MCV4U
Mrs BAHAR

Chapter 3.1 – Higher Order Derivatives, Velocity, and Acceleration

The second derivative of $y = f(x)$ is the derivative of $y = f'(x)$.

- In Newtonian notation, the second derivative is $f''(x)$
- In Leibniz notation, the second derivative is $\frac{d^2y}{dx^2}$

Velocity: If the position of an object, $s(t)$, is a function of time, t, then the first derivative of this function represents the velocity of the object at time t.

- $v(t) = s'(t) = \frac{ds}{dt}$

Acceleration: Acceleration, $a(t)$, is the instantaneous rate of change of velocity with respect to time. Acceleration is the first derivative of the velocity function and the second derivative of the position function

- $a(t) = v'(t) = s''(t)$ or $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

- Negative velocity, $v(t) < 0$ or $s'(t) < 0$, indicates that an object is moving in a negative direction (left or down) at time t, while positive velocity, $v(t) > 0$ or $s'(t) > 0$, indicates that an object is moving in a positive direction (right or up) at time t

- Zero velocity, $v(t) = 0$ or $s'(t) = 0$, indicates that an object is stationary and that a possible change in direction may occur at time t.

- Negative acceleration, $a(t) < 0$ or $v'(t) < 0$ or $s''(t) < 0$ indicates that the velocity is decreasing, while positive acceleration, $a(t) > 0$ or $v'(t) > 0$ or $s''(t) > 0$ indicates that the velocity is increasing.

- Zero acceleration, $a(t) = 0$ or $v'(t) = 0$ or $s''(t) = 0$ indicates that the velocity is constant and that the object is neither accelerating nor decelerating

- An object is accelerating (speeding up) when its velocity and acceleration have the same signs. However, an object is decelerating (slowing down) when its velocity and acceleration have opposite signs.

- The speed of an object is the magnitude of its velocity at time t.
    - Speed $= |v(t)| = |s'(t)|$

Example 1 – Determine the second derivative of $f(x) = \dfrac{x}{1+x}$ when $x = 1$.

Example 2 - The position of an object moving on a line is given by $s(t) = 6t^2 - t^3, t \geq 0$, where s is in metres and t is in seconds.

a) Determine the velocity and acceleration of the object at $t = 2$.
b) At what time(s) is the object at rest?
c) In which direction is the object moving at $t = 5$?
d) When is the object moving in a positive direction?
e) When does the object return to its initial position?

Example 3 /Discuss the motion of an object moving in a horizontal line if its position is given by $s(t) = t^2 - 10t, 0 \leq t \leq 12$, where s is in metres and t is in seconds. Include the initial velocity, final velocity, and any acceleration in your discussion.

Example 4 – A baseball is hit vertically upward. The position function s(t), in metres, of the ball above the ground is $s(t) = -5t^2 + 30t + 1$, where t is in seconds.

a) Determine the maximum height reached by the ball.
b) Determine the velocity of the ball when it is caught 1 m above the ground.