Consolidate and Debrief

Key Concepts

- A probability distribution shows the probabilities of all possible outcomes in an experiment.
- The sum of all probabilities in any distribution is 1.
- A probability histogram graphs the relative frequency of the random variable. The area of each bar represents the probability of the variable.
- Expectation, or expected value, is the weighted average value of the random variable.

$$E(X) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n \cdot P(x_n)$$
$$= \sum_{i=1}^{n} x_i \cdot P(x_i)$$

The expectation can be a non-integer value.

Reflect

R1. The expected number of children in a Canadian family is 1.8. Should this be rounded to 2 or left as is? Explain.

R2. Give two examples of a discrete probability distribution. Explain what makes them discrete.

R3. Describe the steps in setting up a probability distribution for the sum of two 12-sided dice.

Practise

Choose the best answer for #2 and #3.

1. Classify each of the random variables as discrete or continuous:

a) the number of points scored in a basketball game

b) the length of time players played in a basketball game

c) the mass of the weights in a weight room

d) the number of windows in the classrooms in a school

e) the area of the windows in the classrooms in a school

2. Which of the following is a false statement about expectation?

A The sum in the expected value calculations is equal to 1.

B $E(X) = \sum_{i=1}^{n} x_i \cdot P(x_i)$

C It is the predicted average of all possible outcomes.

D It is equal to the mean of the outcomes weighted according to their respective frequencies.

3. In Example 2 on page 148, what is the discrete random variable?

A x

B $P(x)$

C the number of girls in a family of three children

D the expected number of girls in a family of three children

4. Draw a probability histogram for each of the distributions.

a)

x	P(x)
1	0.35
2	0.42
3	0.11
4	0.12

b)

x	P(x)
5	$\frac{1}{8}$
10	$\frac{1}{4}$
15	$\frac{5}{12}$
20	$\frac{1}{12}$
25	$\frac{1}{8}$

5. Calculate the expectation for each of the distributions.

a)

x	P(x)
1	0.3
2	0.2
3	0.1
4	0.4

b)

x	P(x)
0	$\frac{1}{5}$
2	$\frac{3}{10}$
4	$\frac{1}{5}$
6	$\frac{1}{10}$
8	$\frac{1}{10}$
10	$\frac{1}{10}$

Apply

6. **Communication** The distribution of marble sizes in a bag is shown in the table.

Diameter (mm)	Frequency
12.0	5
13.0	11
14.0	24
20.0	15
25.0	5

a) Identify the random variable.

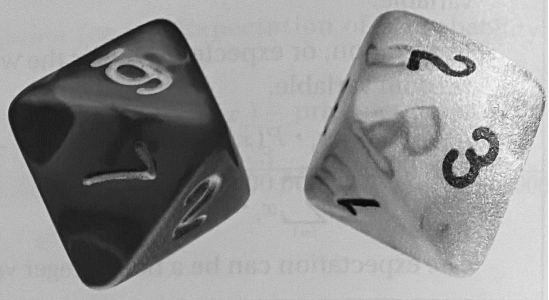
b) Is the random variable discrete? Explain.

c) Draw a probability histogram for this distribution.

d) Describe what each bar in the histogram represents.

e) Calculate the weighted mean of the diameters. How does this relate to the expectation?

7. **Application** Two 8-sided dice are rolled.



a) Show the probability distribution for the sums of the two dice.

b) Draw a probability histogram by hand or using technology.

c) Calculate the expectation. Explain its meaning in this context.

8. A rectangle is to be drawn on a grid with perimeter of 24 cm. The dimensions are integers, and are randomly selected. Show the probability distribution for either the dimensions or the area. Include a probability histogram.

9. **Thinking** A school is holding a fundraising raffle. The first prize is \$500, the three second prizes are \$100 each, and the five third prizes are \$50 each. A total of 2000 tickets were sold at \$5 each.

a) What is the probability of winning a prize?

b) What is the expected payout per ticket?

c) What is the expected profit per ticket?

d) What price should have been charged to have a 90% profit per ticket?

Achievement Check

10. A card is chosen from a standard deck, replaced, then another is chosen. This process is repeated three times.

- **a)** Show the probability distribution for the number of face cards in three trials.
- **b)** Sketch a graph of this distribution.
- **c)** Is the number of face cards a discrete random variable? Justify your response.
- **d)** Calculate the expected value. Explain its meaning.

11. In many games, rolling doubles has beneficial results. Three people are playing a board game in which two dice are rolled.

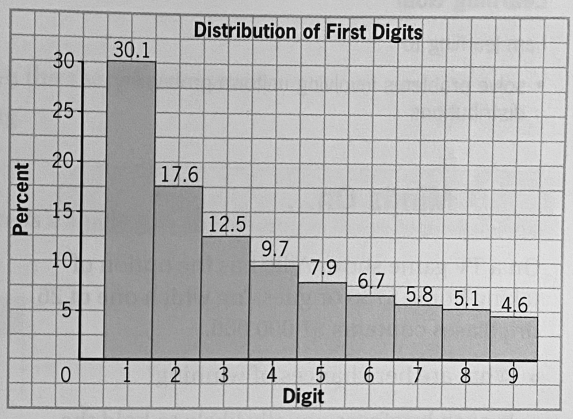
- **a)** Use a tree diagram to illustrate the probability distribution of the number of doubles in three rolls of two dice.
- **b)** Calculate the probability of each outcome in the sample space.
- **c)** What is the expected number of doubles in the three rolls?

12. Build a probability distribution for the sums of three dice. Include all pertinent components of a distribution, and appropriate explanations.

13. Open Question A random device is one that generates a random result. Spinners and dice are typical random devices. Design a random device that has at least four outcomes with non-equal probabilities. Develop the probability distribution for your device and illustrate it using a probability histogram.

> **Project Prep**
> You may need to make a random device in your probability project. Think of an appropriate device that you can use.

14. Communication The graph shows the percent of numbers that start with each digit when applied to many different data sets, such as hydro bills, addresses, stock prices, population sizes, death rates, and lengths of rivers.



- **a)** Why would the distribution look this way?
- **b)** Calculate the expectation. Explain what it means.

Extend

15. When continuously cutting a card from a deck with replacement, what is the probability that the first ace will be cut

- **a)** on the first try?
- **b)** on the second try?
- **c)** on the third try?
- **d)** on the nth try?

16. Use technology to show the probability histogram for a spinner with five unequal sectors, labelled 1 to 5, respectively. The sectors are proportionally equal in arc length to their labelled numbers.

17. What is the expected sum of two weighted dice on which the number 5 occurs twice as often as the other numbers?

4. A jar contains red and green balls. A person reaches in and randomly selects a ball to indicate the number of points earned or lost. There are four red balls, each labelled +3 points. How many green balls, each labelled –2 points, would be required for this to be a fair game?

5. Given the probability distributions, determine the expected values.

a)

x	P(x)
5	$\frac{1}{5}$
10	$\frac{1}{5}$
15	$\frac{1}{5}$
20	$\frac{1}{5}$
25	$\frac{1}{5}$

b)

x	P(x)
0	12.5%
1	12.5%
2	12.5%
3	12.5%
4	12.5%
5	12.5%
6	12.5%
7	12.5%

Apply

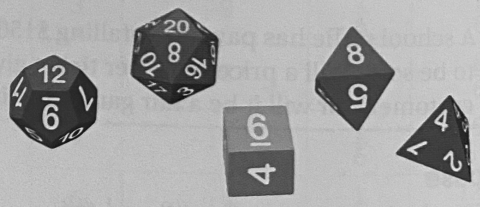
6. A random number between 1 and 12 is generated to decide on the hour during which a special contest will be played on a radio station.
   a) Develop the probability distribution for the contest hour, and calculate the expected outcome.
   b) Does this mean that the time represented by the expectation is the most likely to be selected? Explain.

7. **Communication** A card is randomly selected from a deck.
   a) What is the probability that it is any specific card?
   b) Is this an example of a uniform distribution? Explain.
   c) The card is not placed back into the deck and a second card is selected. What is the probability it is any specific card?
   d) Are the two card choices an example of a uniform distribution? Explain.

8. A multiple choice test has five possible answers, labelled A, B, C, D, E. If the position of the correct answer is to be chosen at random, draw a probability histogram for this distribution.

9. The Prisoner's Dilemma involves two prisoners, P and Q, who are being held for a crime. If both P and Q confess to the crime, each of them goes to prison for two years. If P confesses but Q denies the crime, P will be set free but Q will serve three years in prison (and vice versa). If P and Q both deny the crime, both will serve only one year in prison.
   a) If each prisoner's decision is randomly chosen, show the probability distribution for the number of years in prison for prisoner P.
   b) If you were prisoner P, what would your decision be? Base your decision on mathematical reasoning.

10. **Application** There are only five platonic solids: tetrahedron (4 faces), cube (6 faces), octahedron (8 faces), dodecahedron (12 faces), and icosahedron (20 faces).

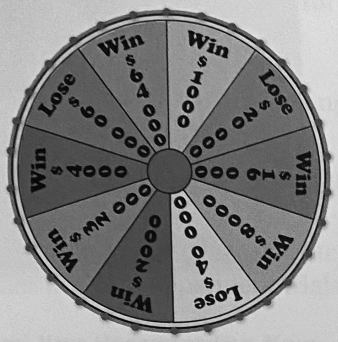


Literacy Link

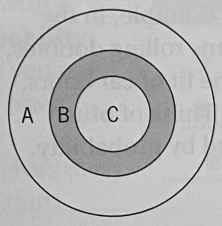
A *platonic solid* is a regular, convex polyhedron with congruent faces.

   a) Predict the expected outcome for each die.
   b) Check your prediction with appropriate calculations for the four smallest platonic solids.
   c) Use your findings to confirm or refine your prediction for the icosahedron die.

11. On a TV game show, a giant wheel has 10 equally spaced sectors as shown. To play the game, contestants must risk a certain amount of their previous winnings. What amount of risk money would make this a fair game?



12. Thinking A target contains circles with radii of 8 cm, 12 cm, and 20 cm.



a) If a dart randomly lands on one of the three regions, show mathematically that this is not a uniform distribution.

b) Assign points to each area to make this a fair game.

c) Create a similar target with a uniform distribution.

13. In its Flip Your Lid contest, a coffee chain offers prizes of 50 000 free coffees, each worth $1.50; two new TVs, each worth $1200; a snowmobile worth $15 000; and a sports car worth $35 000. A total of 1 000 000 promotional coffee cups have been printed for this contest. Coffee sells for $1.50 per cup. What is the expected value of a cup of coffee to the consumer?

14. A charity raffle offers a first prize of $1 000 000, a second prize of $100 000, and a third prize of $10 000. A total of 500 000 tickets will be sold. What price should be charged for a ticket in order for the charity to make a 60% profit on this raffle?

✓ Achievement Check

15. Describe or draw an example of a random number generator to be used in a uniform distribution that

a) has $P(x) = \frac{1}{9}$ for each value of x.

b) has an expected outcome of 8.

c) provides an outcome for an unfair game.

16. The game show Deal or No Deal involves trying to guess which of 26 briefcases contains $1 000 000. Each briefcase contains a different amount of money. Your teacher will direct you to a website where you can read the rules of Deal or No Deal and play the game yourself online.

a) What is the expectation of this game? How does it compare to the offer given to "quit now"?

b) Calculate the expectation after each of the next two rounds. How does it compare to the offers given to quit?

c) Is this a fair game? Explain.

Extend

17. A uniform distribution has possible outcomes from 1 to n. Develop a formula to calculate the expected outcome.

18. A contest involves a contestant choosing a number between 1 and 10. One of two cards, each containing a formula, is selected at random. The first card indicates that the contestant will win $40 plus double the contestant's chosen number. The second card indicates that the contestant will win $100 minus the square of the contestant's chosen number. Describe an appropriate strategy to win the most money.