Unit 3: Probability

Lesson 3.1: Simple Probability

- **Probability**: is the value between 0 and 1 (100%) describes that likelihood of an occurrence of a certain event. A probability of 0 indicates that the event is impossible, and 1 signifies that the event is a certainty.

- The probability of event, A, P(A), is a measure of the likelihood that the certain event will occur.

  If all possible outcomes are equally likely, then **P(A) =** $\frac{n(A)}{n(S)}$, where n(A) is the number of outcomes in which event A can occur, and n(S) is the total number of possible outcomes.

Example 1: Many board games involve a rolling of two six-sided dice to see how far you may move your counters. What is the probability of rolling a total of 7?

	First Die						
		1	2	3	4	5	6
Second Die	1						
	2						
	3						
	4						
	5						
	6						

Let A be the event that the sum of two dice is 7, and let S be the all possible sum of rolling two dice.

Therefore, P(A) = $\frac{n(A)}{n(S)}$ = $\frac{\text{n (rolls totalling 7)}}{\text{n (all possible rolls)}}$ = _____

- **Complement event:** The complement of event A, A' (called A prime), is the event that "event A does not happen". Thus whichever outcomes make up A, all the other outcomes make up A'. Because A and A' together include all possible outcomes, the sum of their probabilities must be 1.

  Thus: **P(A) + P(A') = 1 or P(A') = 1 - P(A)**

Example 2: What is the probability that a randomly drawn integer between 1 and 40 is not a perfect square.

Let event A = {a perfect square} = {1, 4, 9, 16, 25, 36}

The complement of A is the event = {not a perfect square}

S will be all integers from 1 to 40. Therefore, we have 40 numbers in total.

Hence, P (A) = $\frac{n(A)}{n(S)}$ = _______ **and** P(A') = 1 - P(A) = ___________.

Practice:

1. A bag contains three red balls and four green balls. One ball is pulled out. What is the probability of:

a) Pulling out a red ball?

b) Pulling out a green ball?

2. A letter is selected from the word STATISTICS. What is the probability the letter is an S or a T?

3. A jar contains 22 marbles. 4 are red, 3 are green, 7 are blue, and 8 are yellow. If a single marble is pulled out of the jar, find the probability of pulling:

a) a red marble

b) a blue or yellow marble

c) not green

d) purple

4. A 5-digit PIN number can begin with any digit (except zero) and the remaining digits have no restriction. If repeated digits are allowed, find the probability of the PIN code beginning with a 7 and ending with an 8.

Try to determine the number of possible arrangements for the PIN number that satisfy the stated restrictions.

Then determine the total number of arrangements.

Therefore, the probability is:

5. A security code consists of 8 digits, which may be any number from 0 to 9. Repetitions are allowed. Determine the probability a particular code with exactly two 7's, to the nearest hundredth.

6. There are 12 male athletes and 14 female athletes competing in a marathon. Find the probability of three different prizes being awarded to all males or all females.

7. Kevin, Rachel, and 6 other students are standing in a line. Determine the probability Kevin and Rachel are not standing together.

8. A bookcase contains 6 different math books and 12 different physics books. If a student randomly selects two of these books. Determine the probability they are both math or both physics books.

9. A jar contains 5 orange, 3 purple, 7 blue, and 5 green candies if the total number of candies is 20, determine the probability that a handful of four candies contains one of each color.

10. A committee of 4 people is to be formed from a pool of 8 people. What is the probability that James is on the committee?

11. Seven males and nine females are in a selection pool for a committee of 4 people. What is the probability at most 3 males are on the committee?

12. In the card game Bridge, 4 players are each given a hand of 13 cards.

a) What is the probability a player receives all the kings?

b) What is the probability a player does not have any 2's?

c) What is the probability a player receives at least 3 queens?

d) What is the probability a player receives two 4's, four 6's, three 7's, three 10's, and an ace?

Key Concepts

- A probability experiment is a well-defined process in which clearly identifiable outcomes are measured for each trial.
- An event is a collection of outcomes satisfying a particular condition. The probability of an event can range between 0 (impossible) and 1 or 100% (certain).
- The empirical probability of an event is the number of times the event occurs divided by the total number of trials.
- The theoretical probability of an event A is given by $P(A) = \frac{n(A)}{n(S)}$, where $n(A)$ is the number of outcomes making up A, $n(S)$ is the total number of outcomes in the sample space S, and all outcomes are equally likely to occur.
- A subjective probability is based on intuition and previous experience.
- If the probability of event A is given by $P(A)$, then the probability of the complement of A is given by $P(A') = 1 - P(A)$.

Communicate Your Understanding

1. Give two synonyms for the word *probability*.

2. **a)** Explain why $P(A) + P(A') = 1$.

b) Explain why probabilities less than 0 or greater than 1 have no meaning.

3. Explain the difference between theoretical, empirical, and subjective probability. Give an example of how you would determine each type.

4. Describe three situations in which statistical fluctuations occur.

5. **a)** Describe a situation in which you might determine the probability of event A indirectly by calculating $P(A')$ first.

b) Will this method always yield the same result as calculating $P(A)$ directly?

c) Defend your answer to part b) using an explanation or proof, supported by an example.

Practise

A

1. Determine the probability of
- **a)** tossing heads with a single coin
- **b)** tossing two heads with two coins
- **c)** tossing at least one head with three coins
- **d)** rolling a composite number with one die
- **e)** not rolling a perfect square with two dice
- **f)** drawing a face card from a standard deck of cards

2. Estimate a subjective probability of each of the following events. Provide a rationale for each estimate.
- **a)** the sun rising tomorrow
- **b)** it never raining again
- **c)** your passing this course
- **d)** your getting the next job you apply for

3. Recall the sum/product game at the beginning of this section. Suppose that the game were altered so that the slips of paper showed the numbers 2, 3, and 4, instead of 1, 2, and 3.
- **a)** Identify all the outcomes that will produce each of the three possible events

  i) $p > s$ **ii)** $p < s$ **iii)** $p = s$
- **b)** Which player has the advantage in this situation?

Apply, Solve, Communicate

4. The town planning department surveyed residents of a town about home ownership. The table shows the results of the survey.

Residents	At Address Less Than 2 Years	At Address More Than 2 Years	Total for Category
Owners	2000	8000	10 000
Renters	4500	1500	6 000
Total	6500	9500	16 000

Determine the following probabilities.
- **a)** P(resident owns home)
- **b)** P(resident rents and has lived at present address less than two years)
- **c)** P(homeowner has lived at present address more than two years)

B

5. **Application** Suppose your school's basketball team is playing a four-game series against another school. So far this season, each team has won three of the six games in which they faced each other.
- **a)** Draw a tree diagram to illustrate all possible outcomes of the series.
- **b)** Use your tree diagram to determine the probability of your school winning exactly two games.
- **c)** What is the probability of your school sweeping the series (winning all four games)?
- **d)** Discuss any assumptions you made in the calculations in parts b) and c).

6. **Application** Suppose that a graphing calculator is programmed to generate a random natural number between 1 and 10 inclusive. What is the probability that the number will be prime?

7. **Communication**
- **a)** A game involves rolling two dice. Player A wins if the throw totals 5, 7, or 9. Player B wins if any other total is thrown. Which player has the advantage? Explain.
- **b)** Suppose the game is changed so that Player A wins if 5, 7, or doubles (both dice showing the same number) are thrown. Who has the advantage now? Explain.
- **c)** Design a similar game in which each player has an equal chance of winning.

8. a) Based on the randomly tagged sample, what is the empirical probability that a deer captured at random will be a doe?

b) If ten deer are captured at random, how many would you expect to be bucks?

C

9. Inquiry/Problem Solving Refer to the prime number experiment in question 6. What happens to the probability if you change the upper limit of the sample space? Use a graphing calculator or appropriate computer software to investigate this problem. Let A be the event that the random natural number will be a prime number. Let the random number be between 1 and n inclusive. Predict what you think will happen to $P(A)$ as n increases. Investigate $P(A)$ as a function of n, and reflect on your hypothesis. Did you observe what you expected? Why or why not?

10. Suppose that the Toronto Blue Jays face the New York Yankees in the division final. In this best-of-five series, the winner is the first team to win three games. The games are played in Toronto and in New York, with Toronto hosting the first, second, and if needed, fifth games. The consensus among experts is that Toronto has a 65% chance of winning at home and a 40% chance of winning in New York.

a) Construct a tree diagram to illustrate all the possible outcomes.

b) What is the chance of Toronto winning in three straight games?

c) For each outcome, add to your tree diagram the probability of that outcome.

d) Communication Explain how you found your answers to parts b) and c).

11. Communication Prior to a municipal election, a public-opinion poll determined that the probability of each of the four candidates winning was as follows:

Jonsson 10%

Trimble 32%

Yakamoto 21%

Audette 37%

a) How will these probabilities change if Jonsson withdraws from the race after ballots are cast?

b) How will these probabilities change if Jonsson withdraws from the race before ballots are cast?

c) Explain why your answers to a) and b) are different.

12. Inquiry/Problem Solving It is known from studying past tests that the correct answers to a certain university professor's multiple-choice tests exhibit the following pattern.

Correct Answer	Percent of Questions
A	15%
B	25%
C	30%
D	15%
E	15%

a) Devise a strategy for guessing that would maximize a student's chances for success, assuming that the student has no idea of the correct answers. Explain your method.

b) Suppose that the study of past tests revealed that the correct answer choice for any given question was the same as that of the immediately preceding question only 10% of the time. How would you use this information to adjust your strategy in part a)? Explain your reasoning.