

1.4

Rational Numbers

Traditionally, sets of numbers have been represented by letters. The symbol Q is used for rational numbers because they are a *quotient* of two integers.

Set	Definition	Examples	Symbol
Natural Numbers	the counting numbers	1, 2, 3, ...	N
Whole Numbers	the counting numbers and zero	0, 1, 2, 3, ...	W
Integers	positive and negative whole numbers	..., -3, -2, -1, 0, 1, 2, 3, ...	I
Rational Numbers	numbers of the form $\frac{a}{b}$ where a and b are integers and $b \neq 0$	$\frac{3}{4}$, $-\frac{2}{3}$, $-3\frac{5}{8}$, 2.35, -3.921, $-8.\overline{234}$	Q

GOAL

Connect rational numbers to other number systems.

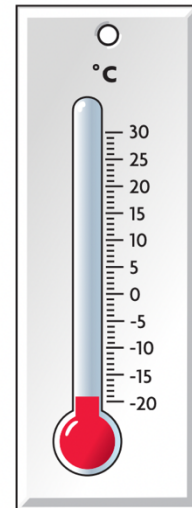
LEARN ABOUT the Math

Shahreen looked at the thermometer outside. The temperature was a **rational number** between -18°C and -19°C . It was closer to -19°C .

? What might the temperature be?

EXAMPLE 1 Using a number line to represent rational numbers

Determine a possible temperature value that is between -18°C and -19°C , but is closer to -19°C .



EXAMPLE 2 Representing rational numbers as decimals

Which of the following represent the same rational number?

$$\frac{-2}{3}, \frac{2}{-3}, -\frac{2}{3}, \frac{3}{-2}, \frac{-3}{-2}, \frac{3}{2}$$

EXAMPLE 3 Representing rational numbers using a number line

Determine which number is greater: $-1\frac{3}{10}$ or $-1\frac{2}{5}$.

Practice:

8. Explain why $-3\frac{1}{4}$ can be renamed as $-3 - \frac{1}{4}$ and not as $-3 + \frac{1}{4}$.

10. True or false? Justify your answer.

- T** a) All mixed numbers can be renamed as decimals.
b) A rational number can be expressed as any integer divided by any other integer.
c) Two rational numbers are opposites if they have different signs.
d) $1 > -1\,000\,000$

11. To write $0.833\,333\dots$ as a fraction, Rhys thought of this as:

A $0.8 + \frac{1}{10}$ of $0.333\,333\dots$

This is the same as:

$$\frac{8}{10} + \frac{1}{10} \times \frac{1}{3}$$

$$= \frac{8}{10} + \frac{1}{30}$$

$$= \frac{24}{30} + \frac{1}{30}$$

$$= \frac{25}{30}$$

$$= \frac{5}{6}$$

Use Rhys's approach to write each of the following as fractions.

a) $0.4333\dots$

b) $0.1\overline{6}$

c) $0.25\overline{3}$

1.5

Rational Number Operations

GOAL

Evaluate expressions involving rational numbers.

LEARN ABOUT the Math

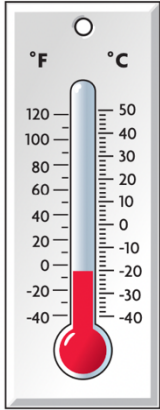
Matthew was chatting online with his friend Bruce, who lives in the United States. Bruce said that the temperature outside was -5.5 degrees Fahrenheit. Matthew was not sure how cold that was because he was used to temperature readings measured in degrees Celsius.

He found the following conversion formula from a weather website:

$$C = \frac{5}{9}[F - 32]$$

where C is the temperature in degrees Celsius and F is the temperature in degrees Fahrenheit.

? What is the Celsius temperature equivalent to -5.5 °F?



$$\begin{aligned} \text{C temperature} \\ = \frac{5}{9} (\text{F temperature} - 32) \end{aligned}$$

EXAMPLE 1

Evaluating a rational number expression

Determine the Celsius temperature equivalent to -5.5 °F.

EXAMPLE 2

Connecting the addition of rational numbers to adding fractions and integers

Calculate $-\frac{4}{5} + \frac{2}{-3}$.

EXAMPLE 3**Using the order of operations to evaluate a rational number expression**

Evaluate $-2\frac{1}{2}x \div y$ when $x = 5\frac{1}{3}$ and $y = -1\frac{7}{9}$.

Practice:

- 13.** Without calculating, determine the sign for each answer. Then, use a **K** calculator to complete the calculation.

a) $-3.2(4.2 - 10)$

d) $6.2(-3.1)(7.3 - 0.9)$

b) $-0.7 - 5.8(12)$

e) $\frac{3.2}{-1.2} + \frac{-4.5}{-6}$

c) $-3.4(-2.3) + 5.7(-9.1)$

f) $\frac{8.5 - (-2.3)}{2(-1.2)}$

- 14.** Evaluate each expression.

a) $-\frac{2}{5} + \frac{3}{-4} - 2\frac{2}{3}$

c) $-2\frac{1}{3} + \left(\frac{3}{-4}\right) \times \left(-1\frac{5}{6}\right)$

b) $-\frac{15}{16} \times 3\frac{1}{5} \div \left(-1\frac{2}{3}\right)$

d) $-2\frac{1}{4} \times \left(1\frac{3}{4} - 5\frac{1}{2}\right)$

Challenge questions:

22. $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}$ is an example of a continued fraction.

- a) Verify that the value of the continued fraction is $1\frac{3}{5}$.

- b) Determine the continued fraction representation for $1\frac{4}{5}$.

Hint: $\frac{4}{5} = \frac{1}{\frac{5}{4}}$

- 23.** The width of a rectangle is $\frac{1}{4}$ of the length. If you increase the width by 12 m and double the length, you obtain a perimeter of 120 m. Determine the dimensions of the original rectangle.