THE ERINDALE ACADEMY
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Course: MDM4U Teacher: Ms. Ella

**Unit 1: Permutation:**

## Lesson 1.3: Pascal's Triangle and its applications

### Part 1: Pascal's Triangle

Pascal's Triangle: The array of numbers shown below is called Pascal's triangle, discovered by Blaise Pascal (1623 - 1662) at the age of 13. Pascal's method for building his triangle is a simple iterative process.

### INVESTIGATE PASCAL'S TRIANGLE

Review the first four rows of Pascal's triangle and determine the missing values for the fifth and sixth rows.

| | |
|---|---|
| Row 0 | 1 |
| Row 1 | 1 1 |
| Row 2 | 1 2 1 |
| Row 3 | 1 3 3 1 |
| Row 4 | 1 4 6 4 1 |
| Row 5 | |
| Row 6 | |

- Each term is equal to the ________ of the two terms immediately ____________ it.
- The first and last terms in each row are _________ since the only term immediately above is one.

**Find a formula for any term in Pascal's triangle.**

| | |
|---|---|
| Row 0 | $t_{0,0}$ |
| Row 1 | $t_{1,0}$ $t_{1,1}$ |
| Row 2 | $t_{2,0}$ $t_{2,1}$ $t_{2,2}$ |
| Row 3 | $t_{3,0}$ $t_{3,1}$ $t_{3,2}$ $t_{3,3}$ |
| Row 4 | $t_{4,0}$ $t_{4,1}$ $t_{4,2}$ $t_{4,3}$ $t_{4,4}$ |
| Row 5 | $t_{5,0}$ $t_{5,1}$ $t_{5,2}$ $t_{5,3}$ $t_{5,4}$ $t_{5,5}$ |
| Row 6 | $t_{6,0}$ $t_{6,1}$ $t_{6,2}$ $t_{6,3}$ $t_{6,4}$ $t_{6,5}$ $t_{6,6}$ |

- If $t_{n,r}$ represent the term is row n, position r, then for any term $t_{n,r} =$ ________________________
- Rearrange the formula to solve for:

  $t_{n-1,r-1} =$ _________________________

  $t_{n-1,r} =$ ___________________________

### INVESTIGATE ROW SUMS IN PASCAL'S TRIANGLE

a) Find the sum of the numbers in each of the first six rows of Pascal's triangle

| | Sum |
|---|---|
| Row 1 | |
| Row 2 | |
| Row 3 | |
| Row 4 | |
| Row 5 | |
| Row 6 | |

b) Predict the sum of the entries in row 7, 8, 9.

Row 7:

Row 8:

Row 9:

c) Determine the formula to predict the sum of the entries in ANY row n of Pascal's triangle.

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**<u>Example 1</u>**: Which row in Pascal's triangle has the sum of tis terms equal to 32 768?

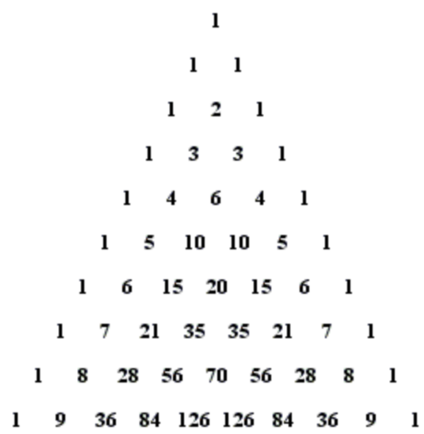
**<u>Example 2</u>**: Coins can be arranged in the shape of an equilateral triangle.

a) Continue the pattern to determine the number of coins in tringles with 4, 5, and 6 rows.

b) Locate these numbers in Pascal's triangle.

c) Related Pascal's triangle to the number of coins in a triangle with "n" rows.

d) How many coins are in triangle with 12 rows?




**MORE PROPERTIES IN PASCLE'S TRIANGLE:**

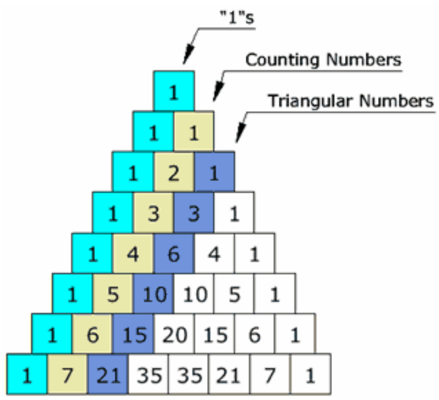
**Diagonals:**

The 1st diagonal is, of course, just "1"s

The 2nd diagonal has the counting numbers (1,2,3,etc)

The 3rd diagonal has the <u>triangular numbers</u> (Notice that the nth triangular number is also the sum of the first n positive integers).

The 4th diagonal, not highlighted in this course, has the tetrahedral numbers.



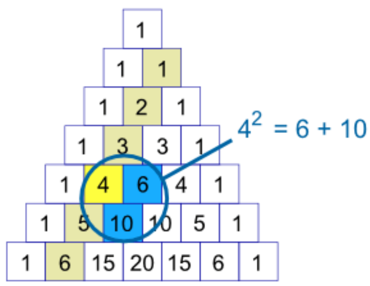

**Squares:**

For the second diagonal, the square of a number is equal to the sum of the numbers next to it and below both of those.

Such as $3^2 = 3 + 6 = 9$

$4^2 = 6 + 10 = 16$

$5^2 = 10 + 15 = 25$......



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## Part 2: Applying Pascal's Method -- Pathway

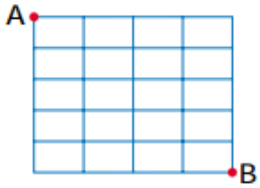
**Example 3**: Fill in the missing numbers using Pascal's method.

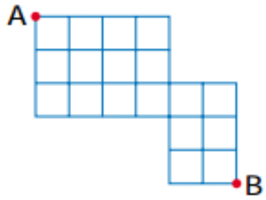
___ ___ 495 ___
___ ___ 825
3003 2112
___

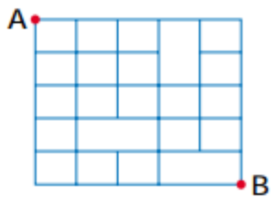
**Example 4**: Determine how many different paths will spell PATTERNS if you start at the top and proceed to the next row by moving diagonally left or right?

P
A A
T T T
T T T T
E E E E E
R R R R R R
N N N N N N N
S S S S S S S S

**Example 5**: Determine the number of possible routes from A to B if you travel only south or east.

a) 

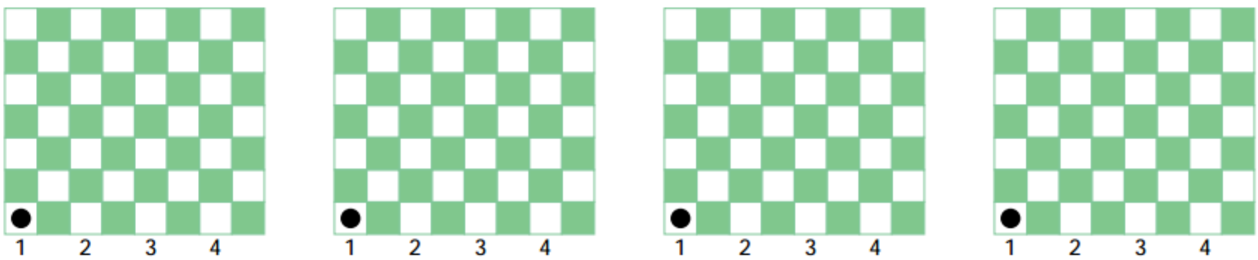

b) 


c) 


**Example 6**:

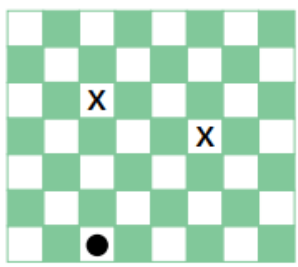
1) If a checker is placed as shown below, how many possible paths are there for that checker to reach the top of the game board? Recall that checkers can travel only diagonally on the white squares, one square at a time, moving upward.

2) when a checker reaches the opposite side, it becomes a "king". If the starting squares are labelled 1 to 4, from left to right, from which starting square does a checker have the most routes to become a king? Verify your statement.




**Practice**:

A checker is placed on a checkerboard as shown. The checker may move diagonally upward. Although it cannot move into a square with an X, the checker may jump over the X into the diagonally opposite square.



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## Part 3: Use Pascal's Triangles to apply Binomial Theorem

**Example 7**: Use Pascal's triangle to expand

a) $(x+y)^4$

b) $(2x-1)^5$

c) $(3x-2y)^4$

d) $(x+\frac{2}{x^2})^4$

**Example 8**: Rewrite $1 + 10x^2 + 40x^4 + 80x^6 + 80x^8 + 32x^{10}$ in the form $(a + b)^n$

**Practice:**

1. Rewrite the following expansions in the form $(a + b)^n$ where n is a positive integer.

1) $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

2) $243a^3 - 405a^4b + 270a^3b^2 - 90a^2b^3 + 15ab^4 - b^5$

2. Use the binomial theorem to expand and simplify the following

1) $(x^2+\frac{1}{x})^5$ 2) $2\ (3m^2-\frac{2}{\sqrt{m}})^4$ 3) $(\sqrt{y}-\frac{2}{\sqrt{y}})^7$

3. Use the binomial theorem to simplify each of the following. Explain your results.

1) $7^9$ - 9 x $7^8$ + 36 x $7^7$ - ....... - $7^0$

2) $(0.7)^7 + 7(0.7)^6(0.3) + 21(0.7)^5(0.3)^2 +$ ........ $+ (0.3)^7$

4. Use the binomial theorem to expand $(x + y + z)^4$.