1 – Algebra Review

EXPANDING AND FACTORING REVIEW

SPECIAL PRODUCT FORMULA
1. $(A - B)(A + B) = A^2 - B^2$
2. $(A + B)^2 = A^2 + 2AB + B^2$
3. $(A - B)^2 = A^2 - 2AB + B^2$
4. $(A + B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$
5. $(A - B)^3 = A^3 - 3A^2B + 3AB^2 - B^3$

FACTORING FORMULAS	
i. $A^2 - B^2 = (A - B)(A + B)$	Difference of Squares
ii. $A^2 + 2AB + B^2 = (A + B)^2$	Perfect Square
iii. $A^2 - 2AB + B^2 = (A - B)^2$	Perfect Square
iv. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$	Difference of Cubes
v. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$	Sum of Cubes

Example 1: Simplify.

a) $(3x^2 + x + 1) - (2x^2 - 3x - 5)$

c) $(3x - 2)(2x + 5)^2$

b) $4(x^2 - 3x + 5) - 3(x^2 - 2x + 1)$

d) $(2x + 3y)^3$

Example 2: Factor fully.

a) $2x^2 - 7x - 4$

d) $27x^3 - y^3$

b) $25s^2 - 10st + t^2$

e) $3x^3 - 12x^2 + 5x - 20$

c) $4t^2 - 9s^2$

f) $x^3(x^2 - 16) + 27(x^2 - 16)$

RADICALS

Two radical expressions are <u>conjugates</u> if their product is a rational number.	
The conjugate of a monomial radical expression $a\sqrt{b}$ is $\sqrt{b}$.	The conjugate of a binomial radical expression $\left(a\sqrt{b} - c\sqrt{d}\right)$ is $\left(a\sqrt{b} + c\sqrt{d}\right)$

Example 3: Rationalize the denominator.

a) $\dfrac{2}{\sqrt{7}}$

c) $\dfrac{7}{\sqrt{3}+\sqrt{2}}$

b) $\dfrac{4}{3\sqrt{2}}$

d) $\dfrac{5+\sqrt{3}}{1-2\sqrt{3}}$

e) Rationalize the numerator: $\dfrac{5+\sqrt{3}}{1-2\sqrt{3}}$

RATIONAL EXPRESSIONS REVIEW

Correct Multiplication Property	Common Error with Addition Counterexample
$(a \cdot b)^2 = a^2 \cdot b^2$	$(a + b)^2 \neq a^2 + b^2$
$\sqrt{a \cdot b} = \sqrt{a}\sqrt{b}, \quad a, b \geq 0$	$\sqrt{a + b} \neq \sqrt{a} + \sqrt{b}$
$\sqrt{a^2 \cdot b^2} = a \cdot b, \quad a, b \geq 0$	$\sqrt{a^2 + b^2} \neq a + b$
$\dfrac{1}{a} + \dfrac{1}{b} = \dfrac{1}{a \cdot b}$	$\dfrac{1}{a} + \dfrac{1}{b} \neq \dfrac{1}{a + b}$
$\dfrac{ab}{a} = b$	$\dfrac{a + b}{a} \neq b$
$a^1 + b^{-1} = (a \cdot b)^{-1}$	$a^{-1} + b^{-1} \neq (a + b)^{-1}$

Example 4: Simplify.

a) $\dfrac{x^2-x-2}{x^2-1}$

b) $\dfrac{y^2-3y-18}{2y^2+5y+3}$

c) $\dfrac{x^2+2xy+y^2}{x^2-y^2} \cdot \dfrac{2x^2-xy-y^2}{x^2-xy-2y^2}$

d) $\dfrac{4y^2-9}{2y^2+9y-18} \div \dfrac{2y^2+y-3}{y^2+5y-6}$

e) $\dfrac{2}{x+1} - \dfrac{5}{x+2}$

f) $x - \dfrac{y}{\frac{x}{y}+\frac{y}{x}}$

ALGEBRA REVIEW WORKSHEET

1. Simplify.
   a. $(3x^2 + x + 1) + (2x^2 - 3x - 5)$
   b. $(x^3 + 6x^2 - 4x + 7) - (3x^2 + 2x - 4)$
   c. $8(2x + 5) - 7(x - 9)$
   d. $2(2 - 5t) + t^2(t - 1) - (t^4 - 1)$
   e. $\sqrt{x}(x - \sqrt{x})$
   f. $y^{1/3}(y^2 - 1)$
   g. $(3t - 2)(7t - 5)$
   h. $(x + 2y)(3x - y)$
   i. $(1 - 2y)^2$
   j. $(2x^2 + 3y^2)^2$
   k. $(2x - 5)(x^2 - x + 1)$
   l. $(x^2 - a^2)(x^2 + a^2)$

2. Factor fully.
   a. $12x^3 + 18x$
   b. $6y^4 - 15y^3$
   c. $x^2 - 2x - 8$
   d. $y^2 - 8y + 15$
   e. $2x^2 + 5x + 3$
   f. $9x^2 - 36x - 45$
   g. $6x^2 - 5x - 6$
   h. $4t^2 - 12t + 9$
   i. $r^2 - 6rs + 9s^2$
   j. $x^2 - 36$
   k. $49 - 4y^2$
   l. $(a + b)^2 - (a - b)^2$
   m. $x^2(x^2 - 1) - 9(x^2 - 1)$
   n. $t^3 + 1$
   o. $8x^3 - 125$
   p. $x^6 - 8y^3$
   q. $x^3 + 2x^2 + x$
   r. $x^4 + 2x^3 - 3x^2$
   s. $y^3 - 3y^2 - 4y + 12$
   t. $2x^3 + 4x^2 + x + 2$

3. Rationalize the denominator.
   a. $\frac{2}{3+\sqrt{5}}$
   b. $\frac{2}{\sqrt{2}+\sqrt{7}}$

4. Rationalize the numerator.
   a. $\frac{1-\sqrt{5}}{3}$
   b. $\frac{\sqrt{3}+\sqrt{5}}{2}$
   c. $\frac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x}\sqrt{x+h}}$

5. Simplify.
   a. $\frac{x-2}{x^2-4}$
   b. $\frac{x^2+6x+8}{x^2+5x+4}$
   c. $\frac{y^2+y}{y^2-1}$
   d. $\frac{t-3}{t^2+9} \cdot \frac{t+3}{t^2-9}$
   e. $\frac{x^2+7x+12}{x^2+3x+2} \cdot \frac{x^2+5x+6}{x^2+6x+9}$
   f. $\frac{2x^2+3x+1}{x^2+2x-15} \div \frac{x^2+6x+5}{2x^2-7x+3}$
   g. $\frac{x^3}{x+1} \div \frac{x}{x^2+2x+1}$
   h. $\frac{x/y}{z}$
   i. $\frac{1}{x+5} + \frac{2}{x-3}$
   j. $\frac{x}{(x+1)^2} + \frac{2}{x+1}$
   k. $\frac{1+\frac{1}{c-1}}{1-\frac{1}{c-1}}$
   l. $\frac{\frac{5}{x-1}-\frac{2}{x+1}}{\frac{x}{x-1}+\frac{1}{x+1}}$

4 | MCV4U: 1 – Algebra Review

DRAFT Answers

1. a. $5x^2 - 2x - 4$ b. $x^3 + 3x^2 - 6x + 11$
   c. $9x + 103$ d. $-t^4 + t^3 - t^2 - 10t + 5$
   e. $x^{3/2} - x$ f. $y^{7/3} - y^{1/3}$
   g. $21t^2 - 2at + 10$ h. $3x^2 + 5xy - 2y^2$
   i. $1 - 4y + 4y^2$ j. $4x^4 + 12x^2y^2 + 9y^4$
   k. $2x^3 - 7x^2 + 7x - 5$ l. $x^4 - a^4$

2. a. $6x(2x^2 + 3)$ b. $3y^3(2y - 5)$
   c. $(x - 4)(x + 2)$ d. $(y - 5)(y - 3)$
   e. $(2x + 3)(x + 1)$ f. $9(x - 5)(x + 1)$
   g. $(2x - 3)(3x + 2)$ h. $(2t - 3)^2$
   i. $(r - 3s)^2$ j. $(x - 6)(x + 6)$
   k. $(7 - 2y)(7 + 2y)$ l. $4ab$
   m. $(x - 1)(x + 1)(x - 3)(x + 3)$
   n. $(t + 1)(t^2 - t + 1)$
   o. $(2x - 5)(4x^2 + 10x + 25)$
   p. $(x^2 - 2y)(x^4 + 2x^2y + 4y^2)$
   q. $x(x + 1)^2$ r. $x^2(x + 3)(x - 1)$
   s. $(y - 2)(y + 2)(y - 3)$
   t. $(x + 2)(2x^2 + 1)$

3. a. $\frac{3-\sqrt{5}}{2}$ b. $\frac{2(\sqrt{7}-\sqrt{2})}{5}$

4. a. $-\frac{4}{3(1+\sqrt{5})}$ b. $\frac{1}{\sqrt{5}-\sqrt{3}}$ c. $-\frac{1}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$

5. a. $\frac{1}{x+2}$ b. $\frac{x+2}{x+1}$ c. $\frac{y}{y-1}$ d. $\frac{1}{t^2+9}$ e. $\frac{x+4}{x+3}$
   f. $\frac{(2x+1)(2x-1)}{(x+5)^2}$ g. $x^2(x+1)$ h. $\frac{x}{yz}$