution 1.5 Equivalent Linear Systems 1.6 Solving Linear Systems: Elimination 1.7 Exploring Linear Systems</p> <p>Chapter 2: Analytic Geometry: Line Segments and Circles</p> <p>2.1 Midpoint of a Line Segment 2.2 Length of a Line Segment 2.3 Equation of a Circle 2.4 Classifying Figures on a Coordinate Grid 2.5 Verifying Properties of Geometric Figures 2.6 Exploring Properties of Geometric Figures 2.7 Using Coordinates to Solve Problems</p> |
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Chapter 3: Graphs of Quadratic

3.1 Exploring Quadratic Relations

3.2 Properties of Graphs of Quadratic Relations

3.3 Factored Form of a Quadratic Relation




3.4 Expanding Quadratic Expressions

3.5 Quadratic Models Using Factored Form

3.6 Exploring Quadratic and Exponential Graphs

Rubrics:

| Category | Level R (0 – 49%) | Level 1 (50-59%) | Level 2 (60-69%) | Level 3 (70-79%) | Level 4 (80-100%) | Level/ Mark |
|---|---|--|---------------------------------------|---|---|-----------------|
| Knowledge – Understanding of: (Mid Term Exam: Topics: 1.1 to 3.6) | demonstrates insufficient understanding | demonstrates limited understanding | demonstrates some understanding | demonstrates considerable understanding | demonstrates thorough understanding | ✓ |
| | | | | | Individual: Assigned: | ✓ _____ - |

| Category | Level R (0 – 49%) | Level 1 (50-59%) | Level 2 (60-69%) | Level 3 (70-79%) | Level 4 (80-100%) | Level/ Mark |
|--|--|---|--|--|--|--|
| Thinking and Inquiry (What if scenarios) of: (Mid Term Exam: Topics: 1.1 to 3.6) | demonstrates insufficient ability to apply different scenarios | demonstrates limited ability to apply different scenarios | demonstrates some ability to apply different scenarios | demonstrates considerable ability to apply different scenarios | demonstrates through ability to apply different scenarios  |  |
| | | | | | Individual: |  _____ |

| Category | Level R (0 – 49%) | Level 1 (50-59%) | Level 2 (60-69%) | Level 3 (70-79%) | Level 4 (80-100%) | Level/ Mark |
|--|---|--|---|---|--|----------------|
| Communication Communicates effectively with the use of (Mid Term Exam: Topics: 1.1 to 3.6) | demonstrates insufficient ability to communicate effectively | demonstrates limited ability to communicate effectively | demonstrates some ability to communicate effectively | demonstrates considerable ability to communicate effectively | demonstrates through ability to communicate effectively | ✓ |
| | | | | | Individual: ✓ | ✓ |

| Category | Level R (0 – 49%) | Level 1 (50-59%) | Level 2 (60-69%) | Level 3 (70-79%) | Level 4 (80-100%) | Level/ Mark |
|--|--|-------------------------------------|----------------------------------|--|--------------------------------------|----------------|
| <p><u>Application:</u></p> <p>Demonstrates the ability to apply mathematical principles to real world situations.</p> <p>(Mid Term Exam: Topics: 1.1 to 3.6)</p> | demonstrates insufficient ability | demonstrates limited ability | demonstrates some ability | demonstrates considerable ability | demonstrates thorough ability | ✓ |
| | | | | | Individual: | ✓ _____ |

PART A: KNOWLEDGE AND UNDERSTANDING (K/U) – 25%

2 Marks Per Question

Instructions:

Question 1: (True or False) equation for midpoint is $M(x,y) = (\frac{run}{2}, \frac{rise}{2})$?

False

Midpoint $M(x,y) = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Note: rise = $\Delta y = y_2 - y_1$; run = $\Delta x = x_2 - x_1$

Question 2: (True or False) An Isosceles triangle has two sides that are the same?

True

Question 3: (True or False) An Isosceles triangle has two angles that are the same?

True

Question 4: (True or False) An equilateral triangle has no sides the same?

False

Question 5: The Centroid of a triangle is a single point outside the triangle? (True or False)

False

Question 6: A Centroid splits the medians of a triangle by a ratio of 2:1? (True or false)

True

Question 7: A scalene triangle has all the sides the same? (True or false)

False

Question 8: When two triangles are the identical, they are said to be congruent? (True or false)

True

Question 9: An Obtuse triangle is a triangle that has one angle equal to 90 degrees? (True or False)

False

Question 10: A square is not a parallelogram? (True or False)

False

Question 11: A parallelogram has two sides that are parallel and the other two sides are not parallel? (True or False)

False

Question 12: (True or False) The equation for the midpoint of a line segment is $M(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

True

Question 13: A trapezoid is a quadrilateral with exactly one pair of parallel lines? (True or false)

True

Question 14: A Rhombus is a parallelogram with all sides equal? (True or False)

True

Question 15: A rectangle is a parallelogram with 4 right angles, but sides has two pair of sides with equal lengths? (True or false)

True

Question 16: A square has 4 equal sides with only two angles that are 90 degrees? (True or False)

False

Question 17: The right bisector of an isosceles triangle splits a triangle into two equal parts? (True or False)

True

Question 18: The equation of $y=ax^2$, if a is negative the parabola has a maximum? (True or False)

True

Question 19: The equation of $y=ax^2$, if a is between $0 < a < 1$, the parabola is widened or compressed? (True or False)

True

Question 20: The equation of $y=a(x-h)^2$, when $h > 0$ the quadratic is shifted or transformed upwards or downwards by the h value? (True or False)

False

Question 21: For the equation $y=x^2$, there is no minimum for y ? (True or False)

False

Question 22: For the equation $y = -x^2$ maximum value for y is zero? (True or False)

True

Question 23: The line of symmetry for a parabola is at x value of the vertex(x, y)? (True or False)

True

Question 24: A minimum or a maximum is on the vertex of a quadratic equations? (True or False)

True

Question 25: The equation of $y=x^2 + k$, the k transforms the quadratic relation left or right? (True or False)

False

PART B: THINKING AND INQUIRY (T/I) – 25 %

5 Marks Per Question

Show your work: If you do not you will get zero.

Question 1: Find the slope between the two Points P(2,6) and Q(8, 12)?

$$\text{Slope} = M_{pq} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 6}{8 - 2} = \frac{6}{6} = \mathbf{1}$$

Question 2: Find the MidPoint between the two points A(2, -5) and B(5, 7)

$$\text{Midpoint } M_{AB}(x,y) = \left(\frac{2+5}{2}, \frac{-5+7}{2} \right) = \left(\frac{7}{2}, \frac{2}{2} \right) = \left(\frac{7}{2}, 1 \right)$$

Question 3: Identify two points that are on the circumference of the circle $x^2 + y^2 = 4^2$

Two points that are on the circumference of the circle are
for example: (0, 4), (0, -4), (4, 0), (-4, 0).

Question 4: Evaluate the algebraic expression when $a = 2$, $b = 3$, $c = -2$, $d = -1$

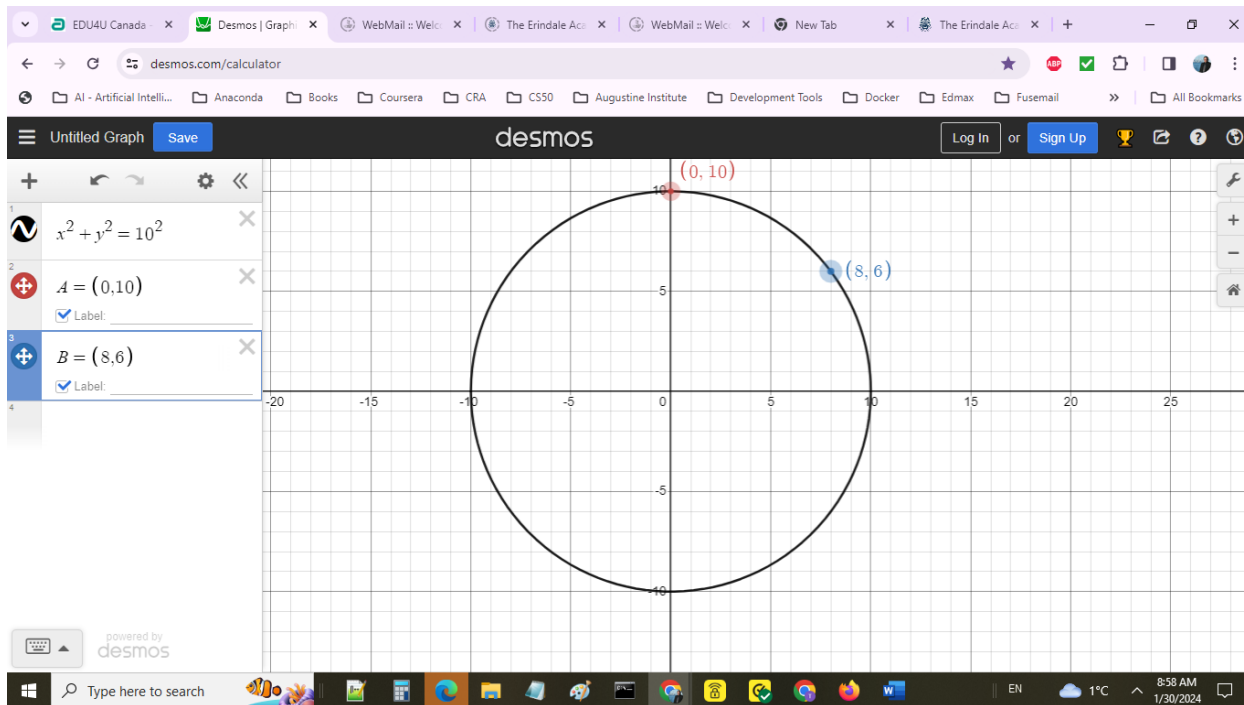
$$\begin{aligned}3a^3 + 4b + 3c - 2d &= 3(2)^3 + 4(3) + 3(-2) - 2(-1) \\ &= 24 + 12 - 6 + 2\end{aligned}$$

$$3a^3 + 4b + 3c - 2d = \mathbf{32}$$

PART C: COMMUNICATION (C) – 25%

10 Marks Per Question

Question 1: Find the equation of line for points A (0,10) and B (8, 6) on the circle $x^2 + y^2 = 10^2$ and perpendicular to the line that connects points A, B?



$$\text{Slope } M_{AB} = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{6 - 10}{8 - 0} = \frac{-4}{8} = \frac{-1}{2}$$

$$\text{Mid Point of } M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{0 + 8}{2}, \frac{10 + 6}{2} \right) = \left(\frac{8}{2}, \frac{16}{2} \right) = (4, 8)$$

Lines that are perpendicular, the product of their slopes is **-1**.

$$M_2 \cdot M_1 = -1.$$

$$M_2 = \frac{-1}{M_1} = \frac{-1}{\left(\frac{-1}{2}\right)} = \frac{\left(\frac{-1}{1}\right)}{\left(\frac{-1}{2}\right)} \cdot \frac{\left(\frac{2}{1}\right)}{\left(\frac{2}{1}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-2}{2}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-2}{2}\right)} = \frac{\left(\frac{-2}{1}\right)}{\left(\frac{-1}{1}\right)} = 2$$

$y = mx + b$ ← substitute slope and a point to find y-intercept.

$$8 = (2)(4) + b$$

$$8 = 8 + b$$



$$8 - 8 = b$$

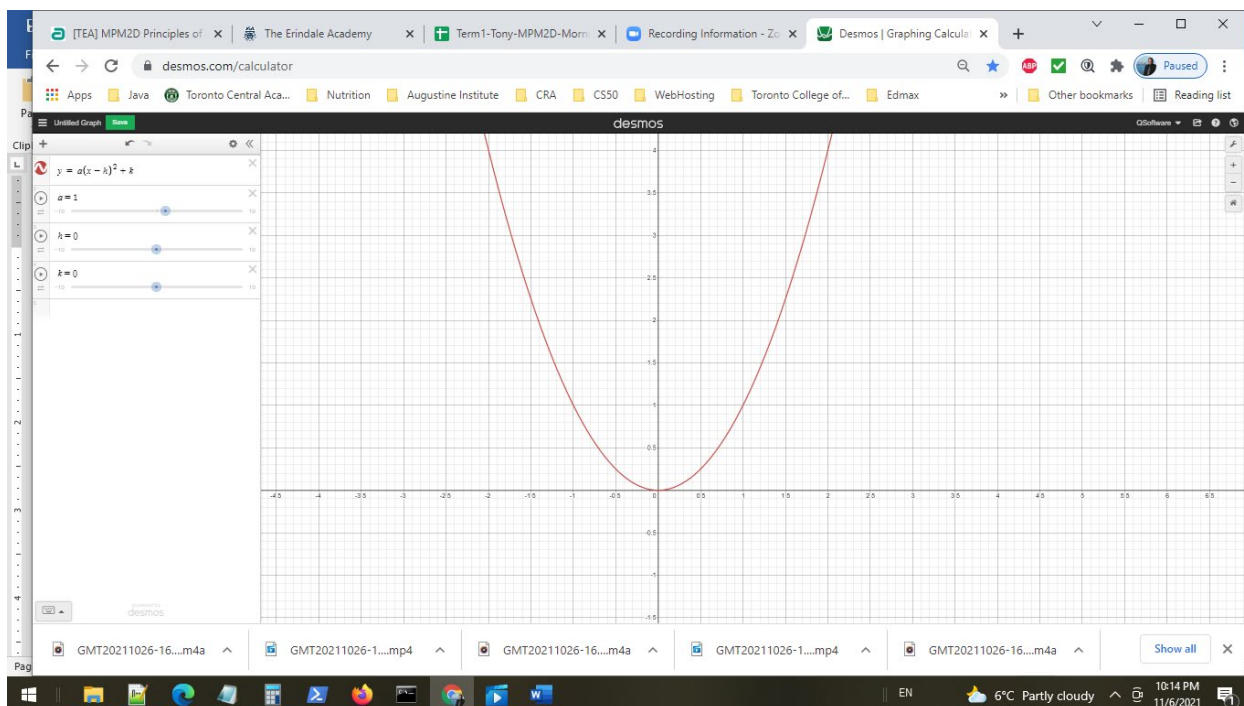
$$0 = b$$

Equation of a line that is perpendicular to points A(0,10), B(8, 6) is $y = 2x$.

Question 2: Quadratic Equation form $y = a(x - h)^2 + k$

Please explain how does variables a , h , and k affect the transformations of a parabola relative to $y = x^2$?

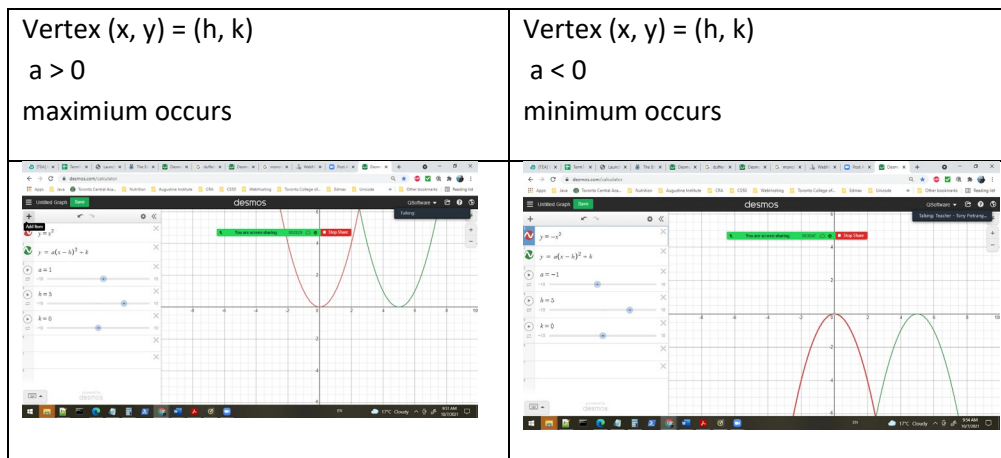
Example the affects of the three variables:



$y = a(x - h)^2 + k$, $a \neq 0$, h and k are real numbers

- (1) h shifts the parabola horizontally.
 - (a) If $h > 0$ parabola shifts to the right,
 - (b) If $h < 0$ parabola shifts to the left
- (2) k shifts the parabola vertically
 - (a) If $k > 0$ the shift is vertically upward.
 - (b) If $k < 0$ the shift is downwards.
- (3) a determines if the parabola, is compressed or stretched.
 - (a) If $a > 0$,
 - (i) the parabola opens upwards
 - (ii) the parabola has a minimum
 - (b) If $a < 0$,
 - (i) the parabola opens downwards.
 - (ii) the parabola has a maximum
 - (c) if $a > 1$
 - (i) the parabola stretches more quickly upwards

- (ii) factored by a
- (d) if $a < -1$
 - (i) the parabola stretches more quickly downwards
 - (ii) factored by a
- (e) if a is a positive fraction
 - (i) the parabola is compressed or flatted
- (f) if a is a negative fraction
 - (i) the parabola is compressed or flatted
- (4) Vertex $(x, y) = (h, k)$



- (5) Axis of symmetry occurs at h .

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PART D: APPLICATION (A) – 25%

10 Marks Per Question

Use your own graph paper?

Question 1: Draw quadrilateral and label the points A(-2, 1), B(3, 3), C(4, -1), and D(-1, -3)

The midpoint to line segment AB call it E.

The midpoint to line segment BC call it F.

The midpoint to line segment DC call it G.

The midpoint to line segment AD call it H.

What type of parallelograms are ABCD, and EFGH?

Points: A(-2, 1), B(3, 3), C(4, -1), D(-1,-3)

| | | |
|--|--|----------------------------------|
| $\text{Mid}_{AB} = \left(\frac{-2+3}{2}, \frac{1+3}{2} \right)$ | $E(x, y) = \left(\frac{1}{2}, \frac{4}{2} \right) = \left(\frac{1}{2}, 2 \right)$ | $E\left(\frac{1}{2}, 2\right)$ |
| $\text{Mid}_{BC} = \left(\frac{3+4}{2}, \frac{3+(-1)}{2} \right)$ | $F(x, y) = \left(\frac{7}{2}, \frac{2}{2} \right) = \left(\frac{7}{2}, 1 \right)$ | $F\left(\frac{7}{2}, 1\right)$ |
| $\text{Mid}_{CD} = \left(\frac{4+(-1)}{2}, \frac{-1+(-3)}{2} \right)$ | $G(x, y) = \left(\frac{3}{2}, \frac{4}{2} \right) = \left(\frac{3}{2}, -2 \right)$ | $G\left(\frac{3}{2}, -2\right)$ |
| $\text{Mid}_{DA} = \left(\frac{-1+(-2)}{2}, \frac{-3+1}{2} \right)$ | $H(x, y) = \left(\frac{-3}{2}, \frac{-2}{2} \right)$ | $H\left(\frac{-3}{2}, -1\right)$ |

Formula for Slope of Line Segment:

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

| Original Points | Midpoints: |
|------------------|---|
| A(-2, 1) | E($\frac{1}{2}$, 2) |
| B(3, 3) | F($\frac{7}{2}$, 1) |
| C(4, -1) | G($\frac{3}{2}$, -2) |
| D(-1, -3) | H($-\frac{3}{2}$, -1) |

| Slopes (M) = (AB, DC) | Slopes (M) = (AD, BC) |
|---|--|
| $M_{AB} = \frac{3 - 1}{3 - (-2)} = \frac{2}{5}$ | $M_{AD} = \frac{-3 - 1}{-1 - (-2)} = -4$ |
| $M_{DC} = \frac{-1 - (-3)}{4 - (-1)} = \frac{2}{5}$ | $M_{BC} = \frac{-1 - 3}{4 - 3} = -4$ |

$$M_{AB} = M_{DC} = \frac{2}{5}$$

$$M_{AD} = M_{BC} = -4$$

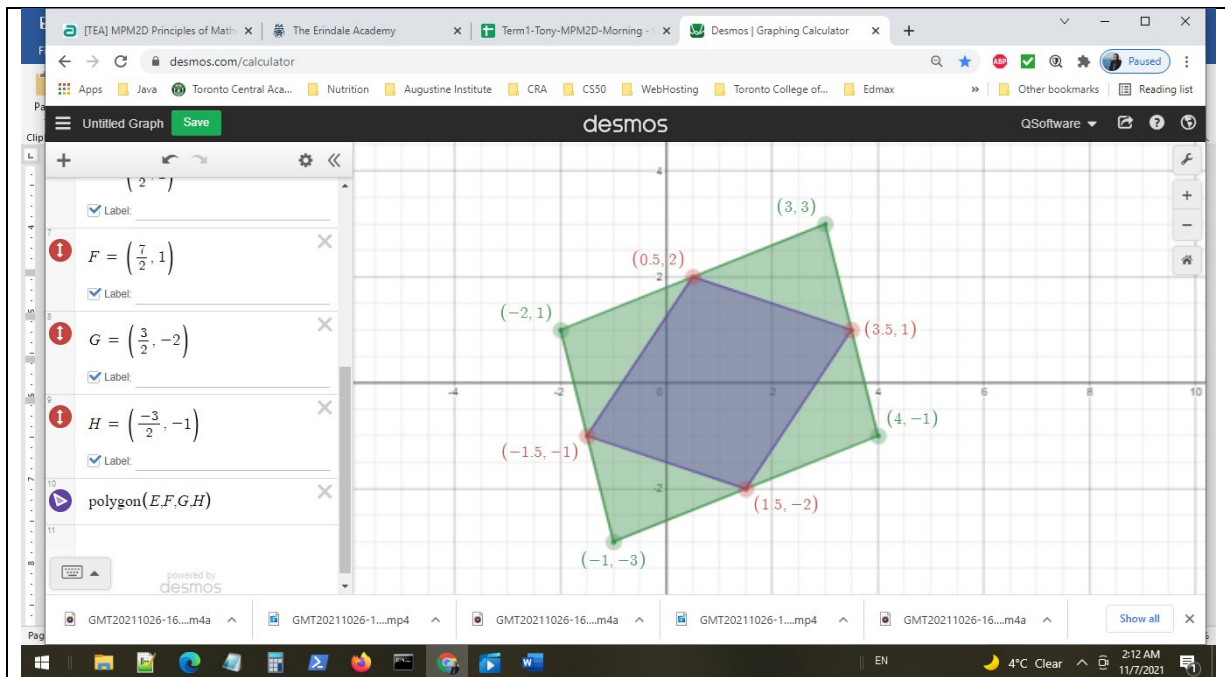
\therefore The outer quadrilateral is a parallelogram since the opposite sides the slopes are the same.

| Slopes (M) = (EH, FG) | Slopes (M) = (EF, HG) |
|---|--|
| $M_{EH} = \frac{-1 - 2}{(\frac{-3}{2} - (\frac{-1}{2}))} = \frac{-3}{-2} = \frac{3}{2}$ | $M_{EF} = \frac{1 - 2}{(\frac{7}{2} - (\frac{-1}{2}))} = \frac{1 - 2}{(\frac{6}{2})} = \frac{-1}{3}$ |
| $M_{FG} = \frac{-2 - 1}{(\frac{3}{2} - (\frac{7}{2}))} = \frac{-3}{(\frac{-4}{2})} = \frac{3}{2}$ | $M_{HG} = \frac{-2 - 1}{(\frac{3}{2} - (\frac{7}{2}))} = \frac{-1}{3}$ |

$$M_{EH} = M_{FG} = \frac{3}{2}$$

$$M_{EF} = M_{HG} = \frac{-1}{3}$$

\therefore The inner quadrilateral is a parallelogram



Both quadrilaterals are parallelograms. If student may further research the lengths and angles to if the quadrilaterals can be special type of other shapes such as: squares, rectangles, rombus, or kites.

Question 2: Use Method of elimination to solve the equations of lines:

1. $4x - 1y = 2$
2. $3x + y = 19$

Solve the equation of the lines algebraically as well as plotting the graphs of the two lines on the same graph.

$$\begin{aligned} \textcircled{1} \quad & 4x - 1y = 2 \\ \textcircled{2} \quad & 3x + y = 19 \quad \leftarrow \text{add equation } \textcircled{1} \text{ and } \textcircled{2} \end{aligned}$$

 $7x + 0y = 21$

$$x = \frac{21}{7} = 3$$

Substitute $x = 3$ into equation $\textcircled{2}$

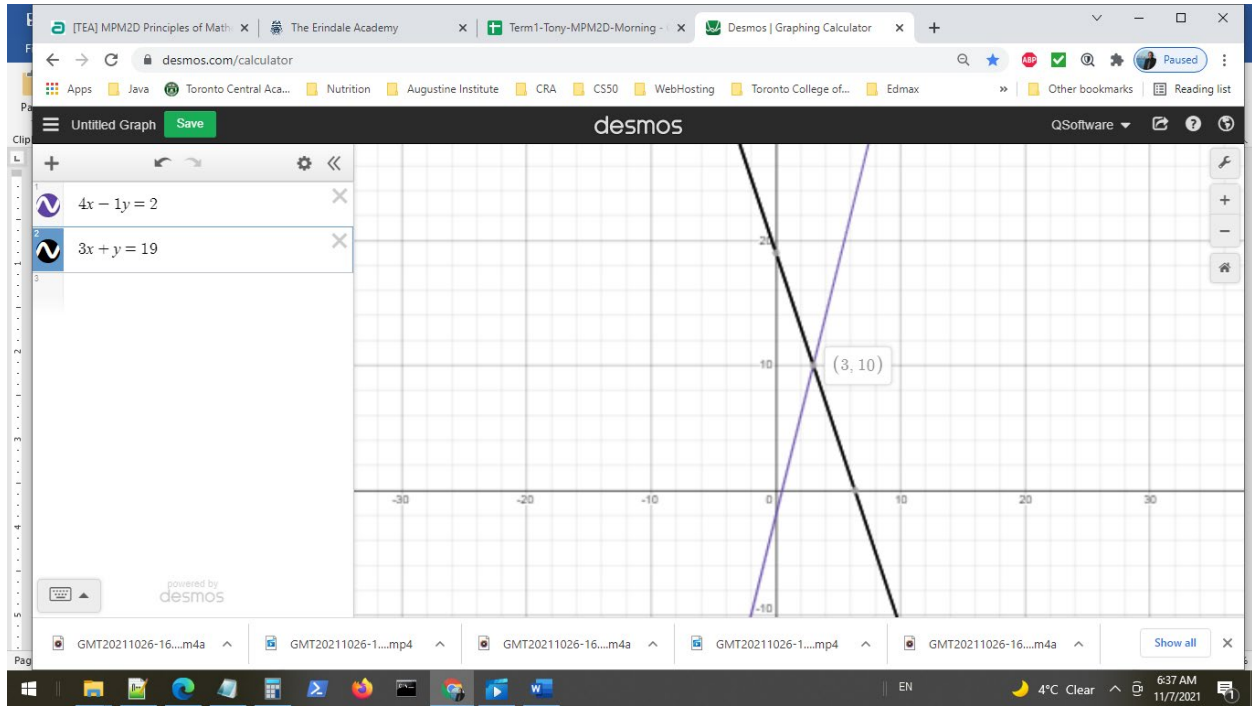
$$\begin{aligned} \textcircled{2} \quad & 3x + y = 19 \\ 3(3) + y &= 19 \\ 9 + y &= 19 \\ y &= 19 - 9 \\ y &= 10 \end{aligned}$$

Point is $P(x, y) = (3, 10)$

substitute into equation $\textcircled{1}$ and $\textcircled{2}$

| | |
|---|--|
| $\textcircled{1} \quad 4x - 1y = 2$ $\text{L.S.} = 4x - 1y$ $\text{L.S.} = 4(3) - 1(10)$ $\text{L.S.} = 12 - 10$ $\text{L.S.} = 2$ $\text{R.S.} = 2$ $\therefore \text{L.S.} = \text{R.S.}$ | $\textcircled{2} \quad 3x + y = 19$ $\text{L.S.} = 3x + y$ $\text{L.S.} = 3(3) + 10$ $\text{L.S.} = 19$ $\text{L.S.} = 19$ $\text{R.S.} = 19$ $\therefore \text{L.S.} = \text{R.S.}$ |
| <p style="color: red;">Since point $(x, y) = (3, 10)$ satisfies both equations.</p> <p style="color: red;">This is the point of intersection.</p> | |

Attach graph here



Graph shows $P(x, y) = (3, 10)$ as point of intersection as well.