Chapter

1

Rational Numbers

GOALS

You will be able to

- Relate rational numbers to decimals, fractions, and integers
- Evaluate expressions involving rational numbers

What might the negative values in the photo represent?

1 Getting Started

WORDS YOU NEED to Know

1. Match each term with the example that most closely represents it.

a) opposite integers	**d)** lowest common denominator	**g)** power
b) numerator	**e)** mixed number	**h)** base of a power
c) denominator	**f)** improper fraction	**i)** exponent

i) $\frac{34}{9}$	**iv)** $+5$ and -5	**vii)** $\frac{2}{3} = \frac{10}{15}$ $\frac{1}{5} = \frac{3}{15}$
ii) $1\frac{2}{3}$	**v)** 7^3	**viii)** 7^3
iii) 7^3	**vi)** $\frac{5}{7}$	**ix)** $\frac{5}{7}$

Study Aid

- For more help and practice, see Appendix A-6 and A-7.

SKILLS AND CONCEPTS You Need

Addition of Fractions

You can add two fractions using fraction strips, number lines, or by using a common denominator.

EXAMPLE

$$\frac{1}{3} + \frac{2}{5}$$

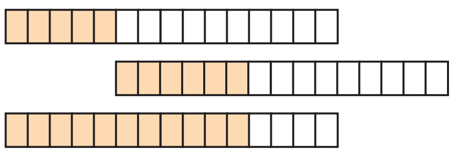
Solution: common denominator

$$= \frac{1 \times 5}{3 \times 5} + \frac{2 \times 3}{5 \times 3}$$

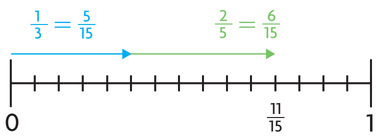
$$= \frac{5}{15} + \frac{6}{15}$$

$$= \frac{11}{15}$$

Solution: fraction strips



Solution: number line



2. Determine each sum.

a) $\frac{1}{2} + \frac{1}{3}$

b) $\frac{3}{4} + \frac{1}{8}$

c) $\frac{3}{10} + \frac{3}{5}$

d) $\frac{2}{5} + \frac{2}{3}$

Subtraction of Fractions

You can subtract two fractions using fraction strips, number lines, or by using a common denominator.

EXAMPLE

$\frac{2}{3} - \frac{1}{5}$

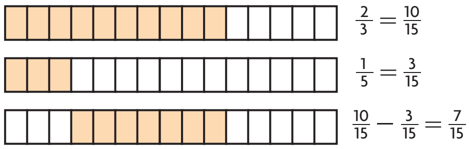
Solution: common denominator

$$= \frac{2 \times 5}{3 \times 5} - \frac{1 \times 3}{5 \times 3}$$

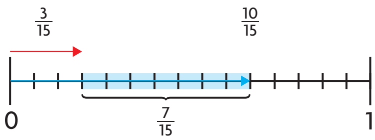
$$= \frac{10}{15} - \frac{3}{15}$$

$$= \frac{7}{15}$$

Solution: fraction strips



Solution: number line



3. Determine each difference.

a) $\frac{1}{2} - \frac{1}{3}$ **b)** $\frac{3}{4} - \frac{1}{8}$ **c)** $\frac{3}{5} - \frac{3}{10}$ **d)** $\frac{6}{7} - \frac{1}{2}$

Multiplication of Fractions

You can use an area model to help you visualize the product of two fractions.

EXAMPLE

$\frac{5}{8} \times \frac{2}{3}$

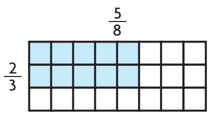
Solution: multiplying

$$= \frac{5 \times 2}{8 \times 3}$$

$$= \frac{10}{24}$$

$$= \frac{5}{12}$$

Solution: area model



4. Determine each product.

a) $\frac{1}{2} \times \frac{1}{4}$ **c)** $\frac{2}{5} \times \frac{3}{10}$

b) $\frac{2}{3} \times \frac{3}{4}$ **d)** $\frac{1}{6} \times \frac{4}{5}$

PRACTICE

Study Aid

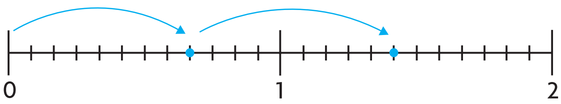
- For help, see the Review of Essential Skills and Knowledge Appendix.

Question	Appendix
6 and 7	A-6
8	A-7
9 and 10	A-3
11	A-4
12	A-9
13	A-1
14	A-2

5. Complete the equation. Use a diagram to show how you got the answer.

$$5\frac{7}{4} = 6\frac{\blacksquare}{4}$$

6. The number line represents the sum of two numbers. Write an equation to show the numbers being added and their sum.



7. a) Use a number line to determine how much greater $4\frac{1}{3}$ is than $1\frac{2}{3}$.

b) Write the subtraction equation that gives the same result as in part a).

8. Evaluate.

a) $\frac{2}{5} \times \frac{9}{8}$ **b)** $\frac{3}{4} \div \frac{9}{10}$

9. Evaluate.

a) $(-3) + 7$ **c)** $11 - (-4)$

b) $(-5) + (-2)$ **d)** $-3 - (-8)$

10. Tiger Woods finished a golf tournament with a score of five under par. Another player finished with a score of two over par. How many shots behind Tiger Woods was the other player?

11. Evaluate.

a) -4×6 **c)** $20 \div (-1)$

b) $(-5) \times (-3)$ **d)** $(-12) \div (-6)$

12. Evaluate.

a) $0.32 + 3.9$ **c)** 0.6×1.1

b) $15.4 - 3.91$ **d)** $24 \div 1.2$

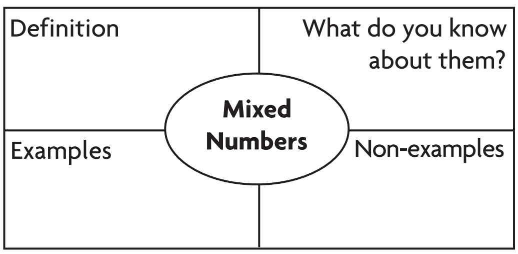
13. Express each power as repeated multiplication, and then, evaluate.

a) 8^2 **b)** 5.2^3

14. Follow the order of operations to determine the value of each expression.

a) $\frac{-12 + 3}{4 + 5}$ **b)** $7^2 - (-6 + 2) \times 4$

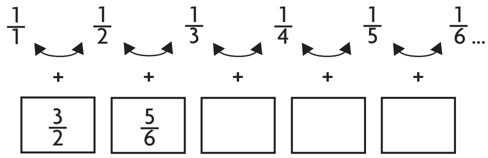
15. Copy and complete the chart to show what you know about mixed numbers.



APPLYING What You Know

Fraction Patterns

Andrew and Kim are exploring a sequence of fractions.



? How can you use patterns to predict the 20th sum, difference, product, and quotient using these fractions?

A. Describe the pattern in Andrew and Kim's original sequence of fractions.

B. Add consecutive terms (pairs of neighbouring fractions) until five sums have been calculated. For example: $\frac{1}{1} + \frac{1}{2} = \frac{3}{2}$, $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$, etc.

C. Describe the pattern and use it to predict the 20th sum.

D. Copy Andrew and Kim's original fraction sequence. Subtract consecutive terms instead of adding them.

E. Describe the pattern and use it to predict the 20th difference.

F. Repeat parts D and E using multiplication.

G. Repeat parts D and E using division.

H. Explain why the values in the pattern decrease for each operation.

I. Which patterns do you think were the easiest to predict? Why?

1.1

Addition and Subtraction of Mixed Numbers

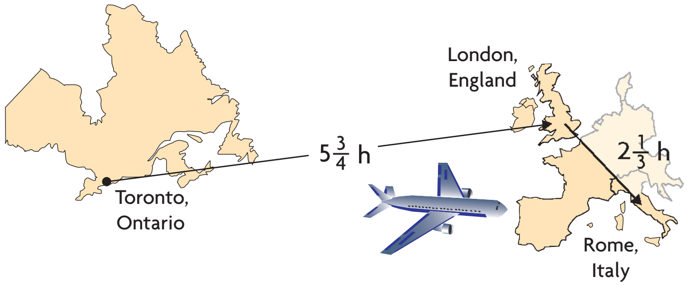
YOU WILL NEED

- fraction strips

GOAL

Add and subtract mixed numbers using a variety of methods.

LEARN ABOUT the Math



Alisa and Greg plan to travel from Canada to Italy for March break. Dawn is driving to her aunt's in Ottawa for March break. According to a travel website, Dawn's trip should take 4 h 36 min.

? How many hours longer is Alisa and Greg's trip than Dawn's trip?

EXAMPLE 1 Selecting a strategy to add mixed numbers

Calculate the time to travel from Toronto to Rome by plane.

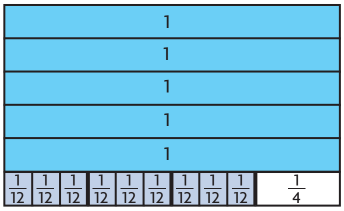
Alisa's Solution: Using a strategy involving equivalent fractions

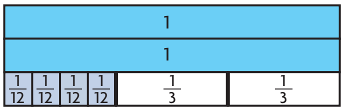
$5\frac{3}{4} + 2\frac{1}{3}$

I estimated the answer to be between 8 h and 9 h because $5 + 2 = 7$ and $\frac{3}{4} + \frac{1}{3}$ is greater than 1 but less than 2.

$= 5\frac{9}{12} + 2\frac{4}{12}$

I knew that I needed a common denominator when adding fractions. I chose 12 since it is the **least common multiple (LCM)** of 4 and 3.

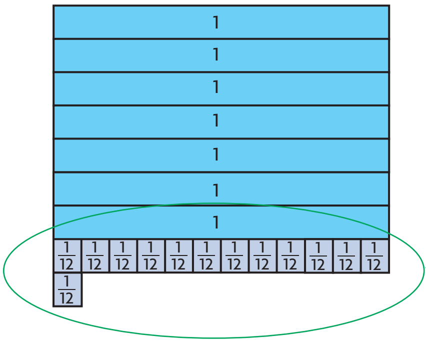




I used fraction strips to determine that $\frac{3}{4} = \frac{9}{12}$ and $\frac{1}{3} = \frac{4}{12}$.

$= 7\frac{13}{12}$

I added the whole parts, and then, I added the twelfths.



13 twelfths was the same as 1 whole and 1 twelfth, so I had 8 wholes and 1 twelfth.

$= 7 + 1\frac{1}{12}$

$= 8\frac{1}{12}$

So, our trip will take $8\frac{1}{12}$ h.

$8\frac{1}{12}$ was between my estimate of 8 h and 9 h, so I thought that my answer was reasonable.

least common multiple (LCM)
the least **whole number** that has all given numbers as factors (e.g., 12 is the least common multiple of 4 and 6)

Greg used a different strategy from Alisa. He changed the mixed numbers into improper fractions in order to add them.

Greg's Solution: Using a strategy involving improper fractions

$5\frac{3}{4} + 2\frac{1}{3}$

$\frac{1}{3}$ is a bit more than $\frac{1}{4}$.

That meant that $5\frac{3}{4} + 2\frac{1}{3}$ had to be a bit more than $5 + 2 + \frac{3}{4} + \frac{1}{4} = 8$.

I estimated our trip would take a bit longer than 8 h.

$= \frac{23}{4} + \frac{7}{3}$

I renamed the mixed numbers as improper fractions.

$= \frac{23 \times 3}{4 \times 3} + \frac{7 \times 4}{3 \times 4}$

$= \frac{69}{12} + \frac{28}{12}$

$= \frac{69 + 28}{12}$

I used a common denominator of 12 to add the fractions. I changed the numerators so that the fractions would be equivalent, and then, I added them.

$= \frac{97}{12}$

$= 8\frac{1}{12}$

I renamed the improper fraction as a mixed number so that I could see how many hours the trip would take.

So, our trip will take $8\frac{1}{12}$ h.

This was close to my estimate, so I thought that it was a reasonable answer.

Both Greg and Alisa have determined the time it will take to fly from Toronto to Rome. Now, they must calculate how much longer this is than Dawn's trip to Ottawa.

EXAMPLE 2 Selecting a strategy to subtract mixed numbers

Calculate the difference in time between the trip to Rome and the trip to Ottawa.

Alisa's Solution: Using a strategy involving equivalent fractions

Dawn's travel time:

4 h 36 min

$= 4\frac{36}{60}$ h

$= 4\frac{3}{5}$ h

Since there are 60 min in an hour, 36 min $= \frac{36}{60}$ h.

$8\frac{1}{12} - 4\frac{3}{5}$

I subtracted to determine the difference in travel times. $8 - 4 = 4$, and $\frac{3}{5}$ is more than $\frac{1}{2}$, so I estimated the difference to be between 3 h and 4 h.

$= 8\frac{1 \times 5}{12 \times 5} - 4\frac{3 \times 12}{5 \times 12}$

I chose 60 as the common denominator since it is the LCM of 12 and 5.

$= 8\frac{5}{60} - 4\frac{36}{60}$

$= 7\frac{60 + 5}{60} - 4\frac{36}{60}$

$= 7\frac{65}{60} - 4\frac{36}{60}$

$= (7 - 4) + \left(\frac{65}{60} - \frac{36}{60}\right)$

I noticed that if I subtracted $\frac{5}{60} - \frac{36}{60}$, the numerator would be negative. I renamed $8\frac{5}{60}$ to make the first numerator greater than the second.

$= 3\frac{(65 - 36)}{60}$

I subtracted the whole numbers, and then, the fractions.

$= 3\frac{29}{60}$

So, we will take $3\frac{29}{60}$ h longer than Dawn will to arrive at our destination.

$3\frac{29}{60}$ h was within my estimate of between 3 h and 4 h, so I thought that my answer was reasonable.

Communication *Tip*

LCM is an abbreviation for Least Common Multiple.

Greg used a different strategy from Alisa. He changed the travel times for each trip into improper fractions in order to subtract them.

Greg's Solution: Using a strategy involving improper fractions

Dawn's travel time:

4 h 36 min

$= 4\frac{36}{60}$ h

$= 4\frac{3}{5}$ h

I decided to express Dawn's travel time as a mixed number measured in hours.

$8\frac{1}{12} - 4\frac{3}{5}$

$8 - 4 = 4$

I knew that $\frac{1}{12}$ was close to zero and $\frac{3}{5}$ was close to $\frac{1}{2}$, so $4 - \frac{1}{2} = 3\frac{1}{2}$. I estimated the answer to be around $3\frac{1}{2}$ h.

$= \frac{97}{12} - \frac{23}{5}$

$= \frac{97 \times 5}{12 \times 5} - \frac{23 \times 12}{5 \times 12}$

$= \frac{485}{60} - \frac{276}{60}$

$= \frac{485 - 276}{60}$

$= \frac{209}{60}$

I renamed the mixed numbers as improper fractions, and created equivalent fractions using a common denominator of 60.

Then, I subtracted the numerators.

$= 3\frac{29}{60}$

I renamed the improper fraction as a mixed number so that I would know the number of hours.

So, we will take $3\frac{29}{60}$ h longer than Dawn will to arrive at our destination.

$3\frac{29}{60}$ h was really close to my estimate of $3\frac{1}{2}$ h, since $3\frac{1}{2} = 3\frac{30}{60}$, so my answer seemed reasonable.

Reflecting

A. Which of Alisa's or Greg's addition strategies would you choose? Why?

B. Which of Alisa's or Greg's subtraction strategies would you choose? Why?

C. How else might you calculate the difference in trip times?

APPLY the Math

EXAMPLE 3 Using a number line to represent addition and subtraction

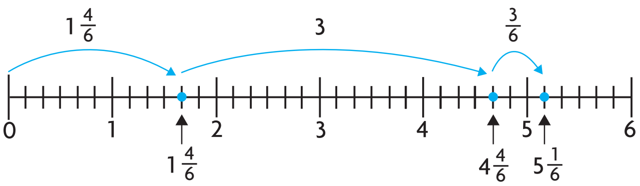
Evaluate.

a) $1\frac{2}{3} + 3\frac{1}{2}$ **b)** $4\frac{1}{8} - 1\frac{3}{4}$

Olecia's Solution

a) $1\frac{2}{3} + 3\frac{1}{2} = 1\frac{4}{6} + 3\frac{3}{6}$

I created equivalent fractions for the mixed numbers because it used smaller numbers than renaming the mixed numbers as improper fractions.



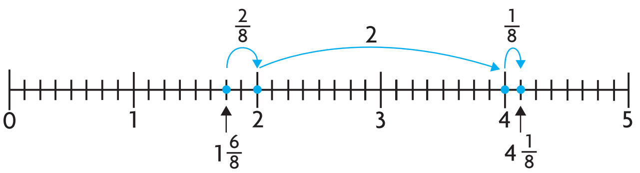
I used a number line to help me visualize the sum. I used intervals of $\frac{1}{6}$ on the number line to match the **lowest common denominator (LCD)** of the fractions.

$1\frac{4}{6} + 3\frac{3}{6} = 5\frac{1}{6}$

I determined that the answer was $5\frac{1}{6}$.

b) $4\frac{1}{8} - 1\frac{3}{4} = 4\frac{1}{8} - 1\frac{6}{8}$

I created equivalent fraction parts for the mixed numbers, using the LCD of 8.



I used a number line with intervals of $\frac{1}{8}$ to help me visualize the difference.

$4\frac{1}{8} - 1\frac{6}{8} = 2\frac{3}{8}$

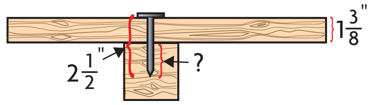
The sum of the "jumps" between $1\frac{6}{8}$ and $4\frac{1}{8}$ on my number line was $2\frac{3}{8}$.

Measurements of length made by carpenters, plumbers, and electricians still use the imperial system: yards, feet, and inches. Construction materials are also sold in imperial units. This leads to calculations involving mixed numbers.

Communication | ***Tip***

In the imperial number system, the symbol " represents inches while the symbol ' represents feet.

EXAMPLE 4 Solving a problem involving mixed numbers



If a $2\frac{1}{2}$ in. nail is hammered through a board $1\frac{3}{8}$ in. thick and into a support beam, how far into the support beam does the nail extend?

Stefan's Solution: Using equivalent fractions

$2\frac{1}{2} - 1\frac{3}{8}$

$= 2\frac{4}{8} - 1\frac{3}{8}$

$= 1\frac{1}{8}$

The nail will extend $1\frac{1}{8}$ in. into the support beam.

To determine the length of the nail in the support beam, I subtracted the thickness of the board from the nail's length.

I created equivalent fraction parts. $2 - 1 = 1$ and $\frac{4}{8} - \frac{3}{8} = \frac{1}{8}$

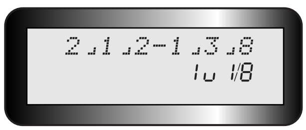
Doug used a calculator that can perform fractional operations.

Doug's Solution: Using technology

$2\frac{1}{2} - 1\frac{3}{8}$

I estimate the answer to be between 1 and 2.

$2 - 1 = 1$ and $\frac{1}{2}$ is a little bit greater than $\frac{3}{8}$.



I used a calculator with a fraction key: [A b/c]

I used this sequence of keystrokes:

[2] [A b/c] [1] [A b/c] [2] [−] [1] [A b/c] [3] [A b/c] [8] [ENTER]

Therefore, the nail will extend $1\frac{1}{8}$ in. into the support beam.

$1\frac{1}{8}$ was within my estimate of between 1 and 2, so I thought that my answer was reasonable.

Tech | ***Support***

Not all calculators with fraction operations use this exact set of keystrokes. Check your user manual to see how to enter fractions in your calculator.

In Summary

Key Idea

- You can add or subtract mixed numbers by dealing with the whole number and fraction parts separately, or by renaming them first as improper fractions.

Need to Know

- Sometimes, the sum of the fraction parts of two mixed numbers is an improper fraction. You can rename the fraction part as a mixed number and add once more.

  For example: $1\frac{2}{3} + 5\frac{2}{3} = 6\frac{4}{3}$
  $= 6 + 1\frac{1}{3}$
  $= 7\frac{1}{3}$

- Sometimes, a mixed number has a lesser fraction part than the number being subtracted. You can rename the mixed number with its whole number part reduced by one and its fraction part increased accordingly.

  For example: $3\frac{1}{3} - 1\frac{2}{3} = 2\frac{4}{3} - 1\frac{2}{3}$
  $= 1\frac{2}{3}$

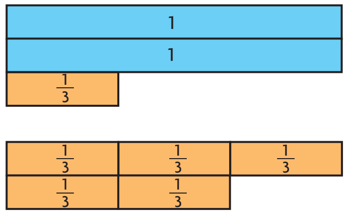
- You may use a number line to visualize the sum or difference of two mixed numbers.
  - Rename each fraction using the lowest common denominator (LCD). Draw a number line with intervals that correspond to the LCD of the fractions.
  - For addition, begin at the location on the number line of one of the fractions. Use the number line to "add on" an amount equal to the second fraction.
  - For subtraction, mark the location of each fraction on the number line and count the intervals between the numbers.
- Most strategies used to add or subtract two mixed numbers with different denominators require you to use equivalent fractions with a common denominator.

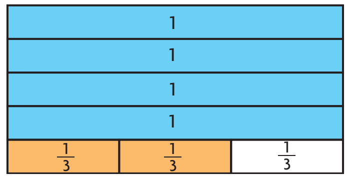
CHECK Your Understanding

1. Use the diagrams to help you evaluate each expression.

a) $2\frac{1}{3} + \frac{5}{3}$

b) $5 - \frac{2}{3}$





2. Use fraction strips to explain how to evaluate each expression. What is the value of each expression?

a) $3\frac{5}{6} + 7\frac{1}{2}$

b) $9\frac{1}{8} - 6\frac{3}{4}$

PRACTISING

3. Use number lines to evaluate the following expressions.

K

a) $7\frac{3}{8} + 4\frac{1}{8}$

b) $7\frac{3}{8} - 4\frac{1}{8}$

c) $6\frac{2}{3} + 5\frac{2}{3}$

d) $3\frac{2}{5} - 1\frac{4}{5}$

4. Between which two whole numbers will each sum lie?

a) $4\frac{1}{2} + 8\frac{1}{6}$

b) $3\frac{3}{4} + 6\frac{1}{5}$

c) $4\frac{1}{3} + 12\frac{5}{8}$

d) $1\frac{4}{5} + 6\frac{2}{3}$

e) $34\frac{7}{10} + 16\frac{3}{4}$

f) $11\frac{1}{2} + 41\frac{3}{5}$

5. Evaluate the expressions in question 4.

6. Estimate.

a) $3\frac{1}{2} - 1\frac{1}{5}$

b) $7\frac{3}{4} - 6\frac{1}{3}$

c) $8\frac{1}{4} - 2\frac{1}{2}$

d) $4\frac{7}{8} - 3\frac{8}{9}$

e) $29\frac{5}{8} - 23\frac{7}{16}$

f) $42\frac{1}{2} - 16\frac{2}{3}$

7. Evaluate the expressions in question 6.

8. Zofia spent $4\frac{1}{3}$ h weeding her garden on Monday and $1\frac{4}{5}$ h on Tuesday. How many hours did she spend weeding her garden altogether?

9. Alexis left her house at 7:45 p.m. to go shopping for clothes. She returned at 10:30 p.m.

a) Express the time in hours that Alexis spent away from home.

b) If Alexis spent $1\frac{1}{2}$ h shopping for clothes, then how much time did she spend doing other things?

10. A recipe for cookies calls for $1\frac{1}{2}$ c chopped dates, $\frac{3}{4}$ c water, $1\frac{1}{2}$ c sugar, $\frac{1}{2}$ c chopped nuts, $\frac{2}{3}$ c butter, and 3 c flour to be mixed together in a bowl. When these ingredients are combined, how many cups will there be altogether?

11. **T** Determine two mixed numbers, with different denominators, that have the following properties.

a) a sum of $3\frac{4}{5}$

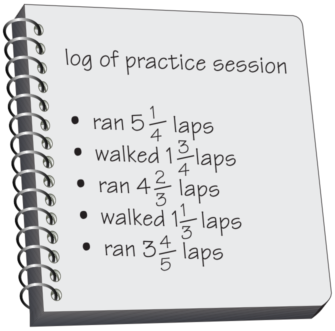
b) a difference of $3\frac{4}{5}$

Explain how you chose your numbers.

12. **C** Explain each of the following. You may use diagrams to show your explanations.

a) Why is $3\frac{2}{5} - 1\frac{4}{7}$ the same as $\frac{3}{7} + 1\frac{2}{5}$?

b) Why is $3\frac{2}{3} - 1\frac{5}{6}$ the same as $3\frac{5}{6} - 2$?



13. John trains by running on the school track. When he gets tired, he walks until he is able to run again. His log for one training day is shown to the left.

A

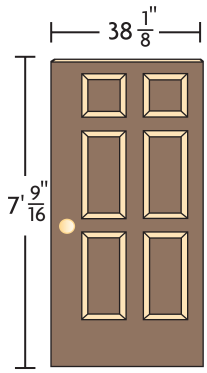
a) Determine how many laps around the track John ran.

b) Determine how many laps he walked around the track.

c) Determine how many more laps John ran than he walked.

d) If one lap around the school track is 400 m, determine the total distance John travelled.

14. Jane is putting wood trim around a doorway like the one shown to the right. How many linear feet of wood will Jane need altogether? (Hint: 1 ft = 12 in.)



15. a) Create an addition or subtraction question involving mixed numbers.

b) How do you think most people would choose to solve your question: by creating equivalent fractions for the fraction parts, or by renaming the mixed numbers as equivalent improper fractions? Explain.

Extending

16. a) Determine the value of $3\frac{1}{4} - 1\frac{1}{2}$.

b) Recalculate $3\frac{1}{4} - 1\frac{1}{2}$ by following the steps below.

i) Find the difference between the whole parts.

ii) Subtract the fraction in the first mixed number from the fraction in the second mixed number.

iii) Subtract the answer in part ii) from the answer in part i).

c) Repeat parts a) and b) for $5\frac{1}{3} - 1\frac{3}{4}$.

d) Repeat parts a) and b) for $4\frac{2}{5} - 2\frac{2}{3}$.

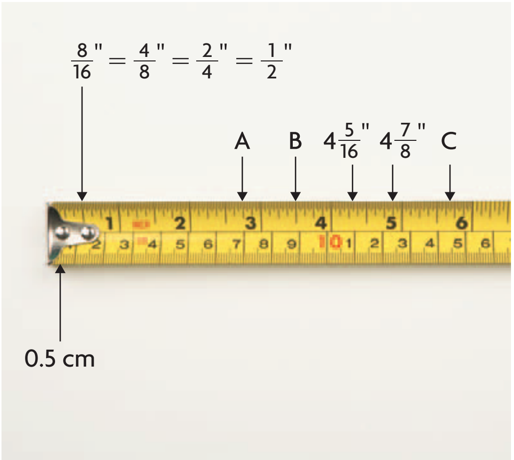
e) Explain why the process in part b) gives the same answer as in part a).

17. The sum of two mixed numbers is $2\frac{1}{2}$ more than the difference. What are the two numbers?

Curious Math

Inches

In Canada, the metric system is the official system of measurement. Before 1970, the imperial system was used, and it is still widely used in construction. Because of this, most measuring tapes found in hardware stores are marked in both inches and centimetres.



Recall that between 0 and 1 cm there are nine markings to create ten equal parts. The distance between adjacent markings represents 1 mm or 0.1 cm. The marking halfway between 0 and 1 cm is a little longer than the other markings. It represents 5 mm or 0.5 cm.

Similarly, between 0 and 1 inch there are 15 markings to create 16 equal parts. The distance between adjacent markings represents $\frac{1}{16}$ of an inch. There are different lengths for the markings to represent measurements of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{16}$ of an inch.

1. What measurement, in inches, is indicated by the arrows A, B, and C?

2. Calculate the total length in inches of A, B, and C.

3. Use the photo of the measuring tape to estimate the number of centimetres in one inch.

4. Most people know their height in feet and inches, but not in centimetres.
   a) Determine your height in inches.
   b) Use your estimate in question 3 to calculate your height in centimetres.

1.2 Multiplication and Division of Mixed Numbers

GOAL

Multiply and divide mixed numbers.

LEARN ABOUT the Math

Mario is using small boxes to transfer cans of soup from $2\frac{2}{3}$ large boxes in the kitchen of the food shelter to the basement.

A large box holds $1\frac{1}{2}$ times as many cans as a small box.

Once the transfer is complete, there are a total of $7\frac{3}{4}$ small boxes full of cans in the basement.

? How many small boxes of cans were moved to the basement? How many large boxes would hold all the cans that are now in the basement?

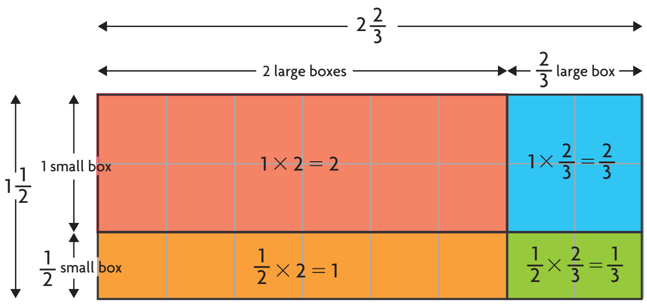
EXAMPLE 1 Selecting a strategy to multiply mixed numbers

Determine how many small boxes the cans in the kitchen of the food shelter will fill.

Carly's Solution: Representing the product as partial areas

$2\frac{2}{3} \times 1\frac{1}{2}$

A large box holds $1\frac{1}{2}$ times as much as a small one, so $2\frac{2}{3}$ large boxes fill $2\frac{2}{3} \times 1\frac{1}{2}$ small boxes.



To multiply, I drew a rectangle $2\frac{2}{3}$ by $1\frac{1}{2}$. The area told me how many small boxes Mario would fill with soup cans.

$$2\frac{2}{3} \times 1\frac{1}{2} = 2 + \frac{2}{3} + 1 + \frac{1}{3}$$
$$= 4$$

I added the partial areas to get the total area.

Four small boxes of cans were moved to the basement.

The cans in the kitchen of the food shelter will completely fill 4 small boxes.

Bobby changed the mixed numbers to improper fractions before he multiplied.

Bobby's Solution: Using a strategy involving improper fractions

I estimate $2\frac{2}{3} \times 1\frac{1}{2}$ to be less than $4\frac{1}{2}$.

$2\frac{2}{3} \times 1\frac{1}{2}$ is less than $3 \times 1\frac{1}{2}$.

$3 \times 1\frac{1}{2} = 3 + \frac{1}{2}$ of 3.

So, the number of small boxes is less than $3 + 1\frac{1}{2}$.

$$2\frac{2}{3} \times 1\frac{1}{2} = \frac{8}{3} \times \frac{3}{2}$$

I decided to rename the mixed numbers as improper fractions because I knew how to multiply entire fractions.

$$= \frac{8 \times 3}{3 \times 2}$$
$$= \frac{24}{6}$$

I multiplied the numerators, and then, multiplied the denominators. I simplified by dividing.

$$= 4$$

4 was less than $4\frac{1}{2}$, so I thought that my answer was correct.

Four small boxes were moved to the basement.

The cans in the kitchen of the food shelter will fill 4 whole small boxes.

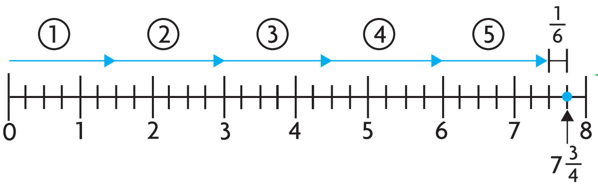
EXAMPLE 2 Selecting a strategy to divide mixed numbers

Determine how many large boxes all the cans in the basement will fill.

Tony's Solution: Using a number line model

$7\frac{3}{4} \div 1\frac{1}{2}$

Since there were $7\frac{3}{4}$ small boxes and it took $1\frac{1}{2}$ small boxes to fill a large box, I had to divide to determine how many $1\frac{1}{2}$s were in $7\frac{3}{4}$.



I created a number line divided into intervals of fourths, since 4 was the lowest common denominator of the fraction parts.
I marked off the number of times I could fit the **divisor** $1\frac{1}{2}$ into the **dividend** $7\frac{3}{4}$.

Therefore, the cans in the basement will fill $5\frac{1}{6}$ large boxes.

One fourth was left on the number line when I needed a jump of $1\frac{1}{2}$ or 6 fourths, so the fraction was $\frac{1}{6}$.

Darla and Enid both used division algorithms they had learned before.

Darla's Solution: Using a strategy involving common denominators

$$7\frac{3}{4} \div 1\frac{1}{2} = \frac{31}{4} \div \frac{3}{2}$$

$$= \frac{31}{4} \div \frac{6}{4}$$

I renamed the fractions as improper fractions with a common denominator.

$$= \frac{31}{6}$$

I divided the numerators to determine how many groups of 6 there were in 31.

$$= 5\frac{1}{6}$$

I renamed the improper fraction as a mixed number so that I could get a better sense of the number of boxes.

Therefore, the cans in the basement will fill $5\frac{1}{6}$ large boxes.

I checked my answer using multiplication.

$$5\frac{1}{6} \times 1\frac{1}{2} = \frac{31}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{2} = \frac{31}{4} = 7\frac{3}{4}$$

Since I got $7\frac{3}{4}$, I knew that my work was correct.

Enid's Solution: Using a strategy involving the reciprocal

$$7\frac{3}{4} \div 1\frac{1}{2} = \frac{31}{4} \div \frac{3}{2}$$

I renamed the mixed numbers as improper fractions.

$$= \frac{31}{4} \times \frac{2}{3}$$

I multiplied the first fraction by the reciprocal of the second fraction.

$$= \frac{31}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{3}$$

Since $\frac{31}{4} \times \frac{2}{3} = \frac{31}{3} \times \frac{2}{4}$, I simplified the fractions before I multiplied.

$$= \frac{31}{6}$$

$$= 5\frac{1}{6}$$

I renamed the improper fraction as a mixed number.

Therefore, the cans in the basement will fill $5\frac{1}{6}$ large boxes.

Reflecting

A. How could Carly determine the partial areas without drawing all the squares?

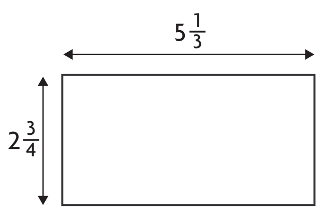
B. How did Bobby's and Enid's rearrangement of the numerators and denominators make the multiplication calculations easier?

APPLY the Math

EXAMPLE 3 Selecting a strategy to determine a product

Calculate $2\frac{3}{4} \times 5\frac{1}{3}$.

Tina's solution: Using an area model

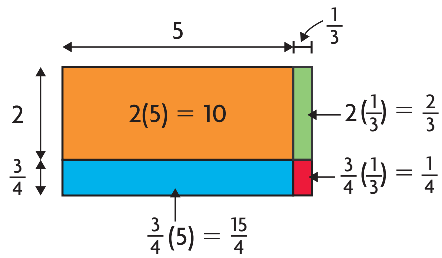


I thought of a rectangle that had a length of $2\frac{3}{4}$ units and a width of $5\frac{1}{3}$ units.

Communication *Tip*

Instead of using the multiplication sign to express a product, brackets may be used around the second factor.

For example: $2 \times 3 = 2(3)$.



I visualized the large rectangle as 4 sections. The dimensions of each of these smaller rectangles were the whole number or fraction part of each mixed number in the product. I multiplied the dimensions of each rectangle to find their areas.

$$= 10 + \frac{15}{4} + \frac{2}{3} + \frac{1}{4}$$

I added the partial areas to find the total area.

$$= 10 + 3\frac{3}{4} + \frac{2}{3} + \frac{1}{4}$$

$$= 10 + 3 + \frac{3}{4} + \frac{1}{4} + \frac{2}{3}$$

$$= 13 + 1 + \frac{2}{3}$$

$$= 14\frac{2}{3}$$

Todd's Solution: using improper fractions

$$2\frac{3}{4} \times 5\frac{1}{3}$$

I expressed each mixed number as an improper fraction.

$$= \frac{11}{4} \times \frac{16}{3}$$

$$= \frac{11}{\cancel{4}_1} \times \frac{\cancel{16}^4}{3}$$

I simplified by dividing 4 into 4 in the denominator and 4 into 16 in the numerator. Then, I multiplied the numerators and the denominators.

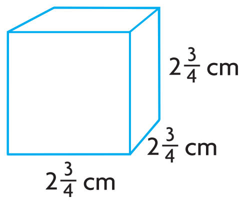
$$= \frac{44}{3}$$

$$= 14\frac{2}{3}$$

I simplified by dividing the numerator by the denominator.

EXAMPLE 4 Connecting products to powers of mixed numbers

Determine the volume of this cube.



Paul's Solution

$$V = s^3$$

$$= \left(2\frac{3}{4}\right)^3$$

$2\frac{3}{4}$ is less than 3. The volume would be less than 3 cubed or 27. I think the volume might be close to 25 cm^3.

$$= 2\frac{3}{4} \times 2\frac{3}{4} \times 2\frac{3}{4}$$

The exponent of 3 told me how many factors of $2\frac{3}{4}$ were multiplied.

$$= \frac{11}{4} \times \frac{11}{4} \times \frac{11}{4}$$

$$= \frac{11 \times 11 \times 11}{4 \times 4 \times 4}$$

$$= \frac{11^3}{4^3}$$

I noticed that the numerators were the same and so were the denominators. I decided to write them each as powers.

$$= \frac{1331}{64}$$

I calculated both powers.

$$= 20\frac{51}{64}$$

This answer seemed reasonable because 21 cm^3 was not that far from my estimate of 25 cm^3.

Therefore, the volume of the cube is $20\frac{51}{64}$ cm^3.

EXAMPLE 5 Problem solving using mixed numbers

Devon's father is installing new wood flooring. He bought boards that are 10 ft long, $\frac{3}{4}$ in. thick, and $\frac{11}{24}$ ft. wide. Determine the number of boards Devon's father will need for a 10 ft by $16\frac{1}{2}$ ft room.

Devon's Solution

I estimate the answer to be a little more than 32.

The boards are as long as the room, so I only had to worry about the width. If the boards were $\frac{1}{2}$ ft wide, I would need 2 boards for every foot of width. For 16 ft, I would need 32 boards.

$$16\frac{1}{2} \div \frac{11}{24}$$

$$= \frac{33}{2} \div \frac{11}{24}$$

$$= \frac{33}{2} \times \frac{24}{11}$$

Instead of dividing, I wrote the equivalent multiplication by multiplying by the reciprocal of $\frac{11}{24}$.

$$= \frac{\overset{3}{\cancel{33}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{12}{\cancel{24}}}{\underset{1}{\cancel{11}}}$$

I simplified by dividing both 2 and 24 by 2, and both 11 and 33 by 11.

$$= 36$$

36 was close to my estimate, so I thought that I was correct.

Therefore, my father will need 36 boards for the room.

In Summary

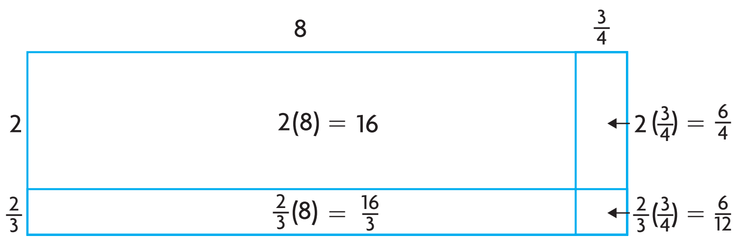
Key Idea

- The strategies you use for multiplying and dividing proper and improper fractions can be used to multiply and divide mixed numbers.

Need to Know

- The most efficient strategy to multiply or divide two mixed numbers is to perform the operations on their equivalent improper fractions.
- You can model the product of two mixed numbers as the area of a rectangle in which the numbers are the length and width. You can then determine the area of each section of the rectangle. The product of the mixed fractions is the sum of these partial areas.

  For example, $8\frac{3}{4} \times 2\frac{2}{3}$ can be calculated as follows:



  Then, the partial areas are added together.

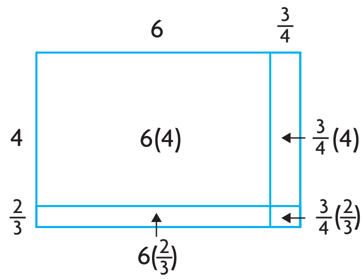
$$\begin{aligned} 8\frac{3}{4} \times 2\frac{2}{3} &= 16 + \frac{6}{4} + \frac{16}{3} + \frac{6}{12} \\ &= 16 + \frac{3}{2} + \frac{16}{3} + \frac{1}{2} \\ &= 16 + 2 + 5\frac{1}{3} \\ &= 23\frac{1}{3} \end{aligned}$$

- One way you can divide one fraction by another is by multiplying the first fraction by the reciprocal of the second fraction.
- Another way you can divide one fraction by another is by renaming the fractions as equivalent fractions with the same denominator, and then, dividing the numerators.

CHECK Your Understanding

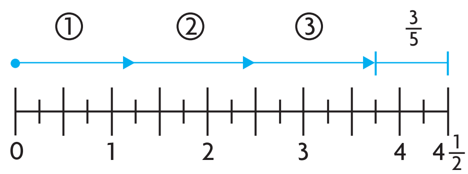
1. a) State the multiplication problem represented by the following area model.

b) Use the model to determine the product.



2. Use improper fractions to calculate $7\frac{1}{2} \times 2\frac{2}{5}$.

3. State the division problem and its answer shown by the number line.



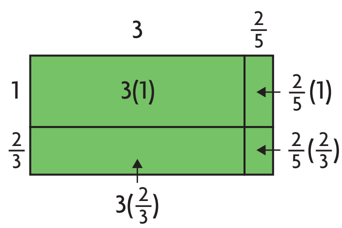
4. Calculate $1\frac{7}{15} \div 1\frac{11}{25}$ by multiplying by the reciprocal.

PRACTISING

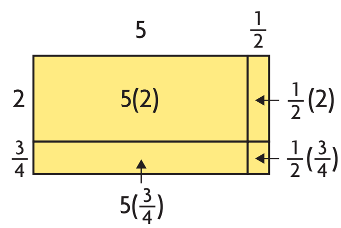
5. a) State the multiplication problem represented by each area model.

b) Use the model to determine each product.

i)



ii)



6. Estimate.

a) $2\frac{1}{5} \times 2\frac{5}{6}$

b) $5\frac{7}{8} \times 6\frac{3}{4}$

c) $5\frac{1}{2} \div \frac{9}{10}$

d) $8\frac{5}{6} \div 1\frac{5}{8}$

7. Calculate each product.

a) $2\frac{1}{6} \times 4\frac{2}{3}$ **c)** $\frac{3}{4} \times 6\frac{11}{12}$ **e)** $4\frac{1}{8} \times 5\frac{1}{3}$

b) $1\frac{3}{4} \times 2\frac{2}{3}$ **d)** $5\frac{1}{9} \times 3\frac{3}{4}$ **f)** $2\frac{2}{7} \times 2\frac{7}{8}$

8. Determine the value that makes each equation true.

a) $\left(\frac{2}{3}\right)^7 = \frac{2^{\blacksquare}}{3^7}$ **b)** $\left(2\frac{1}{2}\right)^{\blacksquare} = \frac{5^4}{2^4}$ **c)** $\left(6\frac{1}{3}\right)^2 = \frac{\blacksquare^2}{3^2}$

9. Calculate each power.

a) $\left(\frac{3}{4}\right)^3$ **b)** $\left(\frac{5}{2}\right)^3$ **c)** $\left(3\frac{1}{5}\right)^3$

10. Calculate each quotient.

a) $2\frac{2}{5} \div \frac{4}{5}$ **c)** $9\frac{2}{3} \div 2\frac{2}{3}$ **e)** $8\frac{2}{3} \div 10\frac{1}{2}$

b) $1\frac{1}{4} \div 3\frac{4}{5}$ **d)** $2\frac{7}{8} \div 3\frac{5}{6}$ **f)** $8\frac{3}{4} \div 5\frac{2}{5}$

11. a) Show that the two calculations are equivalent in each set below.

i) $\frac{4}{9} \div \frac{2}{3}$ and $\frac{4 \div 2}{9 \div 3}$

ii) $\frac{28}{15} \div \frac{4}{5}$ and $\frac{28 \div 4}{15 \div 5}$

iii) $\frac{35}{48} \div \frac{5}{12}$ and $\frac{35 \div 5}{48 \div 12}$

b) Show how you could rewrite $\frac{2}{3}$ as $\frac{8}{12}$, and then, use the strategy in part a) to evaluate $\frac{2}{3} \div \frac{1}{4}$.

12. Calculate.

K **a)** $7\frac{3}{5} \times 3\frac{3}{4}$ **b)** $1\frac{2}{3} \div 5\frac{5}{6}$

13. A farmer made a square chicken coop with a length of $6\frac{1}{2}$ yd.

A **a)** Determine the perimeter of the chicken coop.

b) Determine the area of the chicken coop.

14. Gavin made a patio area out of square blocks that are $1\frac{1}{2}$ ft by $1\frac{1}{2}$ ft. The area of his patio is $175\frac{1}{2}$ sq ft and the length is $19\frac{1}{2}$ ft.

a) Determine the width of his patio.

b) Determine the number of blocks Gavin used to make his patio.

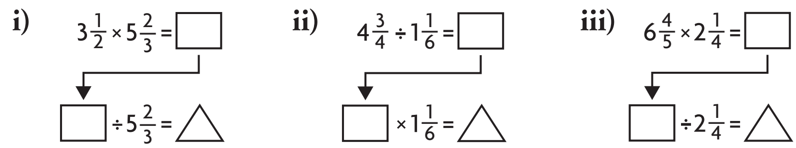
15. Show that dividing a number by $5\frac{1}{2}$ gives the same answer as multiplying the number by $\frac{2}{11}$.

C

16. Which whole numbers can replace the box to make the product of $3\frac{1}{5}$ and $\blacksquare\frac{3}{4}$ greater than 25?

17. **a)** In each case, determine the numbers represented by the rectangle and the triangle.

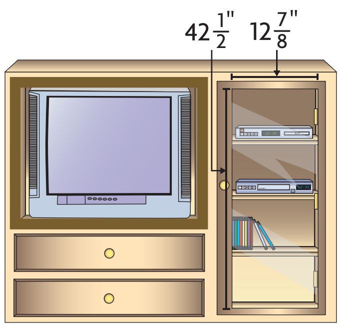
T



b) Describe the connection between the number represented by the rectangle and the number represented by the triangle.

c) Create a question of your own that shows this connection.

18. A large bottle holds $1\frac{3}{4}$ times the amount of liquid of a small bottle. Determine the number of large bottles that would hold the same amount as $10\frac{1}{2}$ small bottles.



19. Determine the replacement cost of glass for the entertainment centre shown if it costs \$3.90 per square foot. (Hint: 1 sq ft = 144 sq in.)

20. Without calculating the quotient, how do you know that $4\frac{2}{3} \div 10\frac{1}{4}$ has to be less than $\frac{1}{2}$?

21. Some people multiply fractions by first renaming them as equivalent fractions with a common denominator. They get the correct answer, but why is this not always an efficient method?

Extending

22. Sherri divided a mixed number by $2\frac{3}{4}$. The quotient was a whole number larger than 10. What are two possibilities for the mixed number?

23. A rectangle measuring $8\frac{1}{4}$ units by $3\frac{3}{4}$ units is to be completely covered by squares that are all the same size. What are the largest possible dimensions of the squares?

1.3 Integer Operations with Powers

GOAL

Evaluate integer expressions involving order of operations and powers.

LEARN ABOUT the Math

Many contests make you answer a skill-testing question before you can claim your prize. Suppose you won a contest and you had to answer this question:

$$-2^4 + (-1 - 1)^3 + 5(-2)^4$$

? What is the answer to the skill-testing question?

EXAMPLE 1 Using the order of operations to evaluate an expression

Determine the correct answer to $-2^4 + (-1 - 1)^3 + 5(-2)^4$.

Michelle's Solution

$-2^4 + (-1 - 1)^3 + 5(-2)^4$

I applied the **order of operations** to calculate the answer.

$= -2^4 + (-2)^3 + 5(-2)^4$

I did the subtraction first because it was in the brackets.

$= -16 + (-2)^3 + 5(-2)^4$

For the first power, since there were no brackets, the base of the power was 2, not −2. So, I treated it as $-(2^4)$.
I calculated $2^4 = 16$, and then, I multiplied it by −1.

$= -16 + (-8) + 5(-2)^4$

I calculated $(-2)^3 = (-2)(-2)(-2)$ or −8.

$= -16 + (-8) + 5(16)$

I calculated $(-2)^4 = (-2)(-2)(-2)(-2)$
$= 16$.

$= -16 + (-8) + 80$

I multiplied before adding.

$= 56$

Reflecting

A. If m is 1, 2, 3, 4, and so on, how can you predict the sign of $(-2)^m$?

B. Why is the value of -2^m never positive for any value of m?

C. How is the use of the order of operations to evaluate an integer expression similar to evaluating a whole number expression? How is it different?

APPLY the Math

EXAMPLE 2 Selecting a strategy to calculate an expression with powers

Calculate $-3^4 + [-2 - (-4)^3] + \sqrt{16}$.

Anthony's Solution: Applying the order of operations

$-3^4 + [-2 - (-4)^3] + \sqrt{16}$ ← In the square brackets, there was a subtraction and a power.

$= -3^4 + [-2 - (-64)] + \sqrt{16}$ ← I calculated the power before the subtraction because I followed the order of operations. I knew that when the base is negative and the exponent is odd, the answer is negative.

$= -3^4 + (-2 + 64) + \sqrt{16}$ ← I did the subtraction in the brackets. To subtract −64, I added its opposite.

$= -3^4 + 62 + \sqrt{16}$

$= -81 + 62 + \sqrt{16}$ ← I calculated the power.

$= -81 + 62 + 4$ ← I calculated the square root before adding.

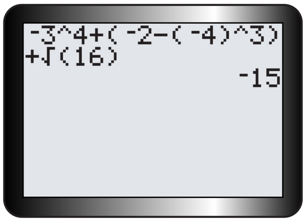
$= -15$

Communication *Tip*

- You can use the memory aid BEDMAS to remember the rules for order of operations.
  Perform the operations in Brackets first.
  Calculate Exponents and square roots next.
  Divide and Multiply from left to right.
  Add and Subtract from left to right.
- When there are multiple brackets, complete the operations in the inner brackets first. For example:
  $[(2 + 3) \times 3]^2$
  $= [5 \times 3]^2$
  $= 15^2$
  $= 225$
- When a square root sign covers an expression, it contains the expression just like brackets.

Many scientific and graphing calculators are programmed to follow the order of operations. Peng used her graphing calculator to calculate the answer.

Peng's Solution: Using a calculator with brackets keys



My calculator only had round brackets so I had to be careful to make sure that the brackets matched the order in the original expression.

I also had to make sure that I used the negative sign key to enter negative numbers instead of the subtraction key.

Tech *Support*

Many graphing calculators use a key with a negative sign inside brackets (-) to distinguish it from subtraction −.

To evaluate an algebraic expression for given values of the variables, substitute these given values into the expression. This results in a numerical expression that can be calculated following the order of operations.

EXAMPLE 3 Evaluating an expression in fraction form

Evaluate the expression $\frac{3x^3 + 16}{-y^3}$ when $x = -4$ and $y = 2$.

Talia's Solution

$$\frac{3x^3 + 16}{-y^3}$$

I knew that a fraction represents a division and that the numerator and denominator have to be evaluated before the division can be done.

$$= \frac{3(-4)^3 + 16}{-(2)^3}$$

I used brackets to show where I substituted the values for the variables.

$$= \frac{3(-64) + 16}{-8}$$

$$= \frac{-192 + 16}{-8}$$

$$= \frac{-176}{-8}$$

I used the order of operations to evaluate the expressions in the numerator and the denominator separately.

$$= 22$$

I knew that dividing integers with the same sign gave a positive result.

In Summary

Key Idea

- You can use the same order of operations (BEDMAS) for integer expressions as you used for whole number expressions.

Need to Know

- For exponent $n = 1, 2, 3, 4$, and so on:
  - $(-a)^n = \underbrace{(-a)(-a)(-a)\ldots(-a)}_{[n \text{ factors}]}$

    For example: $(-2)^3 = (-2)(-2)(-2)$
  - $-a^n = -(a^n)$

    For example: $-2^4 = -(2^4)$
  - If $a < 0$, then $(a)^n$ is positive if n is even and negative if n is odd.

    For example: $(-2)^4 = 16$ and $(-2)^3 = -8$
- You can evaluate an expression for given values for the variable(s) by replacing each variable with its numeric value in brackets. Then, follow the order of operations.

CHECK Your Understanding

1. Without calculating, state whether the answer will be positive or negative.

a) -2^3 **c)** $-(-2)^3$ **e)** $-(-2)^4$

b) $(-2)^3$ **d)** -2^4 **f)** $-(-2^4)$

2. Without using a calculator, determine the answers in question 1.

3. Show the steps required to evaluate the following expressions.

a) $-7^2 - 2(-3)^3$ **b)** $-4^2 - (-4)^2 - 4^2$

PRACTISING

4. Calculate.

a) $\dfrac{-4(15)}{-10}$ **b)** $\dfrac{20}{2(-5)}$ **c)** $\dfrac{-10(6)}{-3(-2)}$

5. Determine the value that makes each equation true.

a) $\blacksquare^3 = 27$ **c)** $-\blacksquare^2 = -25$ **e)** $\blacksquare^5 = 32$

b) $(-3)^{\blacksquare} = -27$ **d)** $-4^{\blacksquare} = -64$ **f)** $\blacksquare^5 = -32$

6. Evaluate.

a) -5^3 **c)** -4^3 **e)** $-(-3)^4$

b) $(-6)^2$ **d)** $(-4)^3$ **f)** $-(-3)^3$

7. Solving the equation $\blacksquare^2 = 64$ gives two possible integer values. Determine the values.

8. Evaluate each expression without using a calculator.

a) $5(-2)^3$ **d)** $-2^3 - (-10 + 5^2)$

b) $-4(-5) - (-3^3)$ **e)** $\dfrac{(-2)^2 - 22}{-3^2}$

c) $[-2(-1)^3]^6$ **f)** $\dfrac{3^3 + 3(7)}{-2^4} + \dfrac{3(-5)^2}{-15}$

9. Find the error in each solution. Explain what was done incorrectly.
A Redo the solution, making the necessary corrections.

a)
$$\begin{aligned} &-4[5 - 2(-3)] \\ &= -4[3(-3)] \\ &= -4(-9) \\ &= 36 \end{aligned}$$

b)
$$\begin{aligned} &-2(3)^2 \\ &= (-6)^2 \\ &= 36 \end{aligned}$$

10. Both Claire and Robin calculated $5(-2) - 3(-2)$.

C Claire's calculations

$5(-2) - 3(-2)$
$= -10 + 6$
$= -4$

Robin's calculations

$5(-2) - 3(-2)$
$= 2(-2)$
$= -4$

Both students are correct. Explain Robin's reasoning.

11. Evaluate each expression when $x = -2$ and $y = -1$.

a) $x^2 + y^3$ **c)** $2y^5 - (3 - x)^2$ **e)** $\frac{8(x + y^2)}{x^2}$

b) $5y^3(-x^4)$ **d)** $x^2 + [5x - 2(y - x)]$ **f)** $\frac{y^5 + y^3 + y}{y^6 + y^4 + y^2}$

12. Evaluate the expression $-y^2 - 4x^3$ when $x = -2$ and $y = 3$.
K

13. Assume b and n are positive integers. For each situation below, decide
T whether $-b^n + (-b)^n$ is positive, negative, or zero. Explain your reasoning.

a) The exponent is an odd number.
b) The exponent is an even number.

Extending

14. a) Use a calculator to calculate the following.

i) $(-2)^3(-2)^4$ **ii)** $(-2)^2(-2)^6$ **iii)** $(-2)(-2)^5$

b) Express each answer in part a) as a power with a base of (-2).
c) Look for patterns. How could you get the power in part b) just by looking at the question in part a)?

15. a) Use a calculator to calculate the following.

i) $\frac{(-3)^9}{(-3)^7}$ **ii)** $\frac{(-3)^8}{(-3)^4}$ **iii)** $\frac{(-3)^5}{(-3)}$

b) Express each answer in part a) as a power with a base of (-3).
c) Look for patterns. How could you get the power in part b) just by looking at the question in part a)?

16. Evaluate $3(2^n) - 2^n$ and 2^{n+1} for various values of n.

a) What pattern do you notice?
b) Why does the pattern work?

1 Mid-Chapter Review

FREQUENTLY ASKED Questions

Q: What strategies can you use to add or subtract mixed numbers?

A: The most efficient method is to create equivalent fractions with the same denominator for the fraction parts of the mixed numbers. You can add or subtract the whole number parts and the fraction parts separately. Rename mixed numbers when necessary.

You can also rename the mixed numbers as improper fractions with a common denominator, but this often results in having to work with large numerators.

Study Aid

- See Lesson 1.1, Examples 1 and 2.
- Try Mid-Chapter Review Questions 1 to 4.

EXAMPLE

$$4\frac{6}{7} + 5\frac{2}{3}$$
$$= 4\frac{18}{21} + 5\frac{14}{21}$$
$$= 9\frac{32}{21}$$
$$= 10\frac{11}{21}$$

$$7\frac{1}{3} - 2\frac{3}{4}$$
$$= 7\frac{4}{12} - 2\frac{9}{12}$$
$$= 6\frac{16}{12} - 2\frac{9}{12}$$
$$= 4\frac{7}{12}$$

Q: What strategies can you use to multiply mixed numbers?

A1: The most efficient method is to write each mixed number as an improper fraction. Then, multiply as if they were ordinary fractions. You may be able to simplify parts of the fractions prior to multiplying.

Study Aid

- See Lesson 1.2, Examples 1 and 3.
- Try Mid-Chapter Review Questions 5 and 7.

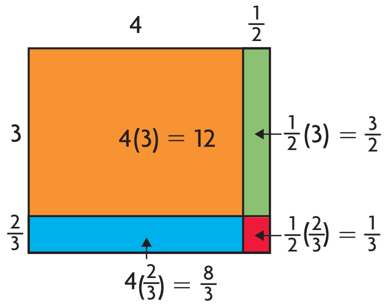
EXAMPLE

$$4\frac{1}{2} \times 3\frac{2}{3}$$
$$= \frac{9}{2} \times \frac{11}{3}$$
$$= \frac{\overset{3}{\cancel{9}}}{2} \times \frac{11}{\underset{1}{\cancel{3}}}$$
$$= \frac{33}{2}$$
$$= 16\frac{1}{2}$$

A2: You can use a rectangular area model to represent the product of two mixed numbers. Use the model to determine the partial areas and add them to calculate the final product.

EXAMPLE

$3\frac{2}{3} \times 4\frac{1}{2}$



$$= 12 + \frac{8}{3} + \frac{3}{2} + \frac{1}{3}$$

$$= 12 + \frac{3}{2} + \frac{8}{3} + \frac{1}{3}$$

$$= 12 + \frac{3}{2} + \frac{9}{3}$$

$$= 12 + 1\frac{1}{2} + 3$$

$$= 16\frac{1}{2}$$

Study *Aid*

- See Lesson 1.2, Example 2.
- Try Mid-Chapter Review Questions 6 and 7.

Q: What strategies can you use to divide mixed numbers?

A1: The most efficient method is to rename the mixed numbers as improper fractions, and then, multiply the dividend by the reciprocal of the divisor. You may be able to simplify parts of the fractions prior to multiplying to get the final result.

EXAMPLE

$$5\frac{1}{2} \div 3\frac{2}{3} = \frac{11}{2} \div \frac{11}{3}$$

$$= \frac{\overset{1}{\cancel{11}}}{2} \times \frac{3}{\underset{1}{\cancel{11}}}$$

$$= \frac{3}{2}$$

$$= 1\frac{1}{2}$$

A2: You can first rename the mixed numbers as equivalent improper fractions with the same denominator. Then, divide the numerators.

EXAMPLE

$$5\frac{1}{2} \div 3\frac{2}{3} = \frac{11}{2} \div \frac{11}{3}$$
$$= \frac{33}{6} \div \frac{22}{6}$$
$$= 33 \div 22$$
$$= 1\frac{11}{22}$$
$$= 1\frac{1}{2}$$

Q: What strategies can you use to evaluate integer expressions with powers?

A: When you evaluate an expression for given values for the variable(s), replace each variable with its numeric value in brackets, and then, follow the order of operations.

Study Aid

- See Lesson 1.3, Examples 1, 2, and 3.
- Try Mid-Chapter Review Questions 12 and 13.

EXAMPLE

Evaluate $(x - y)^2 + (x + y)^3$ when $x = -3$ and $y = 2$.

Solution

$$(x - y)^2 + (x + y)^3$$
$$= [(-3) - (2)]^2 + [(-3) + (2)]^3$$
$$= (-5)^2 + (-1)^3$$
$$= 25 + (-1)$$
$$= 24$$

PRACTICE Questions

Lesson 1.1

1. Calculate.

a) $7\frac{1}{3} + 2\frac{1}{2}$

b) $4\frac{2}{5} + 1\frac{3}{4}$

c) $6\frac{3}{4} - 6\frac{2}{3}$

d) $9\frac{1}{7} - 4\frac{4}{5}$

2. Explain how you can use estimation to tell that your answers in question 1 are reasonable.

3. John works part-time at a restaurant. On Friday he worked $3\frac{1}{4}$ h and on Saturday he worked $6\frac{1}{2}$ h. How many hours did he work altogether?

4. Why does it make sense that $3\frac{1}{5} - 2\frac{1}{4}$ has the same answer as $\frac{3}{4} + \frac{1}{5}$?

Lesson 1.2

5. Calculate each product without using a calculator.

a) $2\frac{5}{8} \times \frac{4}{11}$

b) $1\frac{3}{5} \times 1\frac{2}{7}$

c) $2\frac{3}{5} \times 3\frac{1}{3}$

d) $7\frac{1}{5} \times 4\frac{5}{6}$

6. Calculate each quotient without using a calculator.

a) $5\frac{3}{4} \div \frac{1}{2}$

b) $\frac{1}{2} \div 5\frac{3}{4}$

c) $6\frac{2}{3} \div 2\frac{1}{6}$

d) $10\frac{5}{8} \div 5\frac{1}{3}$

7. Determine the value that makes each equation true.

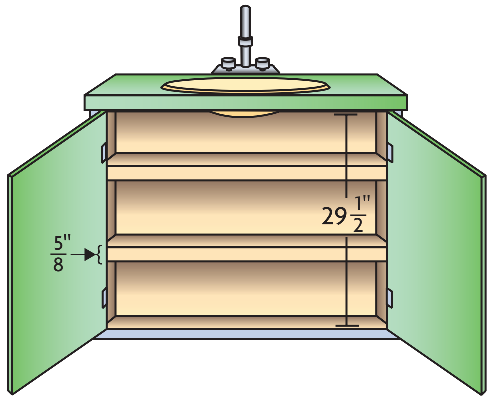
a) $6\frac{3}{4} \times \blacksquare = 19\frac{1}{8}$

b) $\blacksquare \times 1\frac{1}{4} = \frac{5}{8}$

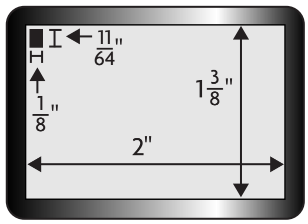
c) $\blacksquare \div 5\frac{1}{3} = 4\frac{2}{3}$

d) $7\frac{3}{4} \div \blacksquare = 5\frac{1}{6}$

8. Melissa is adjusting the two removable shelves in her cupboard. The shelves are to be equally spaced in the cupboard. How much space is above or below each shelf?



9. Suppose the onscreen cursor represents any numeric character. Determine the number of numeric characters that can fit on the calculator screen.



Lesson 1.3

10. Calculate.

a) $(-11)^2$

b) $(-4)^3$

c) -7^2

d) -6^3

11. Determine at least four other powers that have the same value as 8^2.

12. Answer the following skill-testing question.

$-[(5)(-1)]^3 - 2(-4)^3$

13. Evaluate when $x = 2$, $y = -3$, and $z = -1$. Do not use a calculator.

a) $x^2 + y^2 + z^2$

b) $2[x - (y - z)^4]$

c) $\dfrac{2y + 4z}{-x}$

d) $\dfrac{x - y^2}{2z - x + y}$

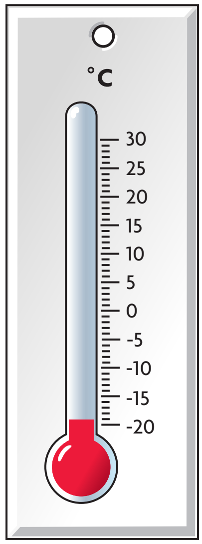
1.4 Rational Numbers

GOAL

Connect rational numbers to other number systems.

LEARN ABOUT the Math

Shahreen looked at the thermometer outside. The temperature was a **rational number** between −18 °C and −19 °C. It was closer to −19 °C.



rational number
a number that can be expressed as the quotient of two integers where the divisor is not 0

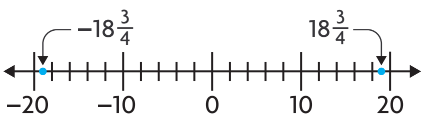
opposites
two numbers with opposite signs that are the same distance from zero (e.g., +6 and −6 are opposites)

? What might the temperature be?

EXAMPLE 1 Using a number line to represent rational numbers

Determine a possible temperature value that is between −18 °C and −19 °C, but is closer to −19 °C.

Mark's Solution: Using fractions and a number line



I marked the approximate position of the temperature value on a number line. Then, I marked the **opposite** of the value because I can read fractions more easily if they are positive. I estimated the positive value to be about $18\frac{3}{4}$.

A possible temperature value between −18 °C and −19 °C is $-18\frac{3}{4}$ °C.

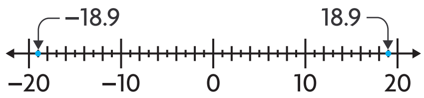
I knew that the opposite of $18\frac{3}{4}$ was $-18\frac{3}{4}$. This meant $-18\frac{3}{4}$ was just as far from zero as $18\frac{3}{4}$ but in the negative direction.

So, $-18\frac{3}{4}$ must be the same as $-18 - \frac{3}{4}$ because it was farther to the left of 0 than −18.

$-18\frac{3}{4}$ is a rational number because it can also be expressed as $-\frac{75}{4}$.

$18\frac{3}{4}$ is $\frac{75}{4}$. Its opposite is $-\frac{75}{4}$.
So, $-18\frac{3}{4} = -\frac{75}{4}$.

George's Solution: Using decimals and a number line



I marked the approximate position of the temperature value on a number line.

Then, I marked the opposite of the value because I can read decimals more easily if they are positive. I estimated the positive value to be about 18.9.

A possible temperature value is −18.9 °C.

I knew that the opposite of 18.9 was −18.9. This meant −18.9 was as far from zero as 18.9 but in the negative direction.

So, −18.9 must be the same as −18 − 0.9 because it was farther to the left of 0 than −18.

−18.9 is a rational number because it can also be expressed as $-\frac{189}{10}$.

As a mixed number, $18.9 = 18\frac{9}{10}$ or $\frac{189}{10}$. Its opposite is $-\frac{189}{10}$. So, $-18.9 = -\frac{189}{10}$.

Reflecting

A. How would you describe how to place a "negative fraction" like $-\frac{2}{5}$ on a number line?

B. Why is −20 less than −18.9 even though 20 is greater than 18.9?

C. How are rational numbers similar to fractions and integers? How are they different?

APPLY the Math

EXAMPLE 2 Representing rational numbers as decimals

Which of the following represent the same rational number?

$\frac{-2}{3}, \frac{2}{-3}, -\frac{2}{3}, \frac{3}{-2}, \frac{-3}{-2}, \frac{3}{2}$

Abby's Solution

$\frac{-2}{3} = -0.\overline{6}$

$\frac{2}{-3} = -0.\overline{6}$

I divided the numerator by the denominator to rename each rational number as a decimal. I put a bar above the digit that repeated.

$-\frac{2}{3} = -0.\overline{6}$

I knew that $-\frac{2}{3}$ was the opposite of $\frac{2}{3}$. As a decimal, $\frac{2}{3} = 0.\overline{6}$, so $-\frac{2}{3} = -0.\overline{6}$.

$\frac{3}{-2} = -1.5$

$\frac{-3}{-2} = 1.5$

$\frac{3}{2} = 1.5$

I knew that having one negative in the fraction would give a negative answer, and a pair of negatives would give a positive answer.

$\frac{-2}{3}, \frac{2}{-3}$, and $-\frac{2}{3}$ represent the same rational number.

$\frac{-3}{-2}$ and $\frac{3}{2}$ represent the same rational number.

I compared the decimal values to determine which rational numbers were equivalent.

When it is necessary to compare the size of two or more rational numbers, a number line is a useful tool to use.

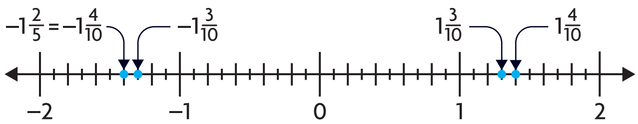
EXAMPLE 3 | Representing rational numbers using a number line

Determine which number is greater: $-1\frac{3}{10}$ or $-1\frac{2}{5}$.

Andrew's Solution

$-1\frac{2}{5} = -1\frac{4}{10}$

I knew that I could compare these numbers if they had equivalent fraction parts. I renamed the fraction part for $-1\frac{2}{5}$.



$-1\frac{3}{10}$ is the opposite of $1\frac{3}{10}$. Since $1\frac{4}{10}$ is farther from zero than $1\frac{3}{10}$, $-1\frac{4}{10}$ is farther from zero than $-1\frac{3}{10}$.

Therefore, $-1\frac{3}{10} > -1\frac{2}{5}$.

Since $-1\frac{3}{10}$ is to the right of $-1\frac{4}{10}$ on the number line, $-1\frac{3}{10}$ is greater than $-1\frac{4}{10}$.

Communication | *Tip*

Traditionally, sets of numbers have been represented by letters. The symbol Q is used for rational numbers because they are a *quotient* of two integers.

Set	Definition	Examples	Symbol
Natural Numbers	the counting numbers	1, 2, 3, …	**N**
Whole Numbers	the counting numbers and zero	0, 1, 2, 3, …	**W**
Integers	positive and negative whole numbers	…, −3, −2, −1, 0, 1, 2, 3, …	**I**
Rational Numbers	numbers of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$	$\frac{3}{4}$, $-\frac{2}{3}$, $-3\frac{5}{8}$, 2.35, −3.921, $-8.\overline{234}$	**Q**

In Summary

Key Idea

- Rational numbers include integers, fractions, their decimal equivalents, and their opposites.

Need to Know

- Rational numbers can be positive, negative, or zero.
- Every integer is a rational number because it can be written as a quotient of two integers: itself as the numerator and 1 as the denominator.

  For example, zero can be expressed as $\frac{0}{1}$.
- To compare rational numbers, it helps to rename them to a common form, either as decimals or as fractions.
- The rules for renaming rational numbers are the same as the rules for positive fractions and decimals.
- When comparing rational numbers, one number is greater than another if it is farther to the right on a number line.
- A negative mixed number is a subtraction of its parts.

  For example: $-7\frac{1}{4} = -7 + \left(-\frac{1}{4}\right)$
  $= -7 - \frac{1}{4}$

  Similarly, for decimals: $-7.25 = -7 + (-0.25)$
  $= -7 - 0.25$

CHECK Your Understanding

1. Rename the following rational numbers as quotients of two integers.

a) $-2\frac{1}{4}$ **b)** $-5\frac{6}{7}$

2. Rename the following rational numbers as decimals.

a) $\frac{1}{5}$ **c)** $\frac{3}{-4}$

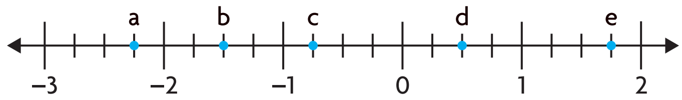
b) $\frac{-4}{7}$ **d)** $-7\frac{5}{6}$

3. Rename the following rational numbers as quotients of integers.

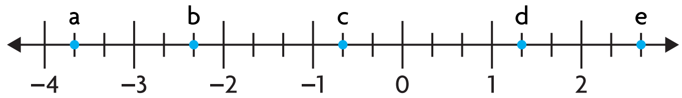
a) -0.35 **b)** 4.625 **c)** -11.46

PRACTISING

4. Identify the values represented by a, b, c, d, and e, in decimal form.



5. Identify the values represented by a, b, c, d, and e, as quotients of two integers.



6. Two students renamed the mixed number $-2\frac{1}{2}$ to its corresponding improper fraction. Who did it correctly, Tammy or Jasmine? Identify the error made in the incorrect solution.

K

Tammy

$$\begin{aligned} -2\frac{1}{2} &= -\left(2\frac{1}{2}\right) \\ &= -\left(\frac{(2 \times 2 + 1)}{2}\right) \\ &= -\left(\frac{(4 + 1)}{2}\right) \\ &= -\frac{5}{2} \end{aligned}$$

Jasmine

$$\begin{aligned} -2\frac{1}{2} &= \frac{(-2 \times 2 + 1)}{2} \\ &= \frac{(-4 + 1)}{2} \\ &= -\frac{3}{2} \end{aligned}$$

Communication ***Tip***

When reading from left to right:
> is the symbol for "greater than"
< is the symbol for "less than."

7. Use $>$, $<$, or $=$ to make true statements. Explain how you know each statement is true.

a) $0 \blacksquare -0.5$

b) $-4.3 \blacksquare -3.4$

c) $-1\frac{2}{5} \blacksquare 1\frac{2}{5}$

d) $-4\frac{1}{2} \blacksquare -\frac{9}{2}$

e) $5.6 \blacksquare 5\frac{3}{5}$

f) $-2\frac{3}{10} \blacksquare -2.\overline{3}$

8. Explain why $-3\frac{1}{4}$ can be renamed as $-3 - \frac{1}{4}$ and not as $-3 + \frac{1}{4}$.

9. **a)** Name three fractions between $\frac{2}{8}$ and $\frac{3}{8}$.

C **b)** How would your answers to part a) help you name three rational numbers between $-\frac{2}{8}$ and $-\frac{3}{8}$?

c) Are your answers in part a) rational numbers? Explain.

10. True or false? Justify your answer.

T **a)** All mixed numbers can be renamed as decimals.

b) A rational number can be expressed as any integer divided by any other integer.

c) Two rational numbers are opposites if they have different signs.

d) $1 > -1\,000\,000$

11. To write 0.833 333 … as a fraction, Rhys thought of this as:

A $0.8 + \frac{1}{10}$ of $0.333\,333\ldots$

This is the same as:

$$\frac{8}{10} + \frac{1}{10} \times \frac{1}{3}$$
$$= \frac{8}{10} + \frac{1}{30}$$
$$= \frac{24}{30} + \frac{1}{30}$$
$$= \frac{25}{30}$$
$$= \frac{5}{6}$$

Use Rhys's approach to write each of the following as fractions.

a) 0.4333 … **b)** $0.1\overline{6}$ **c)** $0.25\overline{3}$

12. Draw a diagram to show the relationship between these sets of numbers: Integers, Whole Numbers, Natural Numbers, and Rational Numbers. Use one circle for each set of numbers.

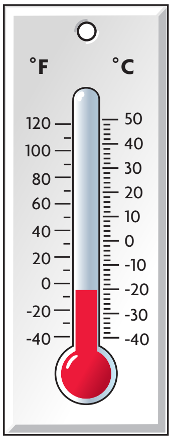
Extending

13. If a and b are positive numbers and $a < b$, how do $-a$ and $-b$ compare? Explain why.

1.5 Rational Number Operations

GOAL

Evaluate expressions involving rational numbers.



C temperature
= $\frac{5}{9}$ (F temperature – 32)

LEARN ABOUT the Math

Matthew was chatting online with his friend Bruce, who lives in the United States. Bruce said that the temperature outside was -5.5 degrees Fahrenheit. Matthew was not sure how cold that was because he was used to temperature readings measured in degrees Celsius.

He found the following conversion formula from a weather website:

$$C = \frac{5}{9}[F - 32]$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Farenheit.

? What is the Celsius temperature equivalent to –5.5 °F?

EXAMPLE 1 Evaluating a rational number expression

Determine the Celsius temperature equivalent to -5.5 °F.

Ishtar's Solution: Connecting to integer and fraction operations

I estimate -5.5 °F to be about -21 °C. ← I estimated by looking at the thermometer.

$$C = \frac{5}{9}[(-5.5) - 32]$$

← I substituted the Fahrenheit temperature value into the formula.

$$= \frac{5}{9}\left[\left(-5\frac{1}{2}\right) - 32\right]$$

← I decided to rename all the numbers as fractions because $\frac{5}{9}$ would become a repeating decimal. This would give me a rounding error in my final answer.

$$= \frac{5}{9}\left[-\left(5 + \frac{1}{2}\right) - 32\right]$$

I followed the order of operations and did the subtraction within the brackets first.

$$= \frac{5}{9}\left(-5 - \frac{1}{2} - 32\right)$$

$$= \frac{5}{9}\left(-5 - 32 - \frac{1}{2}\right)$$

$$= \frac{5}{9}\left(-37 - \frac{1}{2}\right)$$

$$= \frac{5}{9}\left(-37\frac{1}{2}\right)$$

$$= \frac{5}{9}\left(\frac{-75}{2}\right)$$

I decided to multiply using improper fractions.

$$= \frac{5}{\cancel{9}_{3}}\left(\frac{\cancel{-75}^{-25}}{2}\right)$$

$$= -\frac{125}{6}$$

I used what I knew about integers to determine the sign.

$$= -20\frac{5}{6}$$

I renamed the answer as a negative mixed number, so that I could get a better sense of the number of degrees.

$$= -20.8\overline{3}$$

Since the Fahrenheit temperature was expressed in decimal form, I decided to rename my answer as a decimal.

Therefore, -5.5 °F is equivalent to about -20.8 °C.

I thought my answer was reasonable because it was close to my estimate.

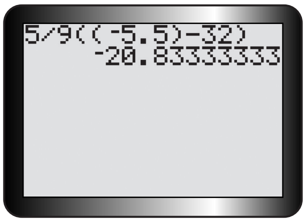
Calculations involving rational numbers can also be performed using scientific and graphing calculators.

Sherry's Solution: Using a calculator

$$C = \frac{5}{9}(F - 32)$$

$$= \frac{5}{9}[(-5.5) - 32]$$

I substituted −5.5 into the conversion formula and entered it into my calculator.



My calculator follows the order of operations, so I assumed it would give the correct answer.

Therefore, −5.5 °F is equivalent to $-20.8\overline{3}$ °C or about −20.8 °C.

Reflecting

A. How did Ishtar use what he knew about integer operations to complete the calculation?

B. How did Ishtar use what he knew about fraction operations to complete the calculation?

APPLY the Math

EXAMPLE 2 Connecting the addition of rational numbers to adding fractions and integers

Calculate $-\frac{4}{5} + \frac{2}{-3}$.

Thai's Solution

I estimate the answer to be between −1 and −2.

I knew that $\frac{4}{5}$ and $\frac{2}{3}$ were each greater than $\frac{1}{2}$, and that $\frac{4}{5} + \frac{2}{3}$ must be between 1 and 2. So, $-\frac{4}{5} + \frac{2}{-3}$ must be between −1 and −2.

$$-\frac{4}{5} + \frac{2}{-3}$$

$$= \frac{-4}{5} + \frac{-2}{3}$$

I am used to adding fractions when both denominators are positive. I knew that $\frac{2}{-3}$ was the same as $\frac{-2}{3}$ because the quotient of a positive and a negative is negative.

$$= \frac{-12}{15} + \frac{-10}{15}$$

I created equivalent fractions using a common denominator of 15.

$$= \frac{-12 + (-10)}{15}$$

I added the numerators.

$$= -\frac{22}{15}$$

$$= -1\frac{7}{15}$$

Since $-1\frac{7}{15}$ was within my estimate, I thought that my answer was reasonable.

To evaluate an algebraic expression whose given values for the variables are rational number, substitute the given values, and then, follow the order of operations.

EXAMPLE 3 Using the order of operations to evaluate a rational number expression

Evaluate $-2\frac{1}{2}x \div y$ when $x = 5\frac{1}{3}$ and $y = -1\frac{7}{9}$.

Uma's Solution

$$-2\frac{1}{2}x \div y$$

$$= -2\frac{1}{2}\left(5\frac{1}{3}\right) \div \left(-1\frac{7}{9}\right)$$

I substituted the given values for the variables.

I estimated the answer by rounding each mixed number to its nearest integer value and got $-3(5) \div (-2) = 7\frac{1}{2}$.

$$= -\frac{5}{\cancel{2}_1}\left(\frac{\cancel{16}^8}{3}\right) \div \left(-1\frac{7}{9}\right)$$

$$= -\frac{40}{3} \div \left(-1\frac{7}{9}\right)$$

I followed the order of operations the same way I would if the numbers were all integers or fractions.

$$= \frac{-40}{3} \div \frac{-16}{9}$$

$$= \frac{-\overset{-5}{\cancel{40}}}{\underset{1}{\cancel{3}}} \times \frac{-\overset{-3}{\cancel{9}}}{\underset{2}{\cancel{16}}}$$

I divided by multiplying by the reciprocal. I simplified before I multiplied.

$$= \frac{15}{2}$$

$$= 7\frac{1}{2}$$

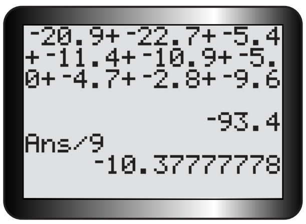
$7\frac{1}{2}$ was the same as my estimate, so I was confident that my answer was correct.

EXAMPLE 4 Solving a problem involving rational numbers

These temperatures were recorded at noon on January 1 from 1998 to 2006 at Ottawa MacDonald-Cartier International Airport. Determine the average noon temperature on January 1 for the years given.

Year	Temperature (°C)
1998	−20.9
1999	−22.7
2000	−5.4
2001	−11.4
2002	−10.9
2003	−5.0
2004	−4.7
2005	−2.8
2006	−9.6

Mark's Solution



I divided the sum of the temperatures by 9, the number of temperature readings, to determine the average noon temperature.

Therefore, the average noon temperature at the Ottawa airport on January 1 is $-10.3\overline{7}$ °C or about -10.4 °C.

I decided to round my answer to one decimal place because all the numbers in the table were given to one decimal place.

In Summary

Key Idea

- The strategies and order of operations you used for calculations with integers, fractions, and decimals can be extended to all rational numbers.

Need to Know

- Calculations with rational number operations may be simpler to perform if you rename mixed numbers as improper fractions, and rewrite negative fractions with the negative sign in the numerator.

CHECK Your Understanding

1. Evaluate without using a calculator.

a) $2.5 - 7.5$

b) $-2(9.5)$

c) $-4.2 + (-2.8)$

d) $\dfrac{8}{-0.5}$

2. Evaluate without using a calculator.

a) $-\dfrac{4}{3} + \dfrac{1}{3}$

b) $\dfrac{3}{4} - \dfrac{5}{4}$

c) $\dfrac{-4}{7} \times \dfrac{6}{-5}$

d) $\dfrac{2}{5} \div \left(-\dfrac{5}{8}\right)$

3. Without evaluating, determine which expressions have the same answer as $\dfrac{3}{4}\left(\dfrac{5}{8}\right)$.

a) $-\dfrac{3}{4}\left(-\dfrac{5}{8}\right)$

b) $\dfrac{-3}{4}\left(\dfrac{5}{-8}\right)$

c) $\dfrac{-3}{4}\left(\dfrac{-5}{8}\right)$

d) $\dfrac{3}{-4}\left(-\dfrac{5}{8}\right)$

PRACTISING

4. Estimate the two consecutive integers between which each answer will lie.

a) $3.64 + 72.9$

b) $-6.5(-10.1)$

c) $-9.37 - 5.93$

d) $-\dfrac{3.046}{10}$

5. Determine the answers in question 4 without using a calculator.

6. The daily changes in selling price for a particular stock during a week were −\$2.78, −\$5.45, \$0.38, \$1.38, and \$2.12.

a) If the selling price of the stock was \$58.22 at the start of the week, then what was the selling price at the end of the week?

b) What was the average daily change in selling price for the stock during this week?

Year	Temperature (°C)
2000	−23.9
2001	−10.0
2002	−7.5
2003	−22.3
2004	−35.7
2005	−14.4

7. The temperature at Moosonee, Ontario on December 25 at 5:00 a.m. from 2000 to 2005 is shown in the table. Determine the average temperature on December 25 at 5:00 a.m. for the given years.

8. Calculate. Show your work.

a) $-\frac{3}{8} + 1\frac{3}{4}$ **c)** $-7\frac{3}{5} + \left(-8\frac{1}{4}\right)$ **e)** $-3\frac{1}{3} - 5\frac{4}{5}$

b) $-5\frac{1}{2} + 2\frac{2}{3}$ **d)** $\frac{6}{5} - \frac{3}{2}$ **f)** $-9\frac{1}{2} - \left(-10\frac{3}{4}\right)$

9. Calculate. Show your work.

a) $\left(\frac{5}{-12}\right)\left(-\frac{8}{15}\right)$ **c)** $3\frac{6}{7}\left(-8\frac{1}{3}\right)$ **e)** $-4\frac{2}{3} \div \frac{7}{12}$

b) $-2\frac{1}{2}\left(-1\frac{3}{5}\right)$ **d)** $\frac{15}{16} \div \left(-1\frac{1}{24}\right)$ **f)** $-2\frac{5}{6} \div \left(-1\frac{1}{12}\right)$

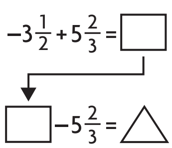
10. Yaroslav takes $\frac{3}{4}$ h to cut his family's front lawn and $1\frac{1}{3}$ h to cut the back lawn. How much longer does it take Yaroslav to cut the back lawn?

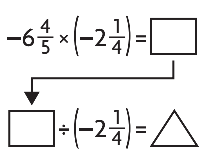
11. Determine the value that makes each equation true.

a) $-1\frac{3}{4} + \blacksquare = 1$ **c)** $-1\frac{3}{4} \times \blacksquare = 1$

b) $-1\frac{3}{4} - \blacksquare = 1$ **d)** $-1\frac{3}{4} \div \blacksquare = 1$

12. a) In each case, determine the numbers represented by the rectangle and the triangle.

i)


ii)


b) Describe the connection between the number represented by the rectangle and the number represented by the triangle.

c) Create a similar question that demonstrates this connection.

13. Without calculating, determine the sign for each answer. Then, use a calculator to complete the calculation.

K

a) $-3.2(4.2 - 10)$

b) $-0.7 - 5.8(12)$

c) $-3.4(-2.3) + 5.7(-9.1)$

d) $6.2(-3.1)(7.3 - 0.9)$

e) $\frac{3.2}{-1.2} + \frac{-4.5}{-6}$

f) $\frac{8.5 - (-2.3)}{2(-1.2)}$

14. Evaluate each expression.

a) $-\frac{2}{5} + \frac{3}{-4} - 2\frac{2}{3}$

b) $-\frac{15}{16} \times 3\frac{1}{5} \div \left(-1\frac{2}{3}\right)$

c) $-2\frac{1}{3} + \left(\frac{3}{-4}\right) \times \left(-1\frac{5}{6}\right)$

d) $-2\frac{1}{4} \times \left(1\frac{3}{4} - 5\frac{1}{2}\right)$

15. The formula to convert temperatures between degrees Fahrenheit and degrees Celsius is $C = \frac{5}{9}(F - 32)$. Apply the formula to convert the following.

a) Miami, Florida's record high of 98 °F to degrees Celsius

b) Anchorage, Alaska's record low of −38 °F to degrees Celsius

c) 0 °C to degrees Fahrenheit

16. The formula to convert Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$. Use this formula to convert the following.

a) The boiling point of water, 100 °C, to degrees Fahrenheit

b) Normal body temperature, 37.0 °C, to degrees Fahrenheit

17. Evaluate each expression for the given values.

a) $x - 2y$ when $x = -9.78$ and $y = 3.2$

b) $(x + y)(x - y)$ when $x = 2.5$ and $y = -7.8$

c) $x(x + y)$ when $x = -2\frac{1}{2}$ and $y = 3\frac{3}{4}$

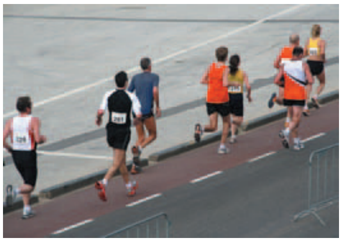
d) $\frac{x}{y} + \frac{y}{x}$ when $x = -1\frac{1}{2}$ and $y = 2\frac{1}{4}$

18. Calculate.

a) $-3.4 + 2\frac{1}{2} - 0.68\left(2\frac{16}{17}\right)$

b) $5.25\left(-2\frac{7}{8}\right) - 8.5\left(-3\frac{3}{4}\right)$

19. James finished a full marathon in a time of 3:57:53.3 (hours:minutes:seconds). The winner of the marathon finished in a time of 2:25:55.6. Determine how much longer James took to complete the marathon than the winner did.



20. **a)** Calculate $-2\frac{3}{5} + 1\frac{1}{4}$ without using a calculator.

b) Calculate $-2\frac{4}{7} + 1\frac{1}{6}$ without using a calculator.

c) Use the decimal equivalents for the fractions in parts a) and b) and evaluate each expression.

d) Would you prefer to do similar calculations using decimal form or fraction form? Explain.

Extending

21. Calculate.

a) $-\left(2\frac{1}{4}\right)^2 + 1.5^3$

b) $-5\frac{2}{3} + 3.\overline{6}(0.\overline{3})^2$

22. $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2}}}$ is an example of a continued fraction.

a) Verify that the value of the continued fraction is $1\frac{3}{5}$.

b) Determine the continued fraction representation for $1\frac{4}{5}$.

Hint: $\frac{4}{5} = \cfrac{1}{\cfrac{5}{4}}$

23. The width of a rectangle is $\frac{1}{4}$ of the length. If you increase the width by 12 m and double the length, you obtain a perimeter of 120 m. Determine the dimensions of the original rectangle.

1.6 Powers of Rational Numbers

GOAL

Evaluate rational number expressions involving powers.

LEARN ABOUT the Math

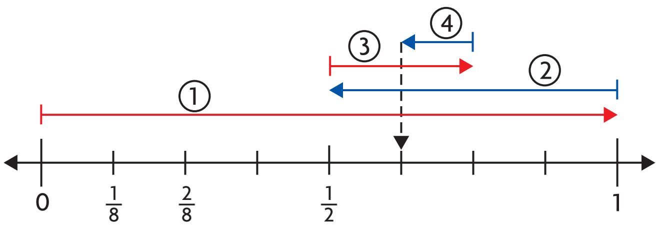
Taylor walks one lap around a track. Then, she turns and walks half as far in the other direction. She changes direction again and walks half as far as her previous distance. She changes direction one last time and again walks only half as far as her last distance.

? What fraction around the track is Taylor's final position?

EXAMPLE 1 Solving a problem involving powers of rational numbers

Determine Taylor's final position on the track.

Haley's Solution: Representing the problem using a number line



I used a number line to model the problem.

I used red arrows to show Taylor's initial direction around the track and blue arrows to show when she walked in the opposite direction.

Therefore, Taylor ended at $\frac{5}{8}$ of the way around the track from where she started.

Keely's Solution: Connecting to powers of integers and fractions

$$1 + \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^3$$

After Taylor's first lap, I expressed each distance as a power of $-\frac{1}{2}$. I used the negative to represent the change in direction. I used powers with base $-\frac{1}{2}$ since she walked half as far each time.

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)$$

I expanded the powers using what I had learned about calculating powers with integer and fraction bases.

$$= 1 + \left(-\frac{1}{2}\right) + \left(\frac{1}{4}\right) + \left(-\frac{1}{8}\right)$$

$$= 1 + \left(-\frac{4}{8}\right) + \left(\frac{2}{8}\right) + \left(-\frac{1}{8}\right)$$

I renamed the fractions using a common denominator of 8 and added them.

$$= 1 + \left(-\frac{3}{8}\right)$$

$$= \frac{5}{8}$$

Therefore, Taylor ended at $\frac{5}{8}$ of the way around the track from where she started.

Because Taylor always walked half of her previous distance, she would end up somewhere in the third quarter of the track. Since $\frac{5}{8}$ was between $\frac{1}{2}$ and $\frac{3}{4}$, my answer seemed reasonable.

Reflecting

A. How is calculating a power with a negative fraction base similar to calculating a power with an integer base?

B. Why did Keely evaluate the powers before she added the rational numbers?

APPLY the Math

EXAMPLE 2 Evaluating an expression with negative decimal bases

Calculate $(-3.2)^2 - 2(-6.5)^3$.

Ahmed's Solution: Using the order of operations

$(-3.2)^2 - 2(-6.5)^3$

$= 10.24 - 2(-6.5)^3$

I followed the order of operations. First I calculated the powers. I knew that multiplying pairs of negatives gave a positive answer.

$= 10.24 - 2(-274.625)$

I knew that, for negative integer bases, if the exponent was odd, the answer would be negative. Also, $6.5^3 = 274.625$.

$= 10.24 - (-549.25)$

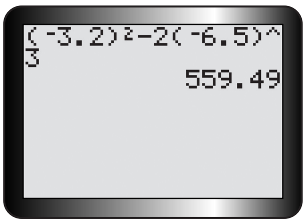
$= 10.24 + 549.25$

$= 559.49$

I multiplied by 2, and then, subtracted by adding the opposite.

When rational numbers are expressed as decimals in an expression, an efficient calculation strategy is to use a calculator.

Faith's Solution: Using a calculator with brackets keys



To calculate $(-3.2)^2 - 2(-6.5)^3$ using my calculator, I needed to put the negative bases within brackets before I could enter the exponents.

Communication *Tip*

If rational numbers are expressed as fractions in an expression, then express your final answer this way as well. Changing fractions into decimal equivalents prior to making calculations can often result in incorrect answers due to rounding errors.

EXAMPLE 3 Evaluating an expression with negative fraction bases

Calculate $-2\frac{2}{3} + \left(-1\frac{3}{4} - \frac{5}{6}\right)^2$.

Ivan's Solution

$-2\frac{2}{3} + \left(-1\frac{3}{4} - \frac{5}{6}\right)^2$ ← To calculate this expression, I needed to follow the order of operations.

$= -2\frac{2}{3} + \left(-\frac{7}{4} - \frac{5}{6}\right)^2$

$= -2\frac{2}{3} + \left(-\frac{21}{12} - \frac{10}{12}\right)^2$

$= -2\frac{2}{3} + \left(-\frac{31}{12}\right)^2$ ← I performed the subtraction within the brackets first.

$= -2\frac{2}{3} + \frac{961}{144}$ ← I calculated the power.

$= -2\frac{2}{3} + 6\frac{97}{144}$ ← I expressed the second fraction as a mixed number.

$= -2\frac{96}{144} + 6\frac{97}{144}$ ← I made sure that the fraction parts had a common denominator before adding.

$= 4\frac{1}{144}$ ← I had to remember that the 96 in the numerator of the first mixed number was also negative.

EXAMPLE 4 Solving a problem involving powers of rational numbers

Josée worked at the mall this summer to help pay for her future university education. She invested \$3000 in an account earning interest at a rate of 3.5% per year. How much money will her investment be worth in 4 years?

Invest to Earn

The Magic of Compounding

$$A = P(1 + i)^n$$

A = future value of investment
P = amount of money invested
i = decimal value of the interest rate used each time interest is earned
n = number of times interest is earned while money is invested

Josée's Solution

$$A = P(1 + i)^n$$
$$= 3000(1 + 0.035)^4$$

I substituted values for the variables.
$P = 3000$ because that is the amount of money that I invested.
$i = 0.035$ because I had to change the interest rate given as a percentage to decimal form.
$n = 4$ because I receive interest once a year for 4 years.

Therefore, my investment will be worth \$3442.57 in four years.

I rounded my answer to the nearest penny because the problem is about money.

In Summary

Key Idea

- Powers with rational bases are calculated in the same way as powers with integer bases.

Need to Know

For exponent $n = 1, 2, 3$, and so on:	Example
$\left(\frac{a}{b}\right)^n = \underbrace{\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\dots\left(\frac{a}{b}\right)}_{[n \text{ factors}]}$	$\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{8}{27}$
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$
$-\left(\frac{a}{b}\right)^n = -\underbrace{\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\dots\left(\frac{a}{b}\right)}_{[n \text{ factors}]}$	$-\left(\frac{2}{3}\right)^3 = -\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = -\frac{8}{27}$
$\left(\frac{-a}{b}\right)^n = \underbrace{\left(\frac{-a}{b}\right)\left(\frac{-a}{b}\right)\dots\left(\frac{-a}{b}\right)}_{[n \text{ factors}]}$	$\left(\frac{-2}{3}\right)^3 = \left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right)\left(\frac{-2}{3}\right) = \frac{-8}{27}$
If $\frac{a}{b} < 0$, then $\left(\frac{a}{b}\right)^n$ is positive if n is even, and negative if n is odd.	$\left(-\frac{2}{3}\right)^2 = \frac{4}{9}$; $\left(-\frac{2}{3}\right)^3 = -\frac{8}{27}$

CHECK *Your Understanding*

1. Use $4.5^2 = 20.25$ and $4.5^3 = 91.125$ to evaluate the powers.

a) $(-4.5)^2$ **b)** -4.5^2 **c)** $(-4.5)^3$ **d)** -4.5^3

2. Without evaluating, state if the answer is positive or negative.

a) $\left(-\frac{2}{3}\right)^2$ **c)** $-\left(-\frac{2}{3}\right)^2$ **e)** $-\left(\frac{2}{3}\right)^3$

b) $-\left(\frac{2}{3}\right)^2$ **d)** $-\left(-\frac{2}{3}\right)^3$ **f)** $\left(\frac{-2}{-3}\right)^5$

3. Evaluate the powers in question 2.

4. Use the expression $-4.5 + 2(3.1 - 9.8)^2$ to answer the following.
a) Describe the order of operations required to calculate the answer.
b) Calculate the answer.

PRACTISING

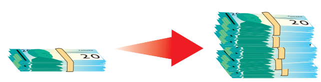
5. Calculate.
K **a)** $8.9 - 3.2^2$
b) $-2(-3.1)^3$
c) $-7.1^2 + 7.1^2$
d) $0.6^2 - 2(3.4 - 5.2)$
e) $-6.02 - 2(-6.71) + 2.3^3$
f) $\dfrac{2.3^3 - 5.4}{-3^2}$

6. Calculate.
a) $2\left(-\frac{1}{3}\right)^2$
b) $-2\frac{1}{3} - \left(-\frac{2}{3}\right)^3$
c) $-\left(\frac{4}{5}\right)^2 + \left(\frac{5}{4}\right)^2$
d) $\left(-4\frac{1}{5}\right)^2\left(\frac{25}{4}\right)^2$
e) $\dfrac{-\frac{1}{3} + \left(\frac{1}{4}\right)^2}{-1\frac{1}{6}}$
f) $\left(-\left(2\frac{1}{2}\right)^2\right)^2$

7. Determine the value that makes each equation true. Explain how you got your answers.
a) $2.4^{\blacksquare} = 5.76$
b) $-2.4^{\blacksquare} = -5.76$
c) $(3.5)^{\blacksquare} = 42.875$
d) $\blacksquare^3 = -42.875$

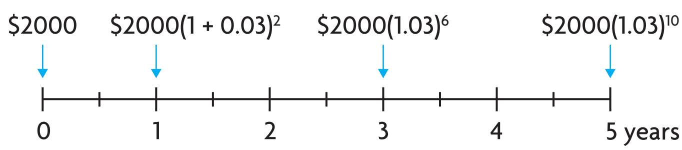
8. Rob invests \$100 in an account earning interest at a rate of 5% per year for 10 years. Sharon invests the same amount of money as Rob but she earns interest at a rate of 10% per year for 5 years.
a) Predict whose investment would be worth more in the end and explain why.
b) Calculate the value of both investments.

Invest your money!



$A = P(1 + i)^n$

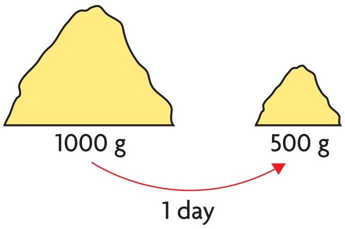
9. Diego invested \$2000. The amount of money in his account is shown over 5 years.
a) Explain why the exponent in the expression at the end of 5 years is 10.
b) Determine the amount in his account at the end of 5 years.



10. Tanjay invests \$100 and earns interest at a rate of 4% per year for 10 years. Eda invests \$100 and earns interest at a rate of 2% every 6 months for 10 years.

a) Calculate the value of both investments.

b) Explain why Eda's investment is worth more at the end of 10 years.



11. A radioactive material has a half-life of 1 day. The material decays according to the equation $M = 1000\left(\frac{1}{2}\right)^t$. Mass M is measured in grams and time t is measured in days.

A

a) What is the mass of the sample after 1 day? 2 days? 10 days?

b) Use a calculator with an exponent key to compute the mass of the sample after 1 year. Explain what the answer means.

12. Evaluate the following expressions.

a) $x^2 - 4x + 3$ when $x = -2.5$

b) $3x^2 + 5x - 12$ when $x = -1.5$

c) $x^3 - 5x$ when $x = 0.5$

d) $4x^3 + 4x^2$ when $x = -4.2$

13. Evaluate the following expressions.

a) $x^2 + x - 5$ when $x = 1\frac{3}{4}$

b) $x^3 + 6x^2$ when $x = -\frac{1}{3}$

c) $5x^2 - 3x + 9$ when $x = -\frac{11}{5}$

d) $x^2 - 3x + 6$ when $x = -2\frac{1}{2}$

14. Explain why $-\left(1\frac{1}{2}\right)^3 = \left(-1\frac{1}{2}\right)^3$, but $-\left(1\frac{1}{2}\right)^4 \neq \left(-1\frac{1}{2}\right)^4$.

15. What must you consider when applying the order of operations to evaluate rational number expressions, that you don't need to consider when evaluating whole number expressions?

C

Extending

16. A small office buys a computer for \$4575. Each year, its value is expected to be 65% of its value the previous year. Find the value of the computer after five years.

T

17. Determine two values for x that will make the following equation true.

$$5\left(\frac{x}{3}\right)^2 - 3\frac{1}{9} = -\frac{8}{9}$$

1 Chapter Review

FREQUENTLY ASKED Questions

Q: What strategies can you use to evaluate an expression involving rational numbers?

A: You can extend the same strategies and order of operations (BEDMAS) used for calculations with integers, fractions, and decimals to all rational numbers.

EXAMPLE

$$\frac{2\left(-1\frac{3}{4}+1\frac{3}{5}\right)^2}{\left(-\frac{1}{10}\right)^3} = \frac{2\left(-1\frac{15}{20}+1\frac{12}{20}\right)^2}{\left(-\frac{1}{10}\right)^3}$$

$$= \frac{2\left(-\frac{3}{20}\right)^2}{\left(-\frac{1}{10}\right)^3}$$

$$= \frac{2\left(\frac{9}{400}\right)}{-\frac{1}{1000}}$$

$$= \frac{\frac{9}{200}}{-\frac{1}{1000}}$$

$$= \frac{9}{\underset{1}{\cancel{200}}} \times \frac{\overset{-5}{\cancel{-1000}}}{1}$$

$$= -45$$

Study *Aid*

- See Lesson 1.6, Examples 1 and 3.
- Try Chapter Review Question 14.

PRACTICE Questions

Lesson 1.1

1. Calculate without using a calculator.

a) $1\frac{2}{3} + 9\frac{1}{2}$

b) $8\frac{1}{6} - 7\frac{2}{3}$

c) $4\frac{3}{8} + 2\frac{1}{4}$

d) $5\frac{5}{6} - 3\frac{3}{4}$

2. A piece of wood $8\frac{7}{8}$ in. long is cut from a piece $45\frac{1}{2}$ in. long. If $\frac{1}{16}$ in. is wasted for the cut, how much wood is left?

3. Stock shares of Champs Sporting Equipment opened at $12\frac{1}{8}$ and closed at the end of the day at $9\frac{1}{2}$. Calculate the change in the stock on this day.

Lesson 1.2

4. Calculate without using a calculator.

a) $1\frac{3}{4} \times 3\frac{1}{2}$

b) $5\frac{7}{9} \times 6\frac{3}{4}$

c) $1\frac{2}{3} \div 4\frac{5}{6}$

d) $4\frac{3}{4} \div 9\frac{1}{2}$

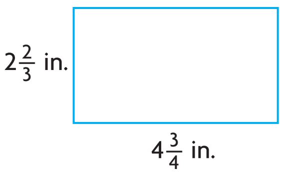
5. Calculate $\left(2\frac{2}{5}\right)^2$.

6. Determine the volume of a cube that has a side length of $1\frac{3}{4}$ m.

7. For the rectangle shown calculate:

a) the perimeter

b) the area



Lesson 1.3

8. Use words to explain the different steps you would take to evaluate -8^2 and $(-8)^2$.

9. Evaluate.

a) $(-8 + 2)^2 \div (-4 + 2)^2$

b) $\dfrac{(-16 + 4) \div 2}{8 \div (-8) + 4}$

c) $16 - [3(6 - 3) - 12]$

d) $\dfrac{20 + (-12) \div (-3)}{(-4 - 12) \div (-2)}$

10. Evaluate.

a) $x^2 - 4x$ for $x = -3$

b) $yx^2 + xy$ for $x = -4$ and $y = 5$

c) $\dfrac{-x^4 - 5x}{x + (-1)^3}$ for $x = -2$

d) $\dfrac{-x^2 - y^2}{x^2 + y^2}$ for $x = 2$ and $y = 3$

Lesson 1.4

11. Explain where each value is located on a number line.

a) -2.6

b) $-\frac{24}{5}$

12. Which is a negative rational number between -10 and -9? How do you know?

a) $-\frac{29}{3}$

b) $-\frac{31}{3}$

13. The temperature in Powassan was -4.8 °C. The temperature in Callander was $-4\frac{5}{6}$ °C. In which town was the temperature colder? Explain.

14. Write these rational numbers in order from least to greatest.

a) $\frac{-3}{5}, \frac{1}{-3}, -1\frac{1}{3}$

b) $-\frac{2}{5}, -2\frac{1}{5}, \frac{4}{5}$

c) $0.7, -0.3, -0.\overline{3}$

d) $0, -1.5, -2$

15. Use $>$, $<$, or $=$ to make true statements. Explain how you know each statement is true.

a) $\frac{-2}{3} \blacksquare -\frac{5}{6}$

b) $\frac{2}{3} \blacksquare \frac{5}{8}$

c) $-2\frac{1}{4} \blacksquare -\frac{9}{4}$

d) $\frac{2}{-5} \blacksquare \frac{3}{10}$

Lesson 1.5

16. The daily changes in selling price for a particular stock during a week were: $-\$4.50$, $-\$0.95$, $\$0.25$, $-\$2.36$, and $-\$3.72$. What was the average daily change in selling price for the stock during this week?

17. Calculate. Show your work.

a) $2\frac{1}{4} - 5\frac{1}{3}$

b) $-5\frac{2}{5} + 2\frac{3}{4}$

c) $-6\frac{3}{4}\left(5\frac{1}{9}\right)$

d) $1\frac{3}{4} \div \left(-\frac{30}{49}\right)$

18. Create two other expressions that give the same answer as $-1\frac{3}{4}\left(5\frac{1}{3}\right)$.

19. Calculate.

a) $6.4 - 4.2 \times 1.5$

b) $-12.4 + (-16.8) \div (-4.2)$

c) $\frac{15.3 + 2.7 \div 3}{-2 \times 8.1}$

d) $\frac{16 - 4.8 \times 2.1}{6 + 6 \div (-6)}$

20. Calculate.

a) $\frac{2}{5} \div \left(\frac{-2}{5} + \frac{1}{10}\right)$

b) $\frac{-5}{6} + \frac{-2}{3} \times \frac{3}{4}$

c) $\left[\frac{1}{8} + \left(\frac{-2}{3}\right)\right] \times \frac{12}{13}$

d) $-1\frac{1}{2} + \frac{-1}{-2} - \frac{-3}{5}$

Lesson 1.6

21. Calculate.

a) $[5.12 - 3(4.1)]^3$

b) $9.1^3 - 6.7^2$

c) $-2\frac{1}{10} + \left(2\frac{3}{5} - 3\frac{1}{4}\right)^3$

d) $-\frac{1}{4} \div \frac{5}{4} - 2\frac{1}{3} \div \left(-\frac{2}{3}\right)^3$

22. Mikka invests \$100 in an account earning interest at a rate of 4% every 6 months. Calculate the value of his investment at the end of 4 years.

23. Use $>$, $<$, or $=$ to make true statements. Explain how you know each statement is true.

a) $\left(\frac{1}{-2}\right)^3 \blacksquare \left(\frac{1}{2}\right)^2$

b) $\left(\frac{3}{4}\right)^2 \blacksquare \left(-\frac{1}{4}\right)^3$

c) $(-0.5)^2 \blacksquare \left(\frac{1}{2}\right)^2$

d) $\left(\frac{3}{2}\right)^3 \blacksquare \left(\frac{3}{-2}\right)^4$

24. The area of a circle can be calculated using the formula $A = \pi r^2$, where $\pi \doteq 3.14$.

Calculate the area of each circle for each of the given radii. Round to the nearest tenth of a square unit.

a) $r = 5.2$ cm

b) $r = 2\frac{5}{8}$ in.

c) $r = 8.9$ m

d) $r = 4\frac{2}{3}$ in.

25. Evaluate each expression for the given values.

a) $4a^2b^2$; $a = \frac{-2}{3}$, $b = -\frac{1}{2}$

b) $(2ab)^2$; $a = -0.5$, $b = 1.2$

c) $\left(\frac{2a}{5b}\right)^2$; $a = 1\frac{1}{2}$, $b = -\frac{2}{5}$

d) $(3a - 2b)^3$; $a = -1.1$, $b = 2.2$

1 Chapter Self-Test

1. Which value is equivalent to $\frac{-4}{-5}$?

A. $\frac{-4}{5}$ **B.** $-\frac{4}{5}$ **C.** $\frac{4}{-5}$ **D.** $\frac{4}{5}$

2. What does $-3^2 - (-1)^2$ equal?

A. -8 **B.** 8 **C.** -10 **D.** 10

3. Which set of numbers is arranged in ascending order (least to greatest)?

A. $-\frac{11}{5}, \frac{-11}{-5}, -2\frac{2}{5}$

B. $\frac{-11}{-5}, -\frac{11}{5}, -2\frac{2}{5}$

C. $-2\frac{2}{5}, -\frac{11}{5}, \frac{-11}{-5}$

D. $-2\frac{2}{5}, \frac{-11}{-5}, -\frac{11}{5}$

4. Calculate without using a calculator.

a) $5\frac{1}{2} + 4\frac{2}{7}$ **b)** $4\frac{7}{12} \times 1\frac{4}{11}$

5. A piece of wood trim is $51\frac{3}{8}$ in. long. A piece $31\frac{5}{6}$ in. long is cut from it. How long is the remaining piece of wood if the cut removes $\frac{1}{8}$ in. of wood?

6. Explain how to calculate $(-2)^2$ and -2^4. Then, calculate each expression.

7. Evaluate the expression $-3x^2 + y^3$ for each situation below.

a) when $x = -4$ and $y = -2$ **b)** when $x = 0.4$ and $y = -1.1$

8. Which value is farther from zero: $-4\frac{1}{3}$ or 4.3? Explain.

9. Calculate.

a) $\left(\frac{-4}{7}\right) - \left(-2\frac{1}{2}\right)$ **b)** $-2\frac{2}{3} \div \frac{3}{4}$

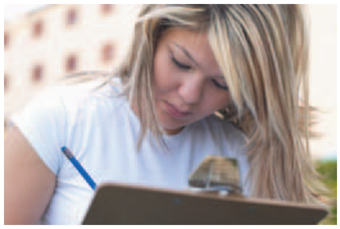
10. The high temperatures for a city during a five-day period were 4.5 °C, 2.3 °C, −3.2 °C, −11.7 °C, and −9.8 °C. Determine the average temperature during the five-day period.

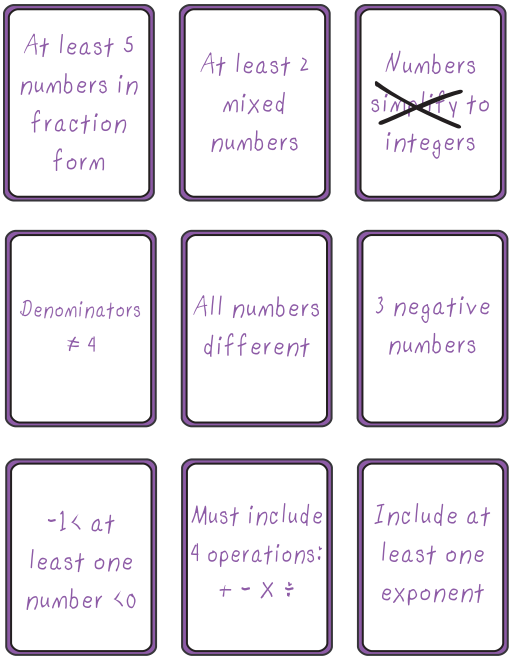
11. Evaluate $\left[-\frac{2}{3}\left(1\frac{4}{5}\right)\right]^2 + 2\left(-1\frac{1}{2}\right)^3$. Show your work.

1 Chapter Task

Be Rational and Concentrate!

Mariel is creating a rational number concentration game. She is making "condition cards" for players to use to create numerical expressions that equal various numbers. She decides to test her cards using the number $-2\frac{3}{4}$. She finds a card whose condition can't be met to create an expression that equals this number. She places an "x" through this card.





? Which of Mariel's condition cards could you use to create an expression that equals $-2\frac{3}{4}$?

A. Create an expression that will equal $-2\frac{3}{4}$ when calculated, using as many of the condition cards above as possible. You get bonus points for using all of the condition cards!

B. Evaluate your expressions. Show your work.

C. Using only decimals, create an expression that evaluates to -5.6. Provide a minimum of five of your own "condition cards" that would describe the numbers and operations used in your expression.

D. Explain the process you used to create your expression.

Task *Checklist*

- ✔ Did you check to make sure that all of the conditions are satisfied?
- ✔ Did you show all of the steps in your calculations?
- ✔ Did you check to make sure that your calculations are correct?
- ✔ Did you explain your thinking clearly?