



Unit 4: Probability Distribution

Lesson 4.2: Binomial Distributions and Geometric Distribution

Binomial Distribution

Minds On*: A report from the Secretary of Health and Human Services stated that 70% of single-vehicle traffic fatalities that occur at night on weekends involve an intoxicated driver. If a sample of 10 single-vehicle traffic fatalities that occur at night on a weekend is selected.

- Find the probability that exactly seven involve a driver that is intoxicated.
- What about the probability of at least 8 involve a driver that is intoxicated?

Definitions:

- Situations where there are exactly two possible Mutually Exclusive outcomes

For example:

- Yes/No to a survey question
 - Product is defective/not defective
 - correct/wrong response to a multiple choice question
- Conditions for binomial distributions:
 - Fixed number of trials, n
 - All trials are identical & independent
 - Each trial has exactly two mutually exclusive outcomes: success (S) or Failure (F)
 - The probability of success or failure is the same in every trial
 - The random variable is the number of successes in a given number of trials

Test your understanding: Which of the following results in a binomial probability distribution?

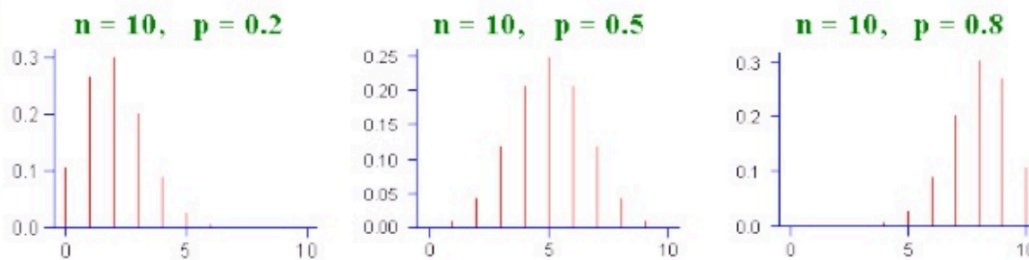
- Out of 300 heart pacemakers, determine the probability of certain number of heart pacemakers are defective.
- Surveying people on their opinions of current prime minister



- Formula for binomial distributions
 - $P(x) = {}_n C_x p^x q^{n-x}$
 - Where p is the probability of success on any independent trial
 - $q = 1 - p$ is the probability of failure
- The expectation or Expected Value in a binomial distribution is:
 - $E(X) = n \times p$

The shape of the binomial distribution depends on the values of n and p .

Fig.1. Binomial distributions for different values of p with $n=10$



Example 1: A manufacturer of electronics components produces precision resistors designed to have a tolerance of $\pm 1\%$. From quality-control testing, the manufacturer knows that about one resistor in six is actually within just 0.3% of its nominal value. A customer needs three of these more precise resistors. What is the probability of finding exactly three such resistors among the first five tested?



Example 2: Tan's family moves to an area with a different telephone exchange, so they have to get a new telephone number. Telephone numbers in the new exchange start with 466, and all combinations for the four remaining digits are equally likely. Tan's favorite numbers are the prime numbers 2, 3, 5, and 7.

- Find the probability distribution for the number of these prime digits in Tan's new phone number. And construct probability histogram.
- What is the expected number of these prime digits in the new telephone number?

Practice 1: A box of Smarties has different colors in it. There is a 15% chance of getting a blue Smarties.

- What is the probability that at least 4 Smarties in a given box are blue out of 10?
- What is the expected number of blue Smarties?

Practice 2: The faces of a 12-sided die are numbered from 1 to 12. What is the probability of rolling 9 at least twice in ten tries?

Practice 3: A certain type of rocket has a failure rate of 1.5%.

- Determine the probability of fewer than 4 failures in 100 launches.
- What is the expected number of failures in 100 launches of the rocket?



7. Suppose that 65% of the families in a town own computers. If eight families are surveyed at random,
- what is the probability that at least four own computers?
 - what is the expected number of families with computers?
8. **Inquiry/Problem Solving** Ten percent of a country's population are left-handed.
- What is the probability that 5 people in a group of 20 are left handed?
 - What is the expected number of left-handed people in a group of 20?
 - Design a simulation to show that the expectation calculated in part b) is accurate.
9. **Inquiry/Problem Solving** Suppose that Bayanisthol, a new drug, is effective in 65% of clinical trials. Design a problem involving this drug that would fit a binomial distribution. Then, provide a solution to your problem.
10. Pythag-Air-US Airlines has determined that 5% of its customers do not show up for their flights. If a passenger is bumped off a flight because of overbooking, the airline pays the customer \$200. What is the expected payout by the airline, if it overbooks a 240-seat airplane by 5%?
11. A department-store promotion involves scratching four boxes on a card to reveal randomly printed letters from A to F. The discount is 10% for each A revealed, 5% for each B revealed, and 1% for the other four letters. What is the expected discount for this promotion?

12. a) Expand the following binomials.
- $(p + q)^6$
 - $(0.2 + 0.8)^5$
- b) Use the expansions to show how the binomial theorem is related to the binomial probability distribution.



ACHIEVEMENT CHECK

Knowledge/ Understanding	Thinking/ Inquiry/ Problem Solving	Communication	Application
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13. Your local newspaper publishes an Ultimate Trivia Contest with 12 extremely difficult questions, each having 4 possible answers. You have no idea what the correct answers are, so you make a guess for each question.
- Explain why this situation can be modelled by a binomial distribution.
 - Use a simulation to predict the expected number of correct answers.
 - Verify your prediction mathematically.
 - What is the probability that you will get at least 6 answers correct?
 - What is the probability that you will get fewer than 2 answers correct?
 - Describe how the graph of this distribution would change if the number of possible answers for each question increases or decreases.



14. The French mathematician Simeon-Denis Poisson (1781–1840) developed what is now known as the *Poisson distribution*. This distribution can be used to approximate the binomial distribution if p is very small and n is very large. It uses the formula
- $$P(x) = \frac{e^{-np}(np)^x}{x!},$$
- where e is the irrational number 2.718 28 ... (the base for the natural logarithm).



Geometric distribution

Minds On: In a certain town, 30% of adults are very familiar with modern CPR techniques. If adults from this town are randomly selected, what is the probability that the 6th person sampled is the first person who is very familiar with CPR techniques?

Definitions:

- How many outcomes must occur to get the “first” success?
- Geometric distribution is the distribution of the first success in repeated trials of two mutually exclusive outcomes, success or failure.
- For the 1st success to occur on the $x + 1^{\text{th}}$ trial, the first x trials must be failures, the $x+1^{\text{th}}$ trial must be a success.
- The comparison between binomial and geometric

The binomial setting	The geometric setting
1. Two discrete outcomes: Success/Failure 2. Probability of success, p , is the same for each observation. 3. Observations are all independent.	
Fixed number of observations, n , is the variable of interest.	Number of trials/failures, x , required for 1 st success is the variable of interest.

- Probability in a geometric distribution:
 - $P(x) = q^{x-1}p$
 - Where p is the probability of success in each single trial, q is the probability of failure and x is the trial in total until get the first number of failures (or waiting time + 1 for the first success/failure to occur)
- Expected value $\sum_{n=0}^{\infty} xP(x) = \frac{q}{p}$
 - For example, $q = P(\text{flight scheduled on time}) = 0.75$ and $p = P(\text{delay}) = 0.25$, then the expected waiting time before a flight delay is $0.75/0.25 = 3$ times.



Example 1: Identify which is binomial, which is geometric:

- a) 25% of the customers entering a grocery store between 5pm and 7pm use an express checkout. Consider five randomly selected customers, and let X denote the number among the five who use the express checkout. What is the probability that two customers used the express checkout?

- b) Suppose that each of three randomly selected customers purchasing a hot tub at a certain store chooses either an electric (E) or a gas (G) model. Assume that these customers make their choices independently of one another and have 40% of all customers select an electric model. What is the probability that two customers choose an electric model?

- c) Suppose we have data that suggest that 3% of a company's hard disc drives are defective. You have been asked to determine the probability that the first defective hard drive is the fifth unit tested.

- d) A basketball player makes 80% of her free throw. We put her on the free throw line and ask her to shoot free throws until she misses one. Let X = the number of free throws the player takes until she misses. What is the probability that she will make 5 shots before she misses?



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Example 2: suppose that an intersection you pass on the way to school has a traffic light that is green for 40s and then amber or red for a total of 60s.

- a) What is the probability that the light will be green when you reach the intersection at least once a week (5 school days)?

Out of 100s, 40s green & 60s not green, so $p = 0.4$ and $q = 0.6$.

- b) What is the expected number of days before the light is green when you reach the intersection?

- c) **Your Turn:** Find the probability distribution, and construct probability histogram. *



Questions #3, 4, 7 – 12

2. Prepare a table and a graph for six trials of a geometric distribution with
- a) $p = 0.2$ b) $p = 0.5$

Apply, Solve, Communicate



3. For a 12-sided die,
- a) what is the probability that the first 10 will be on the third roll?
- b) what is the expected waiting time until a 1 is rolled?
4. The odds in favour of a Pythag-Air-US Airlines flight being on time are 3:1.
- a) What is the probability that this airline's next eight flights will be on time?
- b) What is the expected waiting time before a flight delay?
5. **Communication** To finish a board game, Sarah needed to land on the last square by rolling a sum of 2 with two dice. She was dismayed that it took her eight tries. Should she have been surprised? Explain.
6. In a TV game show, the grand prize is randomly hidden behind one of three doors. On each show, the finalist gets to choose one of the doors. What is the probability that no finalists will win a grand prize on four consecutive shows?
7. **Application** A teacher provides pizza for his class if they earn an A-average on any test. The probability of the class getting an A-average on one of his tests is 8%.
- a) What is the probability that the class will earn a pizza on the fifth test?
- b) What is the probability that the class will not earn a pizza for the first seven tests?
- c) What is the expected waiting time before the class gets a pizza?
8. Minh has a summer job selling replacement windows by telephone. Of the people he calls, nine out of ten hang up before he can give a sales pitch.
- a) What is the probability that, on a given day, Minh's first sales pitch is on his 12th call?
- b) What is the expected number of hang-ups before Minh can do a sales pitch?
9. Despite its name, Zippy Pizza delivers only 40% of its pizzas on time.
- a) What is the probability that its first four deliveries will be late on any given day?
- b) What is the expected number of pizza deliveries before one is on time?
10. A poll indicated that 34% of the population agreed with a recent policy paper issued by the government.
- a) What is the probability that the pollster would have to interview five people before finding a supporter of the policy?
- b) What is the expected waiting time before the pollster interviews someone who agrees with the policy?
11. Suppose that 1 out of 50 cards in a scratch-and-win promotion gives a prize.
- a) What is the probability of winning on your fourth try?
- b) What is the probability of winning within your first four tries?
- c) What is the expected number of cards you would have to try before winning?
12. A top NHL hockey player scores on 93% of his shots in a shooting competition.
- a) What is the probability that the player will not miss the goal until his 20th try?
- b) What is the expected number of shots before he misses?