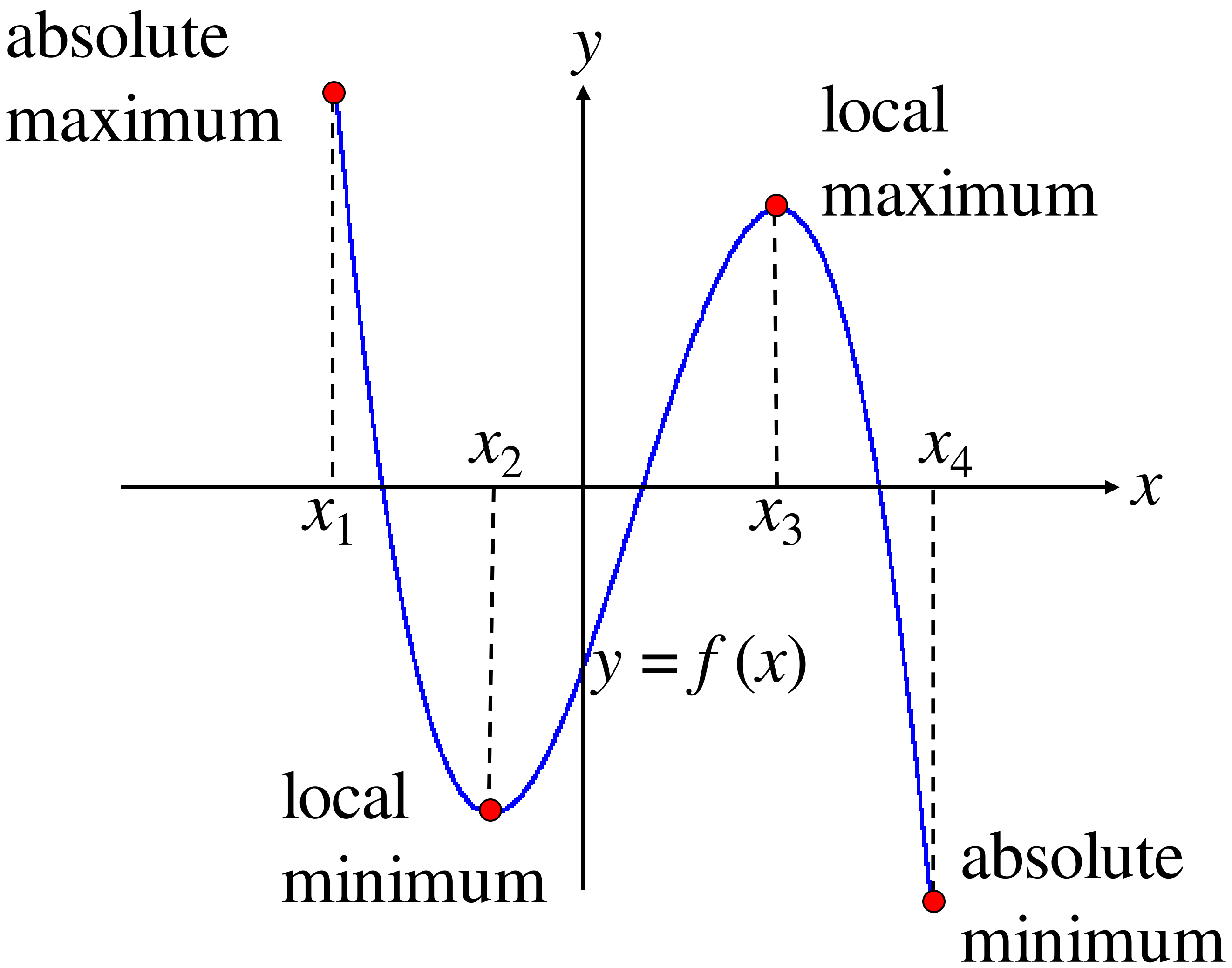
3.2 Maximum and Minimum on an Interval (Extreme Values)

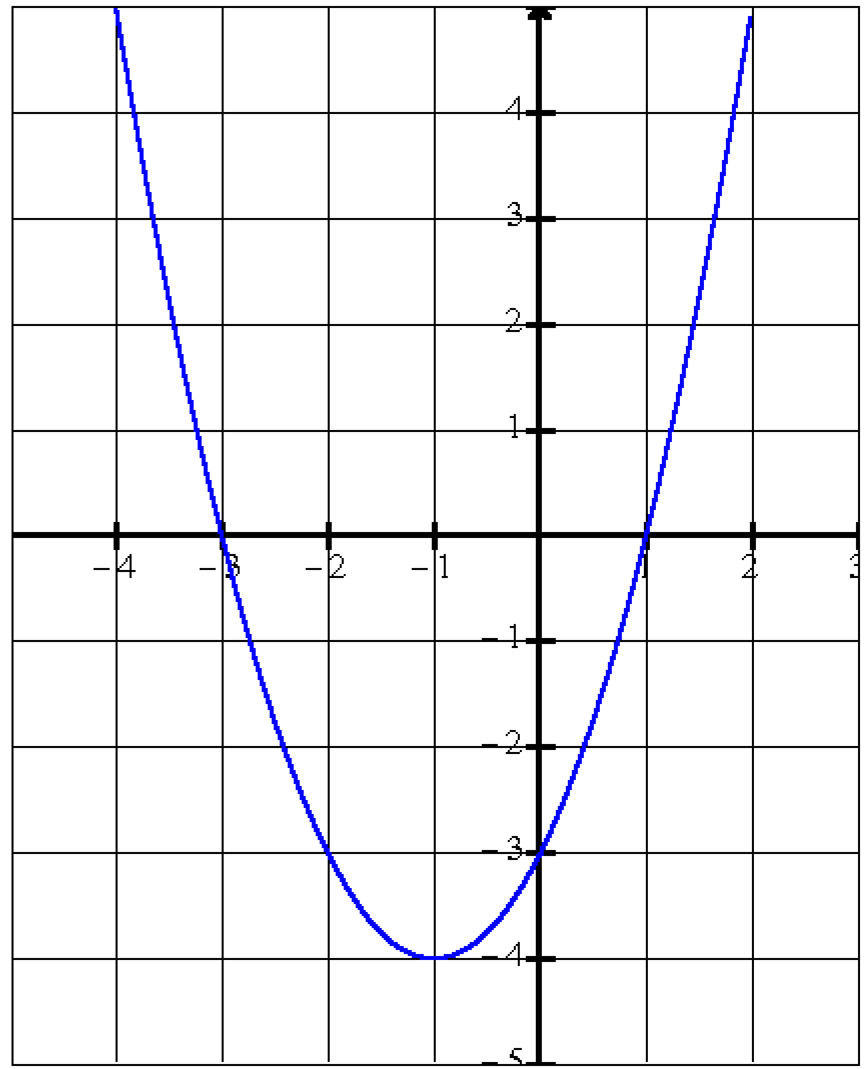


$$y = x^2 + 2x - 3$$

The lowest point on the graph is (– 1, – 4).

$f(x) > f(-1)$ for all values of x.

The point (– 1, – 4) corresponds to the local and absolute minimum point of the function.



Since $f(x) \to \infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, there is no maximum value.

Critical Numbers:

Points on the graph where the slope of the tangent lines are zero.

Points where $f'(x) = 0$.

Example:

$$f(x) = \frac{1}{3}x^3 - x^2 - 3x + 2$$

$$f'(x) = x^2 - 2x - 3$$

$$f'(x) = (x + 1)(x - 3)$$

$f'(x) = 0$ where $x = -1$ and $x = 3$

Finding Absolute Extrema

1) Determine $f'(x)$. Find all critical numbers for the interval $a \leq x \leq b$.

2) Evaluate f at the endpoints a and b and at each critical number c.

3) Compare the values found for step 2.

The largest value is the absolute maximum for the interval $a \leq x \leq b$.

The smallest value is the absolute minimum for the interval $a \leq x \leq b$.

Example 1:

Find all critical numbers for the function and the maximum and minimum values. Graph the function.

$$f(x) = x^3 + 3x^2 - 24x \qquad -5 \le x \le 3.$$

1) Find all critical numbers:

$$f'(x) = 3x^2 + 6x - 24$$

$$f'(x) = 3(x^2 + 2x - 8)$$

$$f'(x) = 3(x - 2)(x + 4)$$

$f'(x) = 0$, where $x = 2$ or 4

sub $x = 2$ and $x = -4$ into

$$f(x) = x^3 + 3x^2 - 24x$$

$$f(2) = (2)^3 + 3(2)^2 - 24(2)$$

$$f(2) = -28$$

$$f(-4) = (-4)^3 + 3(-4)^2 - 24(-4)$$

$$f(-4) = 80$$

critical points are $(2, -28)$ and $(-4, 80)$

Example 1:

Find all critical numbers for the function and the maximum and minimum values. Graph the function.

$$f(x) = x^3 + 3x^2 - 24x \quad -5 \le x \le 3.$$

2) Find the endpoints:

$$f(-5) = (-5)^3 + 3(-5)^2 - 24(-5)$$

$$f(-5) = 70$$

$$f(3) = (3)^3 + 3(3)^2 - 24(3)$$

$$f(3) = -18$$

The endpoints are (–5, 70) and (3, –18)

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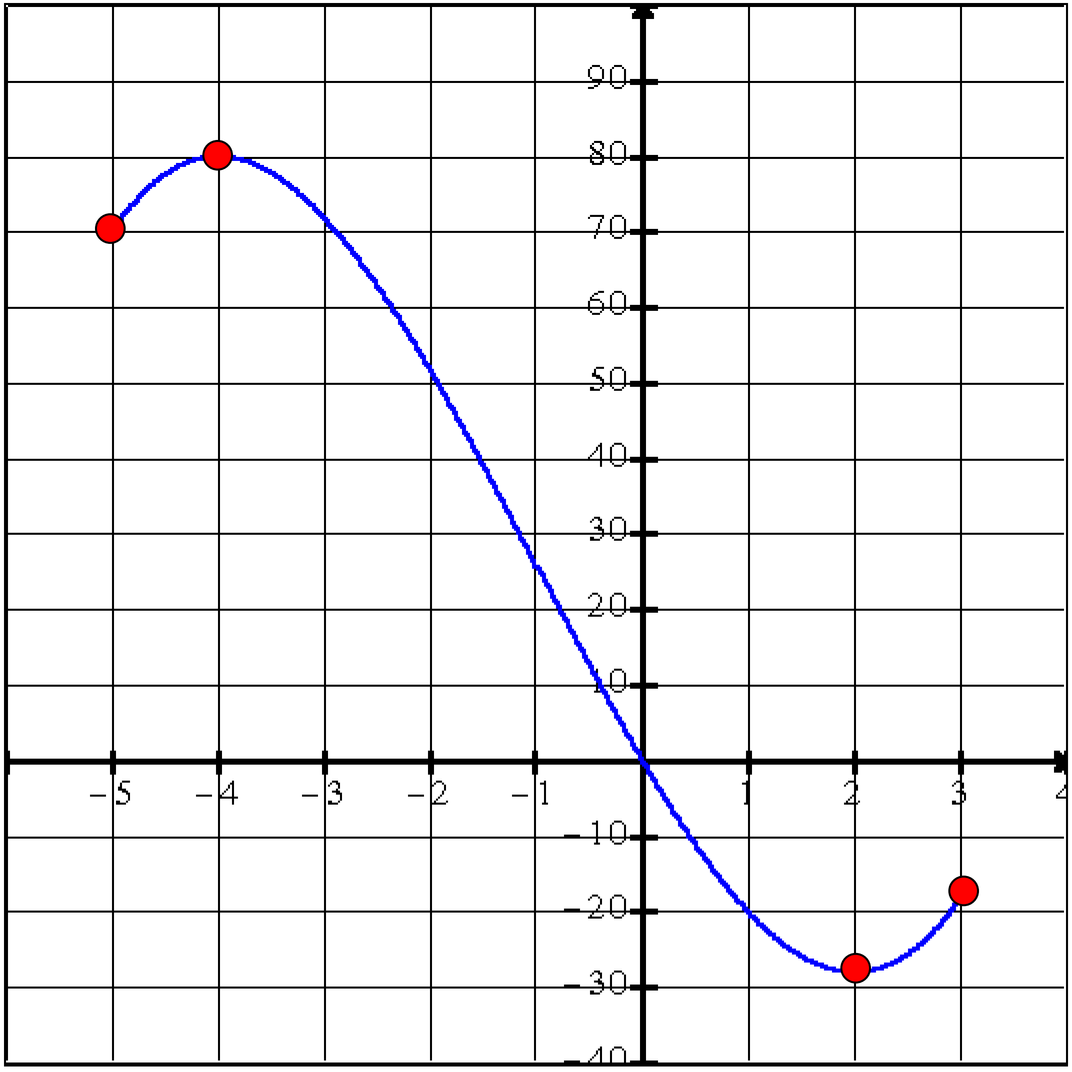
critical points are (–4, 80) and (2, –28)



Example 2: Determine the local and absolute extrema for the function: $y = 2x^3 - 3x^2 - 12x + 1, \; -6 \le x \le 2$

$$y = 2x^3 - 3x^2 - 12x + 1$$

$$y' = 6x^2 - 6x - 12$$

$$y' = 6(x^2 - x - 2)$$

$$y' = 6(x - 2)(x + 1)$$

There are critical points at $x = 2$ and $x = -1$

Sub at $x = 2$ and $x = -1$ into the original equation.

$$y = 2(2)^3 - 3(2)^2 - 12(2) + 1$$

$$y = -19$$

$$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$

$$y = 8$$

(2, –19) and (–1, 8)