



Unit 2: Combination

Lesson 2.1: Venn diagrams

Definitions:

✓ **Venn Diagram:** A useful way of representing various sets.

✓ **Set:** A collection of distinguishable or different elements.

Denoted by circles in a Venn diagram.

✓ **Elements:** The individual members of a set.

✓ **Universal Set:** The set \underline{S} , from which all the elements are derived.

Denoted by a rectangular box enclosing the sets.

✓ **Common Elements:** Elements which belong to more than one set.

Illustrated by using overlapping circles.

✓ **Subset of a Set:** "A" is a subset of "B" if all the elements in A are also in B.

Example: if $S = \{\text{real numbers}\}$, $B = \{\text{whole numbers}\}$, $C = \{\text{even whole numbers}\}$,

then we can say C is the subset of B, and both B and C are subsets of S.

✓ **Union of Sets:** The union of sets A and B is denoted by $A \cup B$.

It consists of all elements which are A or B.

✓ **Intersection of Sets:** The intersection of sets A and B is denoted by $A \cap B$.

It consists of all elements which are in A and B.

✓ **Principle of Inclusion and Exclusion for two sets:** For sets A and B, the total number of elements in either A or B is the number in A plus the number in B minus the number in both A and B.

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

Example 1:

If S = the set of whole numbers less than 12 but greater than or equal to 1 $\rightarrow (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11)$

If A = the set of even numbers less than 12 but greater than or equal to 1 $\rightarrow (2, 4, 6, 8, 10)$

If B = the set of whole numbers less than 3 but greater than 1, and numbers less than 12 but greater than 1 which are divisible by 3 $\rightarrow (2, 3, 6, 9)$

Use Venn Diagram to represent S, A, and B.



Working with three sets

Example 2: Use the principle of inclusion and exclusion to develop a formula for the number of elements in

- a) Three sets

- b) Four sets

Example 3: Of the 140 grade 12 students at The Erindale High School, 52 have signed up for biology, 71 for chemistry, and 40 for physics. The science students include 15 who are taking both biology and chemistry, 8 who are taking chemistry and physics, 11 who are taking biology and physics, and 2 who are taking all three science courses. How many students are not taking any of three science courses? Illustrate the enrolments with a Venn diagram.

☆ **Example 12.** (AMC 10) How many numbers between 1 and 2005 are integer multiples of 3 or 4 but not 12?
(A) 501 (B) 668 (C) 835 (D) 1002 (E) 1169

Problem 6. How many integers between 1 and 600 inclusive are not divisible by neither 3, nor 5, nor 7?
(A) 325 (B) 105 (C) 275 (D) 200 (E) 300



Apply, Solve, Communicate

B

3. Of the 220 graduating students in a school, 110 attended the semi-formal dance and 150 attended the formal dance. If 58 students attended both events, how many graduating students did not attend either of the two dances? Illustrate your answer with a Venn diagram.
4. **Application** A survey of 1000 television viewers conducted by a local television station produced the following data:
 - 40% watch the news at 12 00
 - 60% watch the news at 18 00
 - 50% watch the news at 23 00
 - 25% watch the news at 12 00 and at 18 00
 - 20% watch the news at 12 00 and at 23 00
 - 20% watch the news at 18 00 and at 23 00
 - 10% watch all three news broadcasts
 - a) What percent of those surveyed watch at least one of these programs?
 - b) What percent watch none of these news broadcasts?
 - c) What percent view the news at 12 00 and at 18 00, but not at 23 00?
 - d) What percent view only one of these shows?
 - e) What percent view exactly two of these shows?
6. **Application** There are 900 employees at CantoCrafts Inc. Of these, 615 are female, 345 are under 35 years old, 482 are single, 295 are single females, 187 are singles under 35 years old, 190 are females under 35 years old, and 120 are single females under 35 years old. Use a Venn diagram to determine how many employees are married males who are at least 35 years old.
7. **Communication** A survey of 100 people who volunteered information about their reading habits showed that
 - 75 read newspapers daily
 - 35 read books at least once a week
 - 45 read magazines regularly
 - 25 read both newspapers and books
 - 15 read both books and magazines
 - 10 read newspapers, books, and magazines
 - a) Construct a Venn diagram to determine the maximum number of people in the survey who read both newspapers and magazines.
 - b) Explain why you cannot determine exactly how many of the people surveyed read both newspapers and magazines.