Name: **Date:**

7 – 2.3 The Product Rule

Lesson Goals:

- Be able to apply the product rule and power of a function rule to find a derivative
- Solve problems using derivatives

1) Product Rule

- If $p(x) = f(x)g(x)$, then $p'(x) = f'(x)g(x) + f(x)g'(x)$.
- If u and v are functions of x, $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$.

Example 1: Identify the factors of the function and their derivatives. Then use the Product Rule to find $p'(x)$.

a) $p(x) = (2x^2 + 1)(3 + 5x - 4x^2)$

b) $p(x) = \sqrt{x}(2 - 3x)$

- Proof of the Product Rule:

$$p'(x) = \lim_{h \to 0} \frac{p(x+h) - p(x)}{h}$$

2) Extended Product Rule

- If $p(x) = f(x)g(x)h(x)$, then $p'(x) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$.

Example 2: Identify the factors of the function and their derivatives. Then use the Product Rule to find $\frac{dy}{dx}$.

$y = (x^3 - 5x^4)^3$

3) Power of a Function Rule

- If $f(x) = [g(x)]^n$, then $f'(x) = n[g(x)]^{n-1}g'(x)$.
- If u is a function of x, and n is an integer, then $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$.

Example 3: Use the Power of a Function Rule to find $p'(x)$.

a) $p(x) = (x^3 - 5x^4)^3$

b) $p(x) = \frac{1}{(x^2-5x)^5}$

Example 4 – Tangent/Normal Lines: $k(x) = (3x^2 + 2)(2x^3 - 1)$

a) Find the slope of the tangent of $k(x)$ at $x = 1$.

b) Find the equation of the normal to $k(x)$ at $x = 1$.

Example 5 – Combination of Power and Product Rules: Express $f(x)$ as a product then use the Product Rule to find $f'(x)$.

$$f(x) = \frac{2x^3 - 5x}{x^2 + 3}$$

Example 6 – Velocity: The position s, in centimetres, of an object moving in a straight line is given by $s = t(6 - 3t)^4, t \geq 0$, where t is the time in seconds. Determine the object's velocity at $t = 2$.

Example 7 – Combination Product and Power of a Function: Differentiate.

$$f(x) = (1 - x^2)^4(2x + 5)^3$$

Example 8 – Horizontal Tangents: Determine the point(s) where the tangent to the curve is horizontal.

$$y = (x^2 + 2x - 15)(x^2 + 2x - 15)$$

Homework: Page 90 #1-10, 12-14 (pick and choose)