Name: **Date:**

4 – 2.1 The Derivative Function

Lesson Goals:

- To find the derivative at a specific point
- To find the general derivative for a given function
- Identify the domain where a function is differentiable

1) Derivative

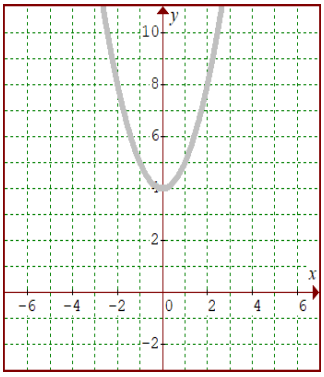
- The derivative has two meanings
  1) At the point $(a, f(a))$ on a function, the derivate is the slope of the tangent line. This is also called the instantaneous rate of change $x = a$.
  2) For the function $f(x)$, the derivative $f'(x)$ is a general formula for all the slopes of tangent lines to the function.
     It can calculate the rate of change at any point.

Function	First Derivative	Second Derivative	Third Derivative
$f(x) = x^4$	$f'(x) = 4x^3$	$f''(x) = 12x^2$	$f'''(x) = 24x$
$y = x^4$	$y' = 4x^3$	$y'' = 12x^2$	$y''' = 24x$
$y = x^4$	$\frac{dy}{dx} = 4x^3$	$\frac{d^2y}{dx^2} = 12x^2$	$\frac{d^3y}{dx^3} = 24x$

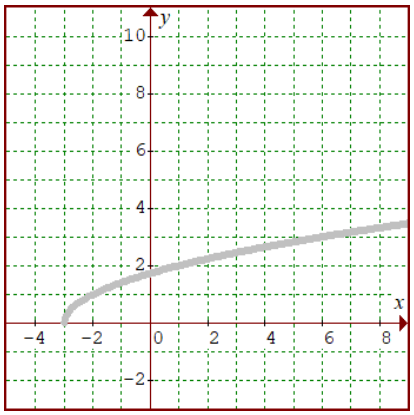
2) First Principles Definition of Derivative

Formula 1	Formula 2
• The derivative at a specific value $x = a$. • Two points are $(a, f(a))$ and $(x, f(x))$. $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}, x \neq a$	• General derivative formula at x. • Two points are $(x, f(x))$ and $(x + h, f(x + h))$. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, h \neq 0$

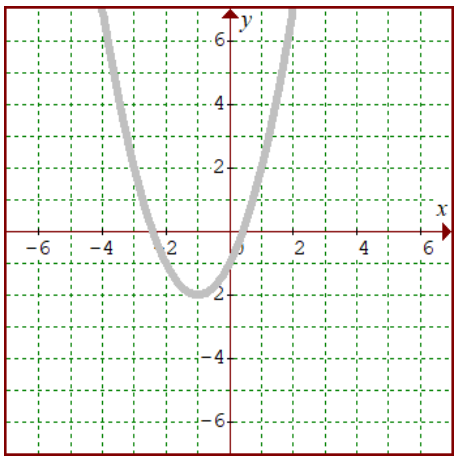
Example 1: Given $f(x) = x^2 + 4$, determine the derivative at $x = -1$ using the First Principles definition.

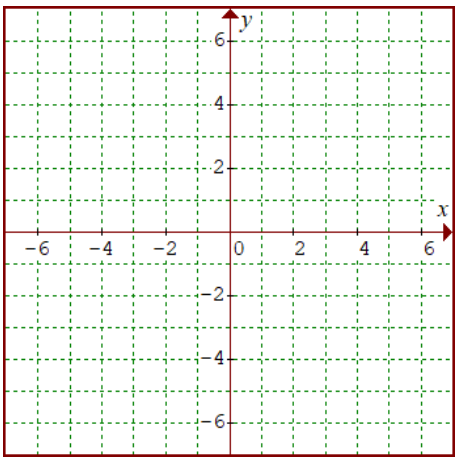


Example 2: Given $g(x) = \sqrt{x+3}$, determine the equation of the tangent line at $x = 6$.



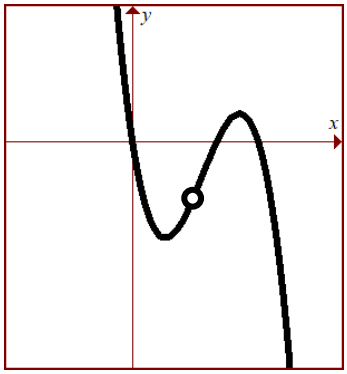
Example 3: Given $f(x) = x^2 + 2x - 1$. Find the derivative $f'(x)$ from First Principles.



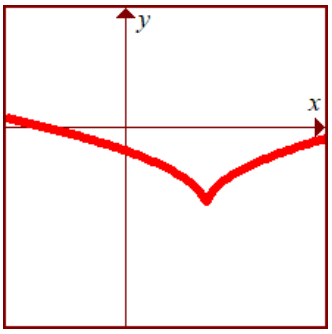


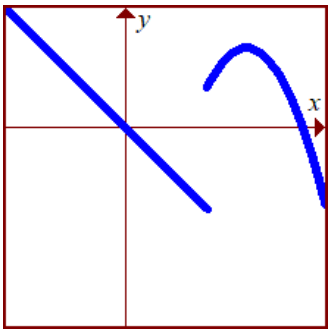
3) Differentiable

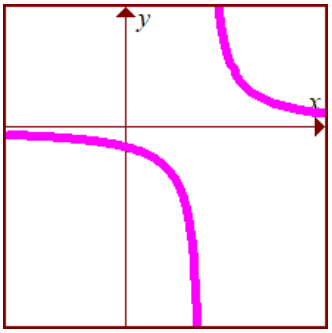
- A function is differentiable at $x = a$ if $f'(a)$ exists.
- That is, if $\lim_{x \to a^-} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^+} \frac{f(x)-f(a)}{x-a}$
- A function is differentiable at $x = a$ or the derivative can be calculated at:

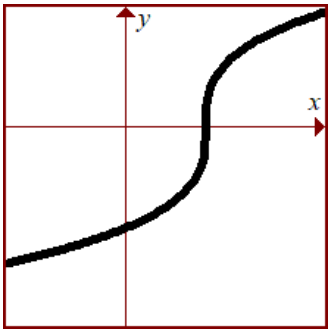


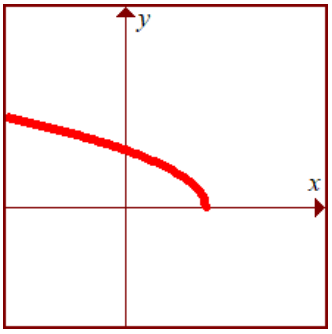
- A function is not differentiable where there is a





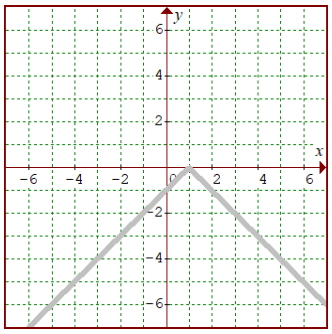




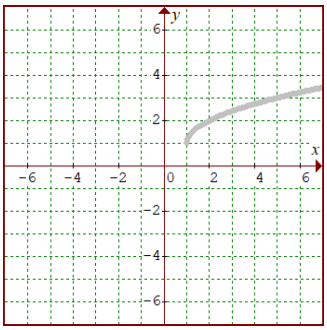


Example 4: For each function, identify the domain where the function is differentiable.

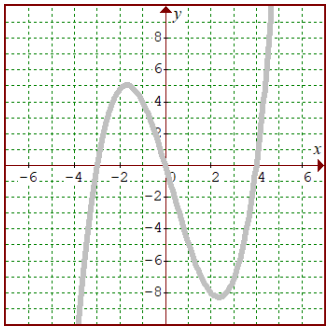
a) $y = -|x - 1|$



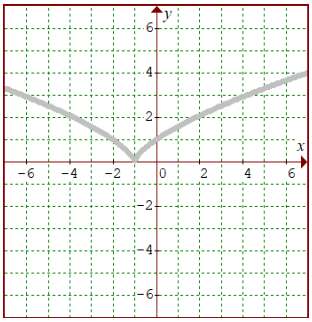
b) $y = \sqrt{x - 1} + 1$



c) $f(x) = 0.4x(x - 4)(x + 3)$



d) $g(x) = (x + 1)^{2/3}$



Example 5:

Given $y = \frac{x}{x+2}$. Find the derivative $\frac{dy}{dx}$ from First Principles.

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