Name: **Date:**

3 – 1.5 Properties of Limits and 1.6 Continuity

Lesson Goals:

- Be able to apply limit properties to evaluate complex limits
- Be able to determine if a function is continuous

1) Limit

- Recall, that the limit of a function $\lim_{x \to a} f(x)$ exists if all 3 criteria are satisfied:
  1) $\lim_{x \to a^-} f(x)$ exists (equals a number)
  2) $\lim_{x \to a^+} f(x)$ exists
  3) $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$
- Limits of functions we know:
  - Constant functions
  - Polynomial functions
  - Exponential functions
  - Sinusoidal functions
  - Radical functions
  - Piecewise functions
  - Rational functions

2) Limit Properties

- For the properties that follow, assume that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist and c is any constant.

Property	Description
1. $\lim_{x\to a} c = c$	The limit of a constant is equal to the constant.
2. $\lim_{x\to a} x = a$	The limit of x as x approaches a is equal to a.
3. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$	The limit of a sum is the sum of the limits.
4. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$	The limit of a difference is the difference of the limits.
5. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$	The limit of a constant times a function is the constant times the limit of the function.
6. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$	The limit of a product is the product of the limits.
7. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$, provided $\lim_{x\to a} g(x) \neq 0$	The limit of a quotient is the quotient of the limits, provided that the denominator does not equal 0.
8. $\lim_{x\to a}[f(x)]^n = \left(\lim_{x\to a} f(x)\right)^n$, where n is a rational number	The limit of a power is the power of the limit, provided that the exponent is a rational number.
9. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$, if the root on the right side exists	The limit of a root is the root of the limit provided that the root exists.

Example 1: Evaluate.

a) $\lim_{n\to\infty} 4$

b) $\lim_{n\to\infty} \left(\frac{27n+1}{8n-3} - \frac{n}{n+1}\right)$

c) $\lim_{n\to\infty} 4\left(\frac{n}{n+1}\right)$

d) $\lim_{n\to\infty} \left(\frac{27n+1}{8n-3} \times \frac{4n}{n+1}\right)$

e) $\lim_{n\to\infty} \left(\frac{27n+1}{8n-3} \div \frac{4n}{n+1}\right)$

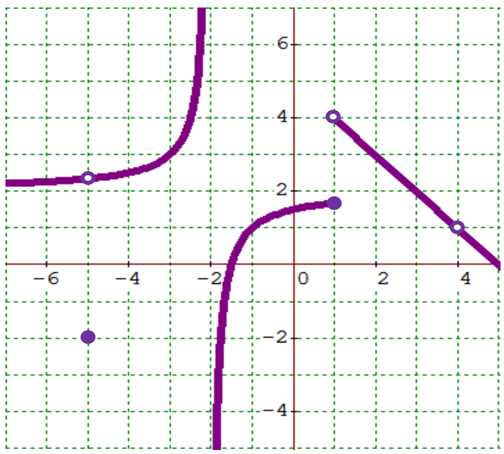
f) $\lim_{n\to\infty} \left(\frac{4n}{n+1}\right)^3$

g) $\lim_{n\to\infty} \sqrt[3]{\frac{27n+1}{8n-3}}$

3) Continuity

- A function $f(x)$ is continuous at $x = a$ if:
    1) $f(a)$ is defined, and
    2) $\lim_{x \to a} f(x)$ exists, and
    3) $\lim_{x \to a} f(x) = f(a)$

Example 2: At what value of x is the function shown discontinuous (not continuous)? Which criteria does it fail?



Example 3:

Given $f(x) = \begin{cases} x^2, & if\ x \leq 2 \\ 6 - x, & if\ x > 2 \end{cases}$

Determine if $f(x)$ is continuous at $x = 2$.

Example 4:

Given $g(x) = \begin{cases} x^2, & if\ x \geq -3 \\ 2x, & if\ x < -3 \end{cases}$

Determine if $g(x)$ is continuous at $x = -3$.

- Continuity of functions we know:
  - Constant functions
  - Polynomial functions
  - Exponential functions
  - Sinusoidal functions
  - Radical functions
  - Piecewise functions
  - Rational functions

Homework: Page 47 #13 and Page 51 #1-17 (pick and choose)