

COURSE NAME: MPM2D – Principles of Mathematics	
Accumulative Activities: 10 AS Learning: Topics: (1.1 to 4.6) Teacher: Antonio Pietrangelo <div style="border: 1px solid black; padding: 2px; width: fit-content;"> Time: Throughout Course </div> <div style="border: 1px solid black; padding: 2px; width: fit-content;"> Pages: 22 </div>	Student's Name: Student#: <div style="border: 1px solid black; padding: 2px; width: fit-content;"> Due Date: Tuesday, February 13th, 2024 2:30 pm EST </div> <div style="border: 1px solid black; padding: 2px; width: fit-content;"> Mark: /100 </div>

Categories	Knowledge/ Understanding	Thinking/Inquiry/ Problem Solving	Communication	Application
Symbol	K/U	T/I	C	A
Weight	25 %	25 %	25 %	25 %
Level	N/A	N/A	N/A	

Overall Expectations:

Expectations as listed in the Ontario Curriculum course outline for your specific course.

Specific Expectations:

Chapter/Unit 1 - Systems of Linear Equations

- 1.1 Representing Linear Relations
- 1.2 Solving Linear Equations
- 1.3 Graphically Solving Linear Systems
- 1.4 Solving Linear Systems: Substitution
- 1.5 Equivalent Linear Systems
- 1.6 Solving Linear Systems: Elimination
- 1.7 Exploring Linear Systems

Chapter 2: Analytic Geometry: Line Segments and Circles, and Advanced Shapes

- 2.1 Midpoint of a Line Segment
- 2.2 Length of a Line Segment
- 2.3 Equation of a Circle
- 2.4 Classifying Figures on a Coordinate Grid
- 2.5 Verifying Properties of Geometric Figures
- 2.6 Exploring Properties of Geometric Figures
- 2.7 Using Coordinates to Solve Problems

Chapter 3: Graphs of Quadratic

- 3.1 Exploring Quadratic Relations
- 3.2 Properties of Graphs of Quadratic Relations
- 3.3 Factored Form of a Quadratic Relation
- 3.4 Expanding Quadratic Expressions
- 3.5 Quadratic Models Using Factored Form
- 3.6 Exploring Quadratic and Exponential Graphs

Chapter 4: Factoring Algebraic

- 4.1 Common Factors in Polynomials
- 4.2 Exploring the Factorization of Trinomials
- 4.3 Factoring Quadratics: $x^2 + bx + c$, where $(a = 1)$
- 4.4 Factoring Quadratics: $x^2 + bx + c$, where $(a \neq 1)$
- 4.5 Factoring Quadratics: Special Cases
- 4.6 Reasoning about Factoring Polynomials

Rubrics:

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Knowledge – Understanding of (Specific Expectations: 1.1 to 4.6 - Accumulative)	demonstrates insufficient understanding	demonstrates limited understanding	demonstrates some understanding	demonstrates considerable understanding	demonstrates thorough understanding	
				Individual: Mark:		

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Thinking and Inquiry (What if scenarios) of: (Specific Expectations: 1.1 to 4.6 - Accumulative)	demonstrates insufficient ability to apply different scenarios	demonstrates limited ability to apply different scenarios	demonstrates some ability to apply different scenarios	demonstrates considerable ability to apply different scenarios	demonstrates through ability to apply different scenarios	
				Individual: Mark:		

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark
Communication Communicates effectively (Specific Expectations: 1.1 to 4.6 - Accumulative)	demonstrates insufficient ability to communicate effectively	demonstrates limited ability to communicate effectively	demonstrates some ability to communicate effectively	demonstrates considerable ability to communicate effectively	demonstrates through ability to communicate effectively	
				Individual: Mark:		

Category	Level R (0 – 49%)	Level 1 (50-59%)	Level 2 (60-69%)	Level 3 (70-79%)	Level 4 (80-100%)	Level/ Mark		
<p><u>Application:</u></p> <p>Demonstrates the ability to apply mathematical principles to real world situations.</p> <p>(Specific Expectations: 1.1 to 4.6 - Accumulative)</p>	demonstrates insufficient ability	demonstrates limited ability	demonstrates some ability	demonstrates considerable ability	demonstrates thorough ability			
				<table border="1" style="width: 100%;"> <tr> <td style="width: 80%;">Individual: Mark:</td> <td></td> </tr> </table>		Individual: Mark:		
Individual: Mark:								



PART A: KNOWLEDGE AND UNDERSTANDING (K/U) – 25% - 100%,
PART B: THINKING AND INQUIRY (T/I) - 25% to 100%, if implemented
PART C: COMMUNICATION (C) – 25 to 100%, if implemented
PART D: APPLICATION (A) – 25% to 100%, if implemented

Each activity will be out of 10 marks, and can be an assessment of one or more of PART A through D. The percentages will be adjusted depending on what sections have been implemented.

PART D: APPLICATION (A) – 100%

Activity 10: Factor these quadratic equations of the form $y=ax^2+bx+c$, where $a \neq 1$

For each quadratic equation find the following (Show all required details of work):

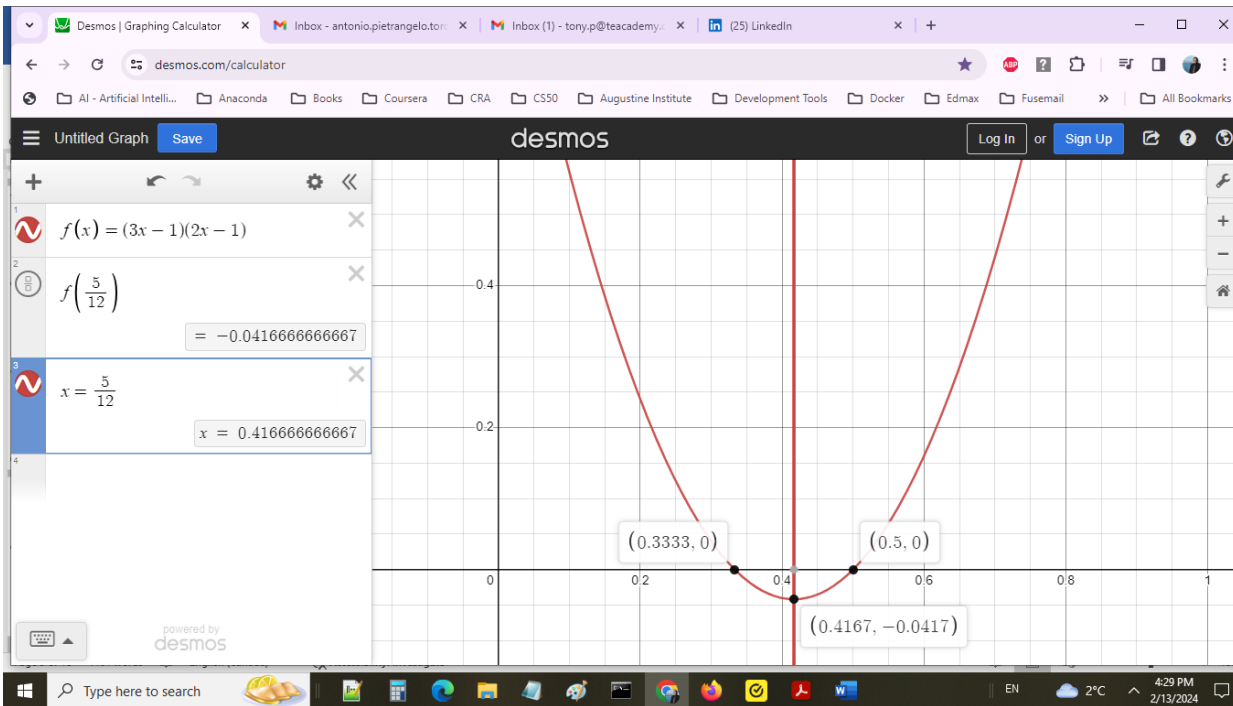
1. Find the factors, if possible and
2. Put the equation into **factored form $y = a(x - r)(x - s)$**
3. Find the axis of symmetry
4. Find the vertex of the parabola
5. Graph the equation using the details above

See example below for $6x^2 - 5x + 1$:

$6x^2 - 5x + 1$ $a = 6$ $b = -5$ $c = 1$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 6</th> <th>Product</th> <th>Sum (b) $b = a + c$</th> </tr> </thead> <tbody> <tr> <td>1, 6</td> <td>6</td> <td>7</td> </tr> <tr> <td>2, 3</td> <td>6</td> <td>5</td> </tr> <tr> <td>-1, -6</td> <td>6</td> <td>-7</td> </tr> <tr> <td>-2, -3</td> <td>6</td> <td>-5</td> </tr> </tbody> </table>	Factors (a x c) = 6	Product	Sum (b) $b = a + c$	1, 6	6	7	2, 3	6	5	-1, -6	6	-7	-2, -3	6	-5	$=6x^2 - 5x + 1$ $=6x^2 - 2x - 3x + 1$ $=2x(3x - 1) - (3x - 1)$ $=(3x - 1)(2x - 1)$ Expand to prove: $=(3x - 1)(2x - 1)$ $=6x^2 - 3x - 2x + 1$ $=6x^2 - 5x + 1$
Factors (a x c) = 6	Product	Sum (b) $b = a + c$															
1, 6	6	7															
2, 3	6	5															
-1, -6	6	-7															
-2, -3	6	-5															
Find axis of symmetry; occurs where $y = 0$	$y = (3x - 1)(2x - 1)$ $0 = (3x - 1)(2x - 1)$ Factor 1: $0 = (3x - 1) \Leftrightarrow$ Solve for x $-3x = -1$ $x_1 = \frac{-1}{-3} = \frac{1}{3}$ Factor 2: $0 = (2x - 1) \Leftrightarrow$ Solve for x $-2x = -1$ $x_2 = \frac{-1}{-2} = \frac{1}{2}$	$x_1 = r = \frac{1}{3} = 0.333$ $x_2 = s = \frac{1}{2} = 0.50$ $x_s = \frac{(r+s)}{2} = \frac{\frac{1}{3} + \frac{1}{2}}{2}$ $x_s = \frac{\frac{2}{6} + \frac{3}{6}}{2}$ $x_s = \frac{5}{6} \times \frac{1}{2}$ $x_s = \frac{5}{12} = 0.4167$															

<p>Find vertex $f(x_s)$, where x_s the axis of symmetry</p> <p>Vertex $(\frac{5}{12}, \frac{-1}{24})$</p>	$Y=f(x_s) = (3x - 1)(2x - 1)$ $f(\frac{5}{12}) = (3(\frac{5}{12}) - 1)(2(\frac{5}{12}) - 1)$ $f(\frac{5}{12}) = ((\frac{15}{12} - \frac{12}{12})(\frac{10}{12} - \frac{12}{12}))$	$f(\frac{5}{12}) = (\frac{3}{12})(\frac{-2}{12})$ $f(\frac{5}{12}) = (\frac{1}{4})(\frac{-1}{6}) = \frac{-1}{24}$ $y = f(\frac{5}{12}) = \frac{-1}{24} = -0.0417$
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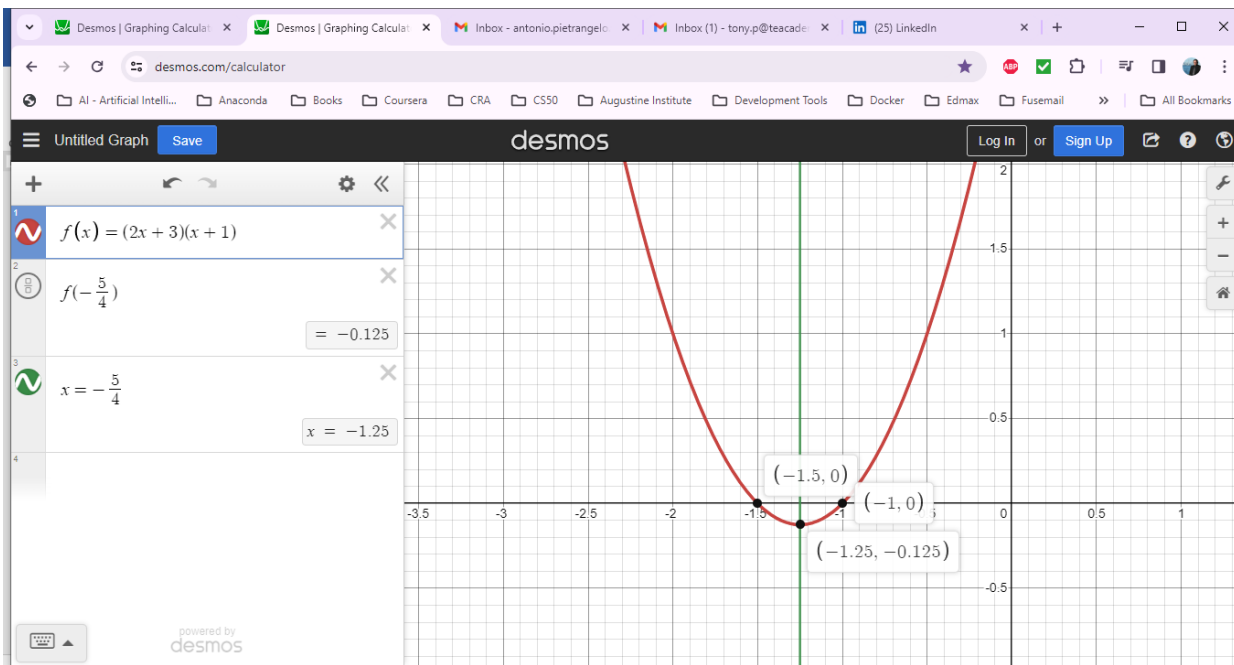
Graph of: $6x^2 - 5x + 1$



Continue with the following quadratic equations as per above

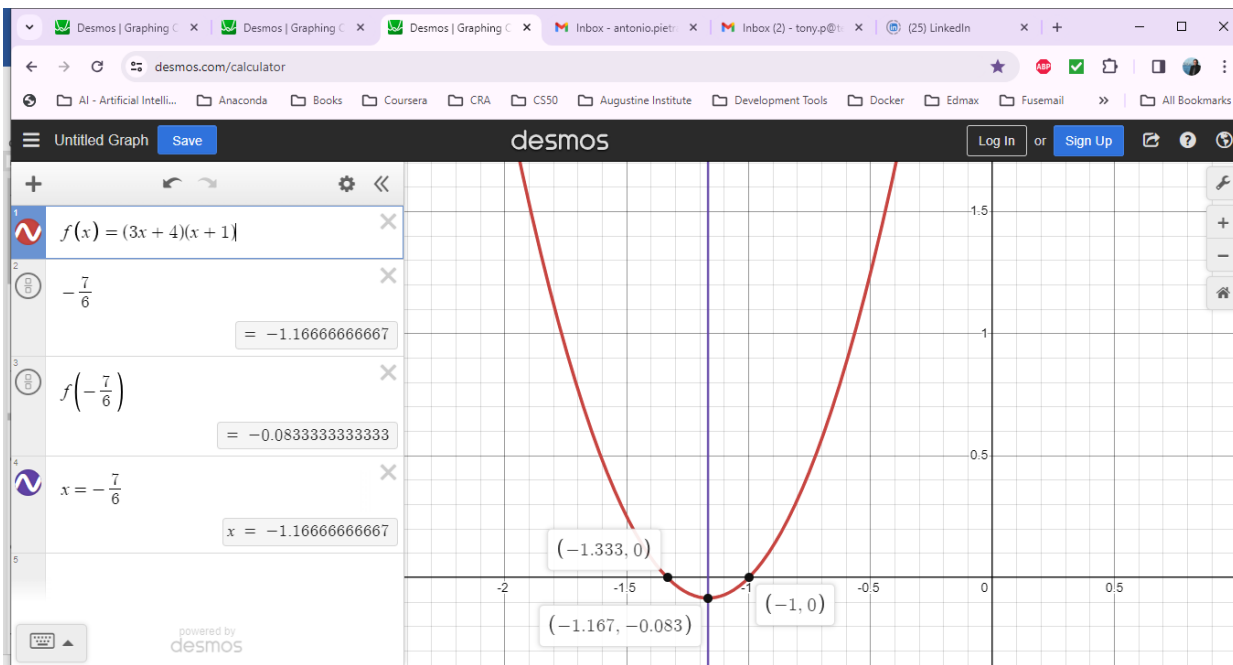
$2x^2 + 5x + 3$ $a = 2$ $b = 5$ $c = 3$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 6</th> <th>Product</th> <th>Sum (b) b = a + c</th> </tr> </thead> <tbody> <tr> <td>1, 6</td> <td>6</td> <td>7</td> </tr> <tr> <td>2, 3</td> <td>6</td> <td>5</td> </tr> </tbody> </table>	Factors (a x c) = 6	Product	Sum (b) b = a + c	1, 6	6	7	2, 3	6	5	$= 2x^2 + 5x + 3$ $= 2x^2 + 2x + 3x + 3$ $= 2x(x + 1) + 3(x + 1)$ $= (x + 1)(2x + 3)$ Expand to prove: $= (x + 1)(2x + 3)$ $= 2x^2 + 3x + 2x + 3$ $= 2x^2 + 5x + 3$
Factors (a x c) = 6	Product	Sum (b) b = a + c									
1, 6	6	7									
2, 3	6	5									
Find axis of symmetry; occurs where $y = 0$	$y = (x + 1)(2x + 3)$ $y = (2x + 3)(x + 1)$ $0 = (2x + 3)(x + 1)$ Factor 1: $0 = (2x + 3)$ $-2x = 3$ $x_1 = -\frac{3}{2}$ Factor 2: $0 = (x + 1)$ $x_2 = -1$	$x_1 = r = -\frac{3}{2}$ Type equation here. $x_2 = s = -1$ $x_s = \frac{(r+s)}{2} = \frac{-\frac{3}{2} + \frac{-2}{2}}{2}$ $x_s = \frac{-5}{2}$ $x_s = \frac{-5}{2} \times \frac{1}{2}$ $x_s = \frac{-5}{4}$									
Find vertex $f(x_s)$, where x_s the axis of symmetry Vertex (x_s, y)	$y = f(x_s) = (2x + 3)(x + 1)$ $f\left(\frac{-5}{4}\right) = \left(2\left(\frac{-5}{4}\right) + 3\right)\left(\frac{-5}{4} + 1\right)$ $f\left(\frac{-5}{4}\right) = \left(\frac{-10}{4} + \frac{12}{4}\right)\left(\frac{-5}{4} + \frac{4}{4}\right) = \left(\frac{2}{4}\right)\left(\frac{-1}{4}\right)$ $f\left(\frac{-5}{4}\right) = \frac{-1}{8} = -0.125$	Vertex $(x, y) = \left(\frac{-5}{4}, \frac{-1}{8}\right)$									

Graph of: $y = 2x^2 + 5x + 3$; $y = (2x + 3)(x + 1)$



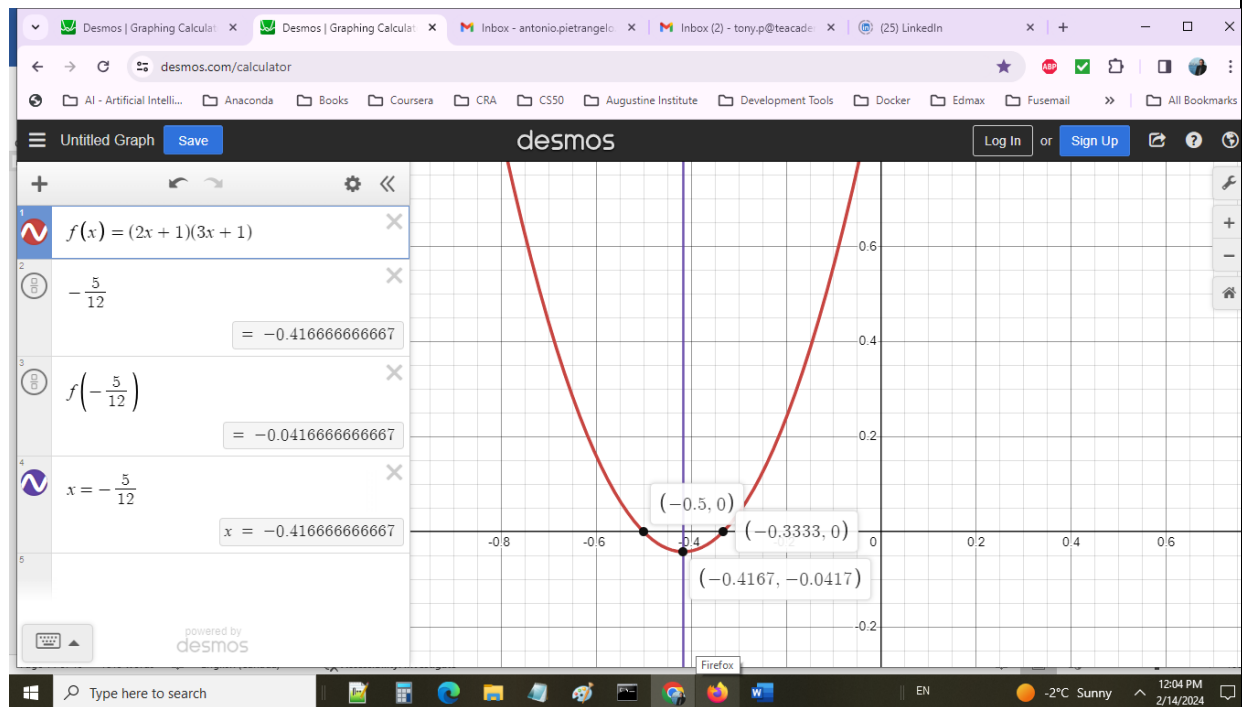
$3x^2 + 7x + 4$ $a = 3$ $b = 7$ $c = 4$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 12</th> <th>Product</th> <th>Sum (b) b = a + c</th> </tr> </thead> <tbody> <tr> <td>1, 12</td> <td>12</td> <td>13</td> </tr> <tr> <td>2, 6</td> <td>12</td> <td>8</td> </tr> <tr> <td>3, 4</td> <td>12</td> <td>7</td> </tr> </tbody> </table>	Factors (a x c) = 12	Product	Sum (b) b = a + c	1, 12	12	13	2, 6	12	8	3, 4	12	7	$= 3x^2 + 7x + 4$ $= 3x^2 + 3x + 4x + 4$ $= 3x(x + 1) + 4(x + 1)$ $= (x + 1)(3x + 4)$ Expand to prove: $= (x + 1)(3x + 4)$ $= 3x^2 + 4x + 3x + 4$ $= 3x^2 + 7x + 4$
Factors (a x c) = 12	Product	Sum (b) b = a + c												
1, 12	12	13												
2, 6	12	8												
3, 4	12	7												
Find axis of symmetry; occurs where $y = 0$	$y = (x + 1)(3x + 4)$ $y = (3x + 4)(x + 1)$ $0 = (3x + 4)(x + 1)$ Factor 1: $0 = (3x + 4)$ $-3x = 4$ $x_1 = -\frac{4}{3}$ Factor 2: $0 = (x + 1)$ $x_2 = -1$	$x_1 = r = -\frac{4}{3} = -1.333$ Type equation here. $x_2 = s = -1$ $x_s = \frac{(r+s)}{2} = \frac{-\frac{4}{3} + \frac{-3}{3}}{2}$ $x_s = \frac{-\frac{7}{3}}{2}$ $x_s = \frac{-7}{3} \times \frac{1}{2}$ $x_s = \frac{-7}{6} = -1.1667$												
Find vertex $f(x_s)$, where x_s the axis of symmetry Vertex (x_s, y)	$Y = f(x_s) = (3x + 4)(x + 1)$ $f\left(\frac{-7}{6}\right) = \left(3\left(\frac{-7}{6}\right) + 4\right)\left(\frac{-7}{6} + 1\right)$ $f\left(\frac{-7}{6}\right) = \left(\frac{-21}{6} + \frac{24}{6}\right)\left(\frac{-7}{6} + \frac{6}{6}\right) = \left(\frac{3}{6}\right)\left(\frac{-1}{6}\right) = \left(\frac{1}{2}\right)\left(\frac{-1}{6}\right)$ $f\left(\frac{-7}{6}\right) = \frac{-1}{12} = -0.0833$	Vertex $(x, y) = \left(\frac{-7}{6}, \frac{-1}{12}\right)$												

Graph of: $y = 3x^2 + 7x + 4$; $y = (3x + 4)(x + 1)$



$6x^2 + 5x + 1$ $a = 6$ $b = 5$ $c = 1$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 6</th> <th>Product</th> <th>Sum (b) b = a + c</th> </tr> </thead> <tbody> <tr> <td>1, 6</td> <td>6</td> <td>7</td> </tr> <tr> <td>2, 3</td> <td>5</td> <td>5</td> </tr> </tbody> </table>	Factors (a x c) = 6	Product	Sum (b) b = a + c	1, 6	6	7	2, 3	5	5	$=6x^2 + 5x + 1$ $=6x^2 + 3x + 2x + 1$ $=3x(2x + 1) + (2x + 1)$ $=(2x + 1)(3x + 1)$ Expand to prove: $=(2x + 1)(3x + 1)$ $=6x^2 + 2x + 3x + 1$ $=6x^2 + 5x + 1$
Factors (a x c) = 6	Product	Sum (b) b = a + c									
1, 6	6	7									
2, 3	5	5									
Find axis of symmetry; occurs where $y = 0$	$y = (2x + 1)(3x + 1)$ $y = (2x + 1)(3x + 1)$ $0 = (2x + 1)(3x + 1)$ Factor 1: $0 = (2x + 1)$ $-2x = 1$ $x_1 = -\frac{1}{2}$ Factor 2: $0 = (3x + 1)$ $-3x = 1$ $x_2 = -\frac{1}{3}$	$x_1 = r = -\frac{1}{2} = -0.50$ Type equation here. $x_2 = s = -\frac{1}{3} = -0.333$ $x_s = \frac{(r+s)}{2} = \frac{-\frac{1}{2} + -\frac{1}{3}}{2}$ $x_s = \frac{-\frac{5}{6}}{2}$ $x_s = \frac{-5}{6} \times \frac{1}{2} = \frac{-5}{12}$ $x_s = \frac{-5}{12} = -0.4167$									
Find vertex $f(x_s)$, where x_s the axis of symmetry Vertex (x_s, y)	$Y=f(x_s) = (2x + 1)(3x + 1)$ $f(\frac{-5}{12}) = (2(\frac{-5}{12}) + 1)(3(\frac{-5}{12}) + 1)$ $f(\frac{-5}{12}) = (\frac{-10}{12} + \frac{12}{12})(\frac{-15}{12} + \frac{12}{12}) = (\frac{1}{6})(\frac{-1}{4})$ $f(\frac{-5}{12}) = \frac{-1}{24} = -0.04167 = -0.042$	Vertex $(x, y) = (\frac{-5}{12}, \frac{-1}{24})$									

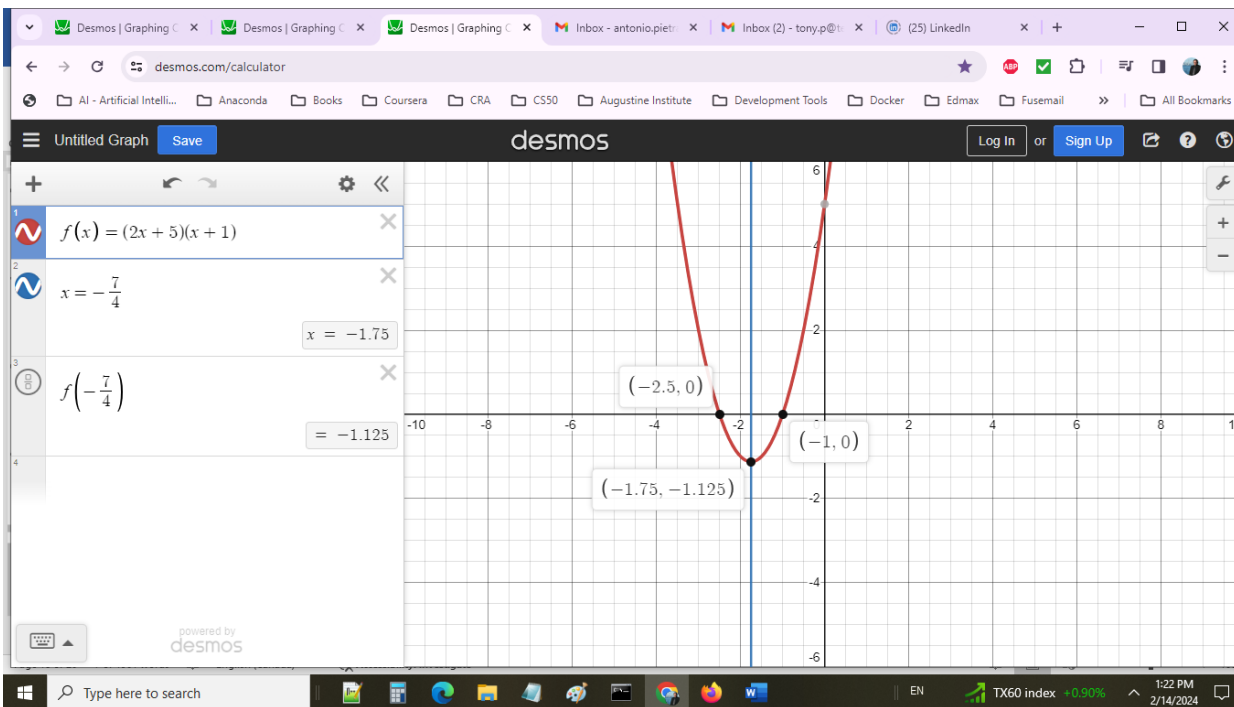
Graph of: $y = 6x^2 + 5x + 1$; $y = (2x + 1)(3x + 1)$



$6x^2 + 11x + 1$ $a = 6$ $b = 11$ $c = 1$	Factors (a x c) = 6	Product	Sum (b) $b = a + c$	Not possible to factor using this method.
	1, 6	6	7	
	2, 3	5	5	
			Can not get two factors of (a x c) equal to the sum of $b = a + c$	

$2x^2 + 7x + 5$ $a = 2$ $b = 7$ $c = 5$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 10</th> <th>Product</th> <th>Sum (b) b = a + c</th> </tr> </thead> <tbody> <tr> <td>1, 10</td> <td>10</td> <td>11</td> </tr> <tr> <td>2, 5</td> <td>10</td> <td>7</td> </tr> </tbody> </table>	Factors (a x c) = 10	Product	Sum (b) b = a + c	1, 10	10	11	2, 5	10	7	$=2x^2 + 7x + 5$ $=2x^2 + 2x + 5x + 5$ $=2x(x + 1) + 5(x + 1)$ $=(x + 1)(2x + 5)$ Expand to prove: $=(x + 1)(2x + 5)$ $=2x^2 + 5x + 2x + 5$ $=2x^2 + 7x + 5$
Factors (a x c) = 10	Product	Sum (b) b = a + c									
1, 10	10	11									
2, 5	10	7									
Find axis of symmetry; occurs where $y = 0$	$y = (x + 1)(2x + 5)$ $y = (2x + 5)(x + 1)$ $0 = (2x + 5)(x + 1)$ Factor 1: $0 = (2x + 5)$ $-2x = 5$ $x_1 = -\frac{5}{2}$ Factor 2: $0 = (x + 1)$ $-x = 1$	$x_1 = r = -\frac{5}{2} = -2.50$ Type equation here. $x_2 = s = -1$ $x_s = \frac{(r+s)}{2} = \frac{-5 + -2}{2}$ $x_s = \frac{-7}{2}$ $x_s = \frac{-7}{2} \times \frac{1}{2} = \frac{-7}{4}$ $x_s = \frac{-7}{4} = -1.75$									
Find vertex $f(x_s)$, where x_s the axis of symmetry Vertex (x_s, y)	$y = f(x_s) = (2x + 5)(x + 1)$ $f\left(\frac{-7}{4}\right) = \left(2\left(\frac{-7}{4}\right) + 5\right)\left(\frac{-7}{4} + 1\right)$ $f\left(\frac{-7}{4}\right) = \left(\frac{-14}{4} + \frac{20}{4}\right)\left(\frac{-7}{4} + \frac{4}{4}\right) = \left(\frac{6}{4}\right)\left(\frac{-3}{4}\right)$ $f\left(\frac{-7}{4}\right) = \frac{-9}{8} = -1.125$	Vertex $(x, y) = \left(\frac{-7}{4}, \frac{-9}{8}\right)$									

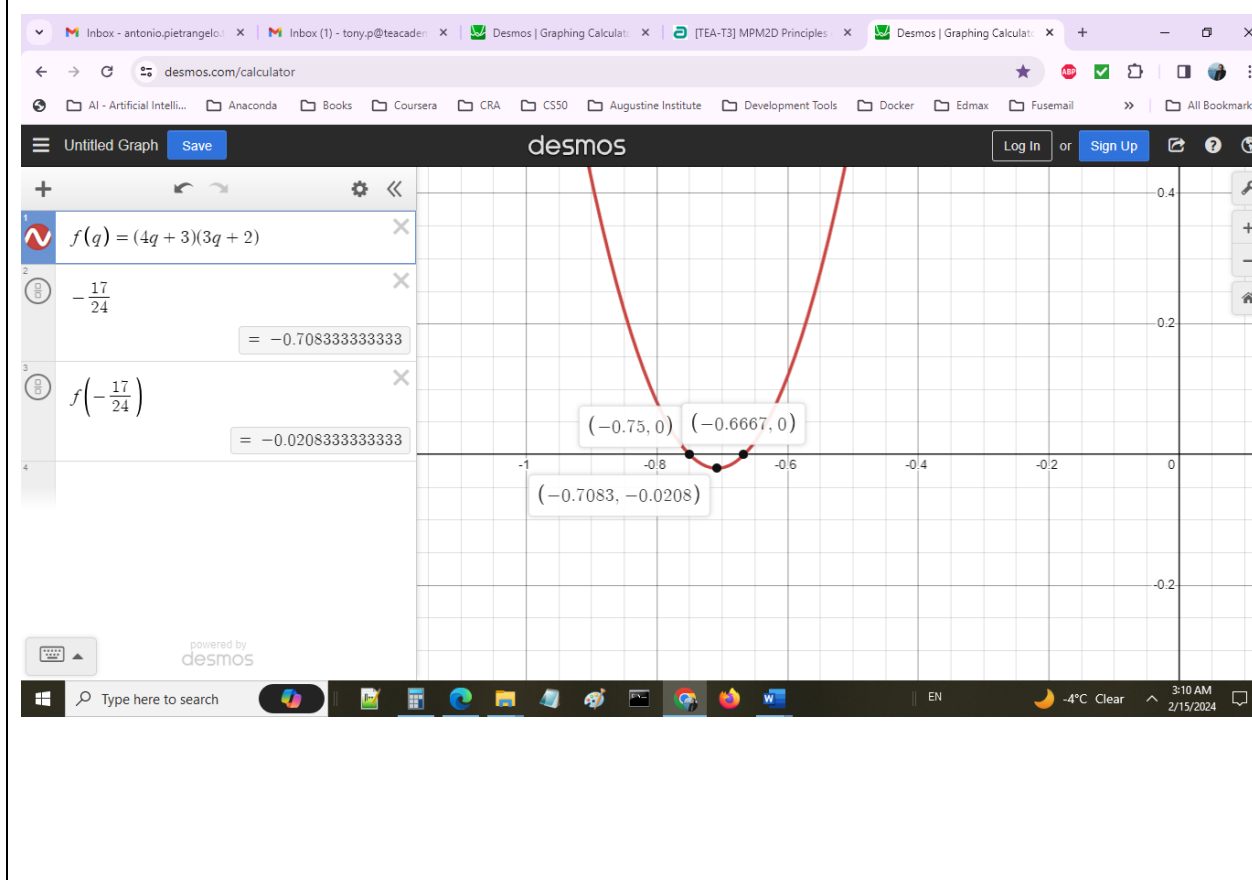
Graph of: $y = 2x^2 + 7x + 5$; $y = (2x + 5)(x + 1)$



<p style="color: red;">$6y^2 + 19y + 18$</p> <p>a = 6 b = 19 c = 18</p>	<p>Factors (a x c) = 108</p>	<p>Product</p>	<p>Sum (b) b = a + c</p>	<p>Not possible to factor using this method.</p>
	1, 108	108	109	
	2, 54	108	56	
	3, 36	108	39	
	4, 27	108	31	
	6, 18	108	24	
	9, 12	108	21	
			<p>Can not get two factors of (a x c) equal to the sum of b = a + c</p>	
<p style="color: red;">Find axis of symmetry; occurs where $y = 0$</p>	<p>Can not solve problem using this method for this quadratic equation.</p>			
<p style="color: red;">Find vertex $f(x_s)$, where x_s the axis of symmetry</p> <p style="color: red;">Vertex (x_s, y)</p>	<p>Can not solve problem using this method for this quadratic equation.</p>			

$12q^2 + 17q + 6$ $a = 12$ $b = 17$ $c = 6$	<table border="1"> <thead> <tr> <th>Factors (a x c) = 72</th> <th>Product</th> <th>Sum (b) b = a + c</th> </tr> </thead> <tbody> <tr> <td>1, 72</td> <td>72</td> <td>73</td> </tr> <tr> <td>2, 36</td> <td>72</td> <td>38</td> </tr> <tr> <td>3, 24</td> <td>72</td> <td>27</td> </tr> <tr> <td>4, 18</td> <td>72</td> <td>22</td> </tr> <tr> <td>6, 12</td> <td>72</td> <td>18</td> </tr> <tr> <td>8, 9</td> <td>72</td> <td>17</td> </tr> </tbody> </table>	Factors (a x c) = 72	Product	Sum (b) b = a + c	1, 72	72	73	2, 36	72	38	3, 24	72	27	4, 18	72	22	6, 12	72	18	8, 9	72	17	$=12q^2 + 17q + 6$ $=12q^2 + 8q + 9q + 6$ $=4q(3q + 2) + 3(3q + 2)$ $=(3q + 2)(4q + 3)$ Expand to prove: $=(3q + 2)(4q + 3)$ $=12q^2 + 9q + 8q + 6$ $=12q^2 + 17q + 6$
Factors (a x c) = 72	Product	Sum (b) b = a + c																					
1, 72	72	73																					
2, 36	72	38																					
3, 24	72	27																					
4, 18	72	22																					
6, 12	72	18																					
8, 9	72	17																					
Find axis of symmetry; occurs where $y = 0$	$y = (3q + 2)(4q + 3)$ $y = (4q + 3)(3q + 2)$ $0 = (4q + 3)(3q + 2)$ Factor 1: $0 = (4q + 3)$ $-4q = 3$ $q_1 = -\frac{3}{4}$ Factor 2: $0 = (3q + 2)$ $-3q = 2$ $q_2 = -\frac{2}{3}$	$q_1 = r = -\frac{3}{4} = -0.75$ $q_2 = s = -\frac{2}{3} = -0.667$ $q_s = \frac{(r+s)}{2} = \frac{-\frac{3}{4} + \frac{-2}{3}}{2}$ $q_s = \frac{\frac{-9}{12} + \frac{-8}{12}}{2} = \frac{-\frac{17}{12}}{2}$ $q_s = \frac{-17}{12} \times \frac{1}{2} = \frac{-17}{24}$ $q_s = \frac{-17}{24} = -0.7083$																					
Find vertex $f(x_s)$, where x_s the axis of symmetry Vertex (x_s, y)	$y = f(q_s) = (4q + 3)(3q + 2)$ $f\left(\frac{-17}{24}\right) = \left(\frac{-68}{24} + \frac{72}{24}\right)\left(\frac{-51}{24} + \frac{48}{24}\right)$ $f\left(\frac{-17}{24}\right) = \left(\frac{4}{24}\right)\left(\frac{-3}{24}\right) = \left(\frac{1}{6}\right)\left(\frac{-1}{8}\right)$ $f\left(\frac{-17}{24}\right) = \frac{-1}{48} = -0.0208$	Vertex $(x, y) = \left(\frac{-17}{24}, \frac{-1}{48}\right)$																					

Graph of: $y = 12q^2 + 17q + 6$; $y = (4q + 3)(3q + 2)$





THANK YOU!!!