## Derivatives of Sine and Cosine Functions

see Geogebra: Sine Slope Demo

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## Derivatives of Sine and Cosine Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

Applying the chain rule:

$$y = \sin(g(x)) \qquad y = \cos(g(x))$$

$$y' = \cos(g(x)) \cdot g'(x) \qquad y' = -\sin(g(x)) \cdot g'(x)$$

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Ex.1 Determine y' and simplify.

(a) $y = \frac{5\sin(4x)}{2}$

$y = \frac{5}{2}\sin(4x)$

$y' = \frac{5}{\cancel{2}_1}\cos(4x)\cdot(\cancel{4})^2$

$= 10\cos(4x)$

(b) $y = \frac{1}{\cos 6x}$

$y = (\cos 6x)^{-1}$

$y' = -1(\cos 6x)^{-2}(-\sin 6x)(6)$

$= \frac{6\sin 6x}{(\cos 6x)^2}$

$= \frac{6\sin 6x}{\cos^2 6x}$ ✓

(c) $y = cos(1 + x^3)$

(d) $y = \frac{\sin\theta}{1+\cos\theta}$

e) $y = \sin 2x \cos 2x$

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Ex.1 Determine y' and simplify.

(c) $y = cos(1 + x^3)$

$y' = -\sin(1+x^3)\cdot(3x^2)$

$= -3x^2\sin(1+x^3)$

(d) $y = \frac{\sin\theta}{1+\cos\theta}$

$y' = \frac{\cos\theta(1+\cos\theta) - \sin\theta(-\sin\theta)}{(1+\cos\theta)^2}$

$= \frac{\cos\theta + \underbrace{(\cos^2\theta + \sin^2\theta)}_{=1}}{(1+\cos\theta)^2}$ Pythag identity

$= \frac{\cancel{(\cos\theta + 1)}}{(1+\cos\theta)^{\cancel{2}}}$

$= \frac{1}{1+\cos\theta}$

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e) $y = \underbrace{\sin(2x)}\underbrace{\cos(2x)}$ $\longrightarrow$ $\sin 2\theta = 2\sin\theta\cos\theta$

$y = \frac{1}{2}\sin 4x$

$y' = (\cos 2x)(2)\cos 2x + \sin 2x\left(-\sin 2x \cdot (2)\right)$

$y' = 2\cos^2 2x - 2\sin^2 2x$ ✓

$y' = 2\left(\cos^2 2x - \sin^2 2x\right)$

or $= 2\left[2\cos^2 2x - 1\right]$

or $= 2\left[1 - 2\sin^2 2x\right]$

Pythag.

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\left[\cos\left(2(2x)\right)\right]$

$y' = 2\cos 4x$

$y = \frac{1}{2}\sin 4x$

$y' = \frac{1}{2}\left[\cos 4x \cdot (4)\right]$

$= 2\cos 4x$

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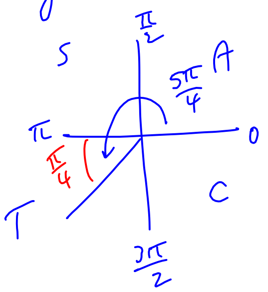
Ex.2 Find the slope of the tangent line to the curve given:

$y = (\sin x - \cos x)^2$ at $x = \dfrac{5\pi}{4}$

$y' = 2(\sin x - \cos x)'(\cos x + \sin x)$




$\sin\frac{5\pi}{4} = -\sin\frac{\pi}{4}$

$= \frac{-\sqrt{2}}{2}$

$\cos\frac{5\pi}{4} = \frac{-\sqrt{2}}{2}$

at $x = \frac{5\pi}{4}$, $y' = 2\left(\frac{-\sqrt{2}}{2} - \left(\frac{-\sqrt{2}}{2}\right)\right)\left(\frac{-\sqrt{2}}{2} + \left(\frac{-\sqrt{2}}{2}\right)\right)$

$y' = 2(0)\left(\frac{-2\sqrt{2}}{2}\right)$

$y' = 0$

$\therefore$ the slope of the tangent line at $x = \frac{5\pi}{4}$ is zero.

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Assigned Work:

p.256 #1, 2, 3, 5, 7, 9, 11, 12

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