Name: **Date:**

6 – 2.2 The Derivatives of Polynomial Functions

Lesson Goals:

- Be able to apply the power, constant multiple, and sum rule to find the derivative of polynomial functions
- Solve problems using derivatives

1) Power Rule

- If $f(x) = x^n$, where n is a real number, then $f'(x) = nx^{n-1}$.
- In Leibniz notation, $\frac{d}{dx}(x^n) = nx^{n-1}$.

Example 1: Use the Power Rule to find the derivative of each function.

a) $f(x) = x^6$

b) $y = \frac{1}{x^5}$

c) $h(t) = t^{\frac{5}{3}}$

d) $y = \sqrt{x}$

e) $g(x) = (x^{20})^{20}$

f) $y = x$

- Proof of the Power Rule:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

2) Constant Multiple Rule

- If $f(x) = kg(x)$, where k is a constant, then $f'(x) = kg'(x)$.
- In Leibniz notation, $\frac{d}{dx}(ky) = k\frac{dy}{dx}$.

Example 2: Find the derivative of each function.

a) $f(x) = -45x^6$

b) $h(t) = 24t^{\frac{5}{3}}$

3) Sum and Difference Rules

- If functions $p(x)$ and $q(x)$ are differentiable, and $f(x) = p(x) + q(x)$, then $f'(x) = p'(x) + q'(x)$.
- In Leibniz notation, $\frac{d}{dx}(f(x)) = \frac{d}{dx}(p(x)) + \frac{d}{dx}(q(x))$.

Example 3: Find the derivative of each function.

a) $f(x) = x^6 + 3x^2 - 6\sqrt{x}$

b) $g(x) = (3x - 5)^2$

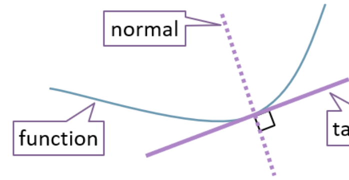
c) $y = \frac{3+6\sqrt{x}-4x^3}{2x}$

d) $h(x) = \begin{cases} x^2 - 9 & \text{if } x \leq 1 \\ \sqrt[5]{x^8} & \text{if } x > 1 \end{cases}$

Example 4: Find the equation of the normal to the function $f(x) = 3x^5 - x^2 + 7$ at $x = -1$.

Definition

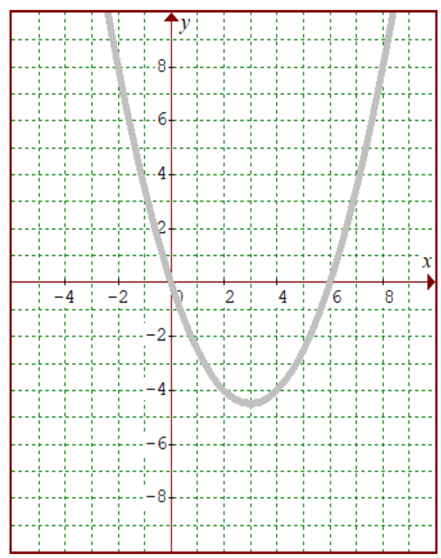
The normal to the graph of a function at point P is the line that is perpendicular to the tangent at P.



Example 5 – Horizontal Tangent: Determine all the points of the function $f(x) = 4x^3 - 3x^2 - 3x + 2\pi$ where the tangent line is horizontal.

Example 6 – Parallel Tangents: Determine the value(s) of x where the tangents of $f(x) = \frac{1}{x}$ and $g(x) = x^3$ are parallel.

Example 7 – Tangent to External Point: Determine the equation of the tangent line(s) of $g(x) = 0.5x^2 - 3x$ that pass through the point A(1,-7).



Homework: Page 82 a, c, e for #2-9, pick and choose #10-25, and 28a