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Course: MDM4U Teacher: Ms. Ella

Unit 2: Combination

Lesson 2.2: Different types of questions involving combination

Definition: A selection from a group of items without regard to order is called a combination.

Combinations of n distinct objects taken r at a time: The number of combinations of r objects chosen from a set of n distinct objects is:

$${}_nC_r \text{ or } C(n,r) = \frac{{}_nP_r}{r!} = \frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{(n-r)!r!}$$

Multiplicative and additive counting principles we learned from permutations in last chapter can also be applied to problems involving combinations.

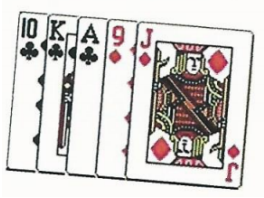
Example 1: 1) If there are 17 randomly placed dots on a circle, how many lines can you form using any 2 dots?

2) If there are 15 dots on a circle, how many triangles can be formed?

Combinations including specific items:

Example 2: A school committee of 5 is to be formed from 12 students. How many committees can be formed if John must be on the committee?

Example 3: From a deck of 52 cards, a 5 card hand is dealt. How many distinct five card hands are there if the queen of spades & the four of diamonds must be in the hand?



Example 4: There are 8 parents and 43 students going on a school trip. Two groups are made, a large one with 30 students and 5 parents, and a small group with 13 students and 3 parents.

1) How many ways can the students be chosen for the large group if Stefan and Dylan must be in the small group?

2) How many ways can students be chosen for the small group if Wade & both his parents must be in the small group?

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Combinations from multiple selection pools:

Example 5: A committee of 3 boys and 5 girls is to be formed from a group of 10 boys and 11 girls. How many committees are possible?

Example 6: From a deck of 52 cards, a 7 card hand is dealt. How many distinct hands are there if the hand must contain 2 spades and 3 diamonds?

Combinations with at least or at most:

Example 7: A committee of 5 people is to be formed from a group of 4 men & 7 women. How many possible committees can be formed if at least 3 women are on the committee?

Example 8: From a deck of 52 cards, a 5 card hand is dealt. How many distinct hands can be formed if there are at least 2 queens?

Practice 1: From a deck of 52 cards, a 7 card hand is dealt. How many distinct hands can be formed if there are at most 6 black cards? (use in-direct method only)

Practice 2: A research team of 6 people is to be formed from 10 chemists, 5 politicians, 8 economists, and 15 biologists. How many teams have:

1) At least 5 chemists?

2) Exactly three economists?

3) Four chemists, but no economists?

4) At least 2 biologists?

5) 4 economists and 2 biologists?

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Permutations & combinations together:

Example 9: How many arrangements of the word TRIGONAL can be made if only two vowels and three consonants are used?

Example 10: There are 7 men and 10 women on a committee selection pool. A committee consisting of President, Vice-President, and Treasurer is to be formed. How many ways can exactly two men be on the committee?

Practice 3: If a sports team with six unique positions is to be formed from 5 men and 7 women, in how many ways can two positions be filled by men and four positions by women?

ALL possible combination of distinct items:

The total number of combinations containing at least one item chosen from a group of n distinct items is $2^n - 1$.

Remember that combinations are subsets of the group of n objects.
A **null set** is a set that has no elements. Thus,

A set with n distinct elements has 2^n subsets including the null set.

$$2^n - 1 = {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n \text{ or } 2^n = {}_nC_0 + {}_nC_1 + {}_nC_2 + \cdots + {}_nC_n$$

Example 11: An Artist has an apple, an orange, and a pear in his refrigerator. In how many ways can the artist choose one or more pieces of fruit for a still-life painting?

Method 1 - …the SUM of the possibility of choosing 1 item OR 2 items OR all 3 items.

Method 2 - …the total number of possible painting minus the possibility of selecting NO fruits.

Practice 4: In how many was can a committee with at least one member be appointed from a board with six members?

ALL possible combinations in which some are alike:

Example 12: Kate is responsible for stocking the coffee room at her office. She can purchase **up to** three cases of cookies, four cases of juice, and two cases of coffee without having to send the order through the accounting department. How many different direct purchases can Kate make?

Can buy # of Cookies:

Juice:

Coffee:

But, you must choose at least ONE combination so, …

If you have to choose **all**, **some**, or **none** of **P available** items you have _____ choices. Using the fundamental counting principle, if you have successive choices of different kinds of items, then the total number of combinations that can be made from p items of one kind, q items of another kind, r items of a third kind, etc. is:

$$(p + 1)(q + 1)(r + 1)$$

BUT…don't forget to think about whether <u>NOT picking</u> any items makes sense or not. If at least one item has to be chosen, then total number of selection is:

$$(p + 1)(q + 1)(r + 1)\ldots - 1$$

Other types:

Handshakes: ${}_nC_2$, n = number of people

Diagonals: ${}_nC_2$ - n , n = number of sides

Multiple combinations: ${}_nC_r$ x k, k = number of times all the possible combinations must happen

Practice 5: 12 people at a party shake hands once with everyone else in the room. How many handshakes took place?

Practice 6: A polygon has 7 sides. How many diagonals can be formed?

Practice 7: If each of the 8 teams in a league must play each other three times, how many games will be played?

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