MPM1D

UNIT #2 PART 1: POWERS

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GETTING READY SECTION – PART 1

I) Words You Need To Know / Definitions :

1. **Power** : A power represents a repeated multiplication.
   Ex. $2 \times 2 \times 2 \times 2 = 2^4$ ➜ 2^4 is a power

2. **Exponent** : The number of times a number is multiplied.
   Ex. 2^4 ➜ 4 is the exponent

3. **Base** : The number that is repeatedly multiplied.
   Ex. 2^4 ➜ 2 is the base

4. **Perfect Square** : A number whose square root is a whole number.
   Ex. 16 is a perfect square because $\sqrt{16} = 4$, since $4 \times 4 = 16$

Examples :

1. Represent each repeated multiplication as a power.

   a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ b) $(-6)(-6)(-6)(-6)$ c) $4 \times 3 \times 4 \times 3 \times 4 \times 3$

2. Represent each power using repeated multiplication.

   a) 2^6 b) $(-2)^6$ c) -2^6

3. Evaluate the following powers.

   a) 3^4 b) $(-3)^4$ c) -3^4 d) $\left(\frac{1}{3}\right)^4$

4. Write the first 15 perfect squares.

2.1 REPRESENTING POWERS UP TO DEGREE 3

Connecting Squares and Square Roots :

Squares : When you square a number, you multiply it by itself.

Ex. $5^2 = 5 \times 5$

$= 25$

Square Roots : A number that when multiplied by itself produces the given number.

Ex. The square root of 9 is 3, because $3 \times 3 = 9$. However, $-3 \times -3 = 9$ as well.

So the square root of 9 is actually 3 and -3.

Radical sign $\sqrt{\ }$: The $\sqrt{\ }$ sign indicates only the positive square root.

Ex. $\sqrt{9} = 3$

* All positive numbers have square roots. The square root of a negative number is undefined.

$\sqrt{-x} =$ undefined

Perfect Squares : Numbers with square roots that are integers (positive and negative whole numbers)

Ex. $\pm\sqrt{64} = \pm 8$, therefore 64 is a perfect square.

Ex. $\pm\sqrt{60} \nabla \pm 7.745966692$, therefore 60 is NOT a perfect square.

Examples :

1. Simplify the following. Round answers to 2 decimal places where necessary.

a) $\sqrt{81} + \sqrt{25}$ b) $\sqrt{81 + 25}$ c) $\sqrt{\sqrt{81} + \sqrt{25}}$ d) $\sqrt{0.81}$

e) $\sqrt{0.04}$ f) $\sqrt{1.44}$ g) $-\sqrt{49}$ h) $\sqrt{-49}$

i) $3\sqrt{49}$ j) $\left(\sqrt{4}\right)\left(\sqrt{9}\right)$

2. State the pair of whole numbers that each of the following square roots lie between. Then estimate the value of the square root to one decimal place. Check your answer with a calculator.

   a) $\sqrt{50}$ b) $\sqrt{95}$ c) $\sqrt{12}$ d) $\sqrt{120}$

3. Determine the area of a square with side length of 6cm.

4. Calculate the length of the side of the square with each of the following areas. Round to one decimal place where necessary.

   a) $A = 3600mm^2$ b) $A = 2.25m^2$ c) $A = 40cm^2$

5. The side length of a square is $\sqrt{7.5}$ cm. What is the area of the square?

6. Determine the volume of the cube with a side length of 5cm.

2.2 MULTIPLYING & DIVIDING POWERS

Recall : $2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Expanded Form — Product Form

I) Multiplying Powers :

Example : Fill in the chart below.

PRODUCT OF POWERS	EXPANDED/PRODUCT FORM	POWER FORM
$2^4 \times 2^3$	$(2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2)$	2^7
$3^2 \times 3^6$		
$m^5 \times m$		

* **Exponent Law for Multiplying Powers :**

➜ To multiply powers with the same base, keep the base and add the exponents.

$m^a \times m^b = m^{a+b}$, where a and b are integers

Example : Simplify the following. (i.e. Write the following powers as a single power.

a) $a^5 \times a^3$ b) $(x^7)(x^4)$

II) Dividing Powers :

Example : Fill in the chart below.

DIVISION OF POWERS	EXPANDED/PRODUCT FORM	POWER FORM
$10^5 \div 10^3$	$\frac{10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$	10^2
$6^7 \div 6^2$		
$m^6 \div m^5$		

* **Exponent Law for Dividing Powers :**

➜ To divide powers with the same base, keep the base and subtract the exponents.

$m^a \div m^b = m^{a-b}$, where a and b are integers

Example : Simplify the following. (i.e. Write the following powers as a single power.

a) $m^7 \div m^3$ b) $\frac{x^5}{x^2}$

Examples :

1. Write the following powers as a single power. Then evaluate.

a) $2^2 \times 2^6$ b) $3^7 \div 3^4$ c) $(-5)^3(-5)$ d) $\frac{(-7)^7}{(-7)^5}$

e) $\frac{8^2}{8^2}$ f) $\frac{\left(\frac{3}{4}\right)^7\left(\frac{3}{4}\right)}{\left(\frac{3}{4}\right)^6}$ g) $\frac{4^3 \times 4^6}{4^8}$ h) $\frac{6^9}{6^7} \times \frac{6^4}{6^3}$

2. Simplify first, then evaluate for $m = 2$ and $n = -3$.

a) $(a)(b^2)(a^3)(b^0)$ b) $\frac{4^2(m^6)(n^5)}{(-2)^3(m^2)(n^3)}$ c) $\frac{\left(\frac{2}{3}\right)^5 x^4 y^2}{\left(\frac{2}{3}\right)^2 xy}$

3. Express each number as a power of 2.

a) 4 b) 64 c) 256 e) $\frac{1}{4}$ f) $\frac{1}{64}$

4. Given that the product of two powers is 6^8 and the quotient is 6^2, determine what the two powers could be. How could you verify your answer?

ZERO AND NEGATIVE EXPONENTS

Recall : $m^a \times m^b = m^{a+b}$ and $m^a \div m^b = m^{a-b}$

Example : Simplify the following using the Exponent Laws.

a) $(x^{-3})(x^3)$ b) $\frac{a^5}{a^5}$ c) $(n^6)(n^{-8})$ d) $\frac{c^4}{c^7}$

I) Exponent of Zero :

Example : Evaluate first, then simplify by using the Exponent Laws. Compare your answers.

a) $\frac{2^6}{2^6}$ b) $\frac{5^2}{5^2}$

* **Zero Exponent** :

➜ Anything to the exponent of zero is equal to one.

$m^0 = 1$, where $m \neq 0$

II) Negative Exponents :

Example : Evaluate first, then simplify by using the Exponent Laws. Compare your answers.

a) $\frac{2^2}{2^5}$ b) $\frac{3^3}{3^4}$

* **Negative Exponents** :

➜ To evaluate a negative exponent, take the reciprocal of the base (i.e. "flip" the base) and change the exponent to a positive.

$m^{-a} = \left(\frac{1}{m}\right)^a$ or $\frac{1}{m^{-a}} = m^a$, where $m \neq 0$

Examples :

1. Evaluate the following powers. NO decimals!

a) $(-11)^0$ b) 2^{-7} c) $(-2)^{-7}$ d) $\frac{1}{8^{-2}}$

e) $\frac{1}{(-8)^{-2}}$ f) $\left(\frac{3}{4}\right)^{-3}$ g) $\left(\frac{-2}{5}\right)^{-2}$ h) $(-4)^{-1}+9^0$

2. Write the following powers as a single power first, then evaluate. NO decimals!

a) $8^5 \times 8^{-6}$ b) $6^{-7} \div 6^{-9}$

c) $\frac{5^2 \times 5^0 \div 5^{-3}}{5^3 \times 5^4}$ d) $\frac{3^3}{3^9} \times \frac{3^7}{3}$

ZERO AND NEGATIVE EXPONENTS WORKSHEET

1. Evaluate the following powers.

   a) 7^0 b) -7^0 c) $(-7)^0$

2. Write each power with a positive exponent. Then evaluate the power. NO decimals!

   a) 3^{-1} b) 6^{-2} c) -2^{-4} d) $(-2)^{-4}$

   e) $\left(\frac{1}{5}\right)^{-3}$ f) $\frac{1}{4^{-4}}$ g) $\left(\frac{-1}{5}\right)^{-3}$ h) $\frac{-1}{4^{-4}}$

   i) $\left(\frac{-3}{4}\right)^{-2}$ j) $\left(\frac{-3}{4}\right)^{-3}$ k) $\frac{-1}{(-2)^{-3}}$ l) $\frac{-5}{(-2)^{-4}}$

3. Write each expression as a single power with positive exponents. Then evaluate. NO decimals!

   a) $8^{-5} \times 8^{-2} \times 8^{9}$ b) $\frac{3^{-3} \times 3^{0}}{3}$

Answer Key :

1. a) 1 b) -1 c) 1
2. a) 1/3 b) 1/36 c) -1/16 d) 1/16 e) 125 f) 256 g) -125 h) -256 i) 16/9 j) -64/27 k) 8 l) -80
3. a) 64 b) 1/81

2.3 POWER OF A POWER

Recall : Repeated multiplication can be written as a single base to an exponent, i.e. it can be written as a power.
Ex. $2 \times 2 \times 2 = 2^3$

Example : Fill in the chart below.

POWER OF A POWER	EXPANDED/PRODUCT FORM	POWER FORM
$(5^3)^2$	$(5^3)(5^3) = (5 \times 5 \times 5)(5 \times 5 \times 5)$	5^6
$(4^2)^5$		
$(m^4)^3$		

* **Exponent Law for a Power of a Power :**

➜ To evaluate the power of a power, leave the base and multiply the exponents.

$(m^a)^b = m^{ab}$, where a and b are integers

➜ To evaluate the power of a product/quotient, leave the base and multiply the exponents of the product/quotient with the outside exponent.

$(m^a n^b)^c = m^{ac} n^{bc}$, where $a, b,$ and c are integers

or

$$\left(\frac{m^a}{n^b}\right)^c = \frac{m^{ac}}{n^{bc}}$$

Examples :

1. Simplify the following powers, i.e. write as a single power.

a) $(9^3)^5$ b) $(x^{-2})^7$ c) $(-4m^3)^2$ d) $(a^4b^5)^6$ e) $(2^2xy^3)^4$

2. Simplify first, i.e. write as a single power. Then evaluate. NO decimals!

a) $(2^4)^2$ b) $(4^{-1})^3$ c) $(7^0)^{-6}$

d) $(3^{-3})^{-2}$ e) $[(-1)^5]^7$ f) $[(-5)^2]^{-1}$

g) $(5^2)^4 \times (5^3)^{-2}$

h) $\dfrac{(8^{-4})^4}{(8^5)^{-3}}$

i) $\dfrac{(4^{12}4^5)^3}{(4^74)^6}$

3. Simplify the following. Then evaluate. NO decimals!

a) $(-3m^5)^4$

b) $\dfrac{(2a^5)^3(2^7ab^6)^2}{(2^3a^2b)^4}$

c) $(7^4)^5 - (7^2)^{10}$

d) $\dfrac{(4^3)^5(6^8)}{(4^6)^2(6^3)^3}$

e) $\dfrac{(5m^{-2}n^{-7})^{-3}}{(5^{-1}mn^6)^4}$

f) $\dfrac{(2^5)^{-2}}{(2^{-3})^{-4}} \times \left(\dfrac{2^7}{(2^{10})^2}\right)^{-1}$

4. Find each value of x.

a) $(x^2)^3 = 64$

b) $(x^{-5})(x) = \dfrac{1}{81}$

c) $(3^3)^x = 729$

d) $\dfrac{4^x}{4^7} = 16$

SCIENTIFIC NOTATION

* Scientific notation is used to represent **very large** or **very small** numbers.

* To write a number in scientific notation, write it as the **product** of :
  - ➜ a number between 1 and 10 or -1 and -10, and
  - ➜ a power of 10

$m \times 10^n$

where m is a number between 1 and 10 or -1 and -10, and n is an exponent

Examples :

1. Write the following numbers in scientific notation.

   a) 120 000 000 000
   (# of stars in our galaxy)

   b) 0.000 000 040 3

   c) 0.000 000 000 000 000 000 000 001 67
   (mass, in grams, of a hydrogen atom)

   d) 70 000

   e) 405 000 000

   f) 0.000 009

2. Write the following in decimal/numeral notation.

   a) 4.91×10^{7}

   b) 6.08×10^{-5}

   c) -5×10^{10}

   d) -1.32×10^{-4}

3. Simplify using the Exponent Laws and Scientific Notation. Make sure your final answer is in proper scientific notation form.

   a) $(4.0 \times 10^{-3}) \times (3.6 \times 10^{-5})$

   b) $\dfrac{1.768 \times 10^{17}}{5.2 \times 10^{6}}$

SCIENTIFIC NOTATION WORKSHEET

Simplify using the Exponent Laws and Scientific Notation. Make sure your final answer is in proper scientific notation form.

1. $(5\times10^{3})\times(3\times10^{4})$

2. $(9.2\times10^{-2})\times(3.6\times10^{10})$

3. $\dfrac{4.5\times10^{5}}{6.0\times10^{8}}$

4. $(8.2\times10^{-2})\div(1.6\times10^{4})$

5. $(-7.05\times10^{11})\times(15.2\times10^{4})$

6. $\dfrac{-0.065\times10^{-12}}{-8.0\times10^{-7}}$

Answer Key :

1. 1.5×10^{8} 2. 3.312×10^{9} 3. 7.5×10^{-4} 4. 5.125×10^{-6} 5. -1.0716×10^{17} 6. 8.125×10^{-8}

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UNIT #2 REVIEW 1 WORKSHEET:

(Zero and Negative Exponents & Scientific Notation)

1. Write the following powers with a positive exponent. Then evaluate. No decimals!

   a) 5^{-3} b) $(-7)^{-2}$ c) -8^{-1} d) $\dfrac{1}{3^{-4}}$

2. Simplify first, i.e. write as a single power. Then evaluate. NO decimals!

   a) $\dfrac{(6^7)(6^8)}{(6^3)^5}$ b) $\dfrac{(4^{-1})^6}{(4^7)^{-4}} \times \left[\dfrac{(4^{-2})^{-5}}{4}\right]^{-3}$

3. Write the following in scientific notation.

   a) 290 000 000 000 000 b) 0.000 004 07

4. Simplify. Make sure your final answers are in proper scientific notation form.

   a) $(4.9\times10^9)\times(6.5\times10^7)$ b) $\dfrac{3.8\times10^{-18}}{7.6\times10^{-6}}$

Answer Key :

1. a) 1/125 b) 1/49 c) –1/8 d) 81 2. a) 1 b) 1/1024 3. a) 2.9×10^{14}
3. b) 4.07×10^{-6} 4. a) 3.185×10^{17} b) 5×10^{-13}

GETTING READY SECTION – PART 2

I) Words You Need To Know / Definitions :

1. **Variable** : A letter or symbol representing one or more numbers.
   Examples : x, m, π

2. **Term** : An expression involving numbers or variables or both.
   Examples : $3x, -7ab^2, y, 5$

3. **Algebraic Expression** : An expression combining terms by adding or subtracting.
   Ex. $2x^2 + 3x - 5$ Ex. $4a - 7b$

4. **Coefficient** : The factor by which a variable is multiplied, i.e. the number in front of the variable in a term.
   Ex. $m^2 - 5m + 3$ ➜ coefficient of the m^2 term is 1
   ➜ coefficient of the m term is -5

5. **Constant** : A number in an algebraic expression with no variable.
   Ex. $m^2 - 5m + 3$ ➜ constant term is 3

6. **Degree of a term** :
   a) For a power with one variable, the degree is the variable's exponent.
      Ex. $2x^3$ ➜ degree is 3 Ex. $5m$ ➜ degree is 1
   b) For a power with more than one variable, the degree is the sum of the exponents of the variables.
      Ex. $3x^2y^3$ ➜ degree is 5 Ex. ab^4c^3 ➜ degree is 8

7. **Degree of an algebraic expression** : The term with the highest degree is the degree of the expression.
   Ex. $2x^4 - 7x^2$ ➜ degree is 4
   Ex. $m^2n^3 + 4mn^2 - 2n$ ➜ degree is 5

II) Skills and Concepts You Need To Know :

Evaluating algebraic expressions by substituting values for the variables.
- ➜ Always substitute in brackets.
- ➜ Remember to use BEDMAS.

Examples :

1. Given the following algebraic expressions, fill in the chart below.

ALGEBRAIC EXPRESSION	a) $-3x^2+4x+5$	b) $4m^4n^3-6m^3n^2+7m^2n-2m$	c) a^4-16
NUMBER OF TERMS			
VARIABLES			
COEFFICIENT OF THE FIRST TERM			
CONSTANT TERM			
DEGREE OF THE EXPRESSION			

1. The perimeter of a rectangle is $P=2l+2w$. Determine the perimeter of a rectangle with a width of 3cm and a length of 5cm.

2. Given the algebraic expression $2a^2-4b$, evaluate if $a=-3$ and $b=-2$.

3. The volume of a cylinder is $V=\pi r^2h$. Determine the volume of a cylinder with a height of 9cm and a radius of 4cm, to one decimal place. Use the π button on your calculator.

2.4 ADDING AND SUBTRACTING POLYNOMIALS

A **polynomial** is an algebraic expression with **more than one** term.

- A **monomial** is an algebraic expression with **one** term.
  Examples of monomials : $3x^2$, $-4ab$, 8

- A **binomial** is an algebraic expression with **two** terms.
  Examples of binomials : $3x+7$, m^2+5 , $a-2b$

- A **trinomial** is an algebraic expression with **three** terms.
  Examples of trinomials : x^2+2x+6 , $a^2-3ab+5b^2$, $2p+9q-4r$

Collecting/Combining Like Terms :

Like Terms : Terms that have the same variables with the same exponents.
Ex. $3x$ and $2x$ are like terms Ex. $2ab^2$ and $-5ab^2$ are like terms

To collect/combine like terms :
add/subtract the coefficients
leave the variable and exponents as is

Examples :

1. Simplify the following expressions, i.e. collect like terms.

   a) $2x+5x$

   b) $3y^2-7y^2-y^2$

   c) $-4ab+9ab$

   d) $4m-3n+m+2n$

   e) $8p+1-3-4p$

   f) $2x^2-5+3x+1+4x^2-2x$

   g) $5a-3b+c-4a-2b+6c$

2. Simplify the following.

   a) $(2x + 5y) + (3x - 6y)$

   b) $(m - 2n) - (4m - 7n)$

   c) $(5a^2 + 4ab - 8b^2) - (3a^2 - ab + 2b^2) + (b^2 - 2a^2 + 3ab)$

3. Determine the polynomials that need to be added to each of the initial polynomials to get the final algebraic expression.

   a) $(2x - 3y) + (\_\_\_\_\_) = 6x - 5y$

   b) $(a^2 + 7a - 9) + (\_\_\_\_\_) = -a^2 + 2a - 1$

4. An air conditioning repair man charges $40 for a service call plus $28/h for the labor. A mechanic charges $30 for a maintenance check plus $45/h for the labor on any repairs.

   a) Write a polynomial to represent the cost of an air conditioning repair bill.

   b) Write a polynomial to represent the cost of a mechanic's repair bill.

   c) Write a single polynomial to represent a combined repair bill.

   d) Calculate a combined repair bill of both the air conditioning repair man and the mechanic work for 5 hours.

2.5 MULTIPLYING A POLYNOMIAL BY A MONOMIAL

* **Distributive Law :** $a(b+c) = ab + ac$

Examples :

1. Simplify the following by using the Distributive Law.

   a) $2(x+3)$ b) $3(4m-5n)$

* **Remember your Exponent Laws!**

2. Expand the following.

   a) $2a(a-9)$ b) $-(m^2-6m+11)$ c) $5x^3(3x^2-7x+8)$

   d) $\frac{1}{4}c(12c-4)$ e) $3p(pq-2qr)$

3. Determine the missing factor and verify your answer.

   a) $\_\_\_\_\_(2x^2-5x+3) = 8x^3-20x^2+12x$

   b) $-6m^4(\_\_\_\_\_+\_\_\_\_\_) = 12m^7+6m^5$

4. Evaluate each expression for $p = -3$ once before you expand it and once after you expand it.

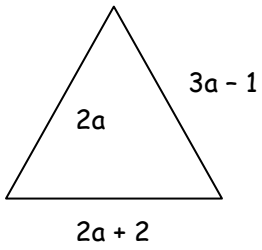
   a) $5(2p + 3)$

   b) $-p(p^2 - 4p - 15)$

5. Write a simplified algebraic expression for the perimeter, P, and the area, A, of the following figures.

   a)

   b)



6. Given that $30 \times 52 = 30 \times 50 + 30 \times 2$, rewrite the following products and evaluate. How is this like using the Distributive Property?

   a) 40×78

   b) 15×22

   c) 20×136

2.6 SIMPLIFYING POLYNOMIAL EXPRESSIONS

Recall :

To simplify a polynomial expression :
1) Use the Distributive Property to expand brackets
2) Collect Like Terms to simplify

Examples :

1. Simplify the following polynomial expressions.

   a) $4x(x+1)+3(2x^2-3x+2)$

   b) $3a^2(a-5)-2a(3a^2-4a+2)$

   c) $2m^4(m^2+3m)+m^3(5m^2-m^3)$

   d) $\frac{3}{4}\left(\frac{1}{6}a-\frac{2}{5}b\right)+\frac{1}{2}\left(2\frac{1}{3}a+\frac{4}{5}b\right)$

   e) $3x(x-2y)-y(4y-5x)$

   f) $2n(5n^3-4n)+n^2(n+6)-3n^3(2n-1)$

2. Expand and simplify the following expression, and then evaluate for $m=4$.

   $4m(m^2+3m-2)-m^2(2-m)+3(m-5m^2-m^3)$

* We use the Distributive Property to expand two binomial brackets.
  ➜ We can use **FOIL** for an order in which to expand.

  F ➜ first
  O ➜ outside
  I ➜ inside
  L ➜ last

$$(a+b)(c+d) = ac + ad + bc + bd$$

3. Apply the Distributive Property to simplify the following.

   a) $(x+1)(x+5)$

   b) $(a-3)(a+4)$

4. Simplify the following polynomial expressions.

   a) $(x-5)(x+2)+3(x^2-2x+1)$

   b) $(2a+b)(3a-4b)-(4a^2-3b^2)$

   c) $(4m+5)(2m-1)+(6-m)(5m+2)$