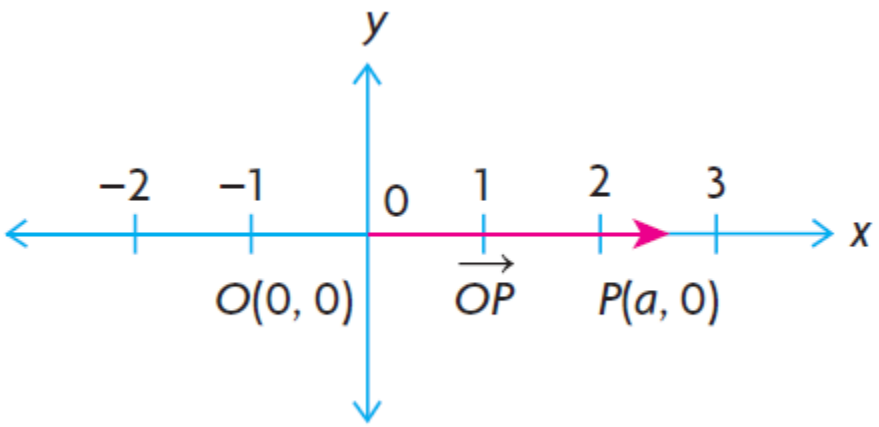
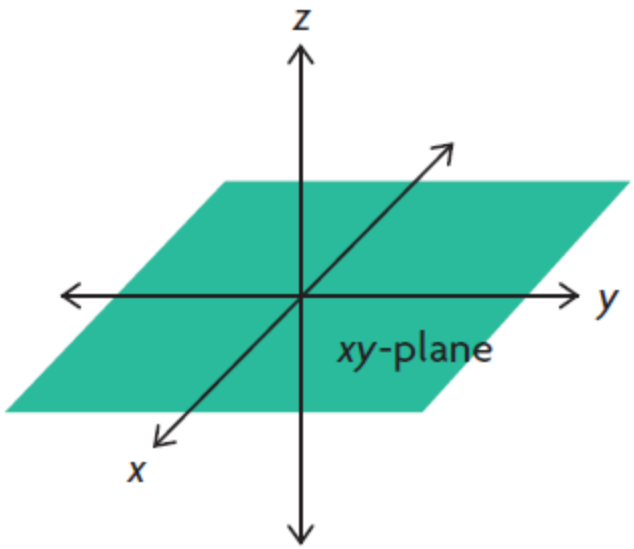
Section 6.5—Vectors in R^2 and R^3

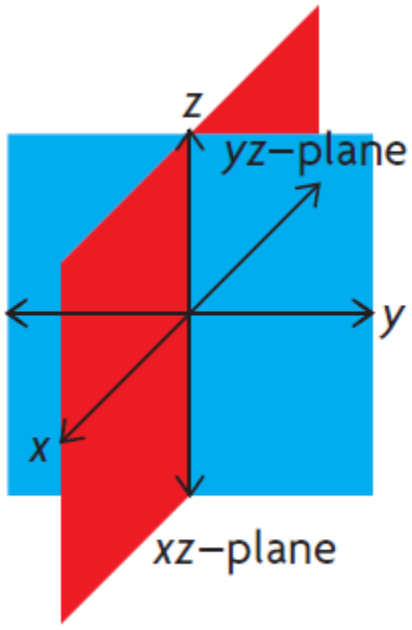
One of the most important ideas that we must consider is that of the **unique** representation of vectors in the xy-plane. The unique representation of the vector $\overrightarrow{OP}$ is a matter of showing the unique representation of the point P because $\overrightarrow{OP}$ is determined by this point. The uniqueness of vector representation will be first considered for the **position vector** $\overrightarrow{OP}$, which has its head at the point $P(a, 0)$ and its tail at the origin $O(0, 0)$ shown on the x-axis below. The x-axis is the set of real numbers, **R**, which is made up of rational and irrational numbers.

Points and Vectors in R^2

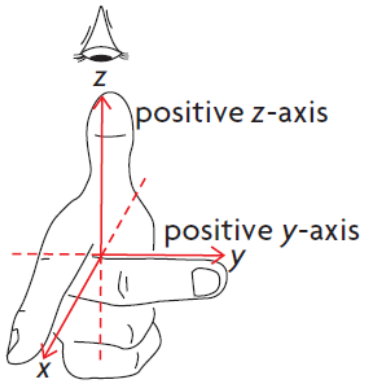


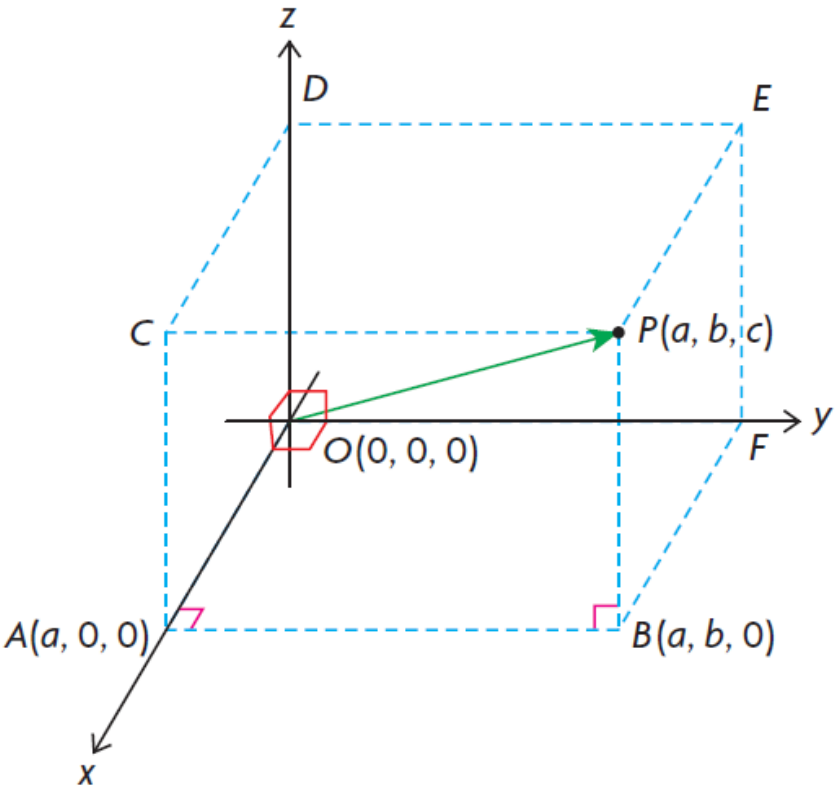
Points and Vectors in R^3


z
y
xy-plane
x


z
yz–plane
y
x
xz–plane

Right-Handed System of Coordinates



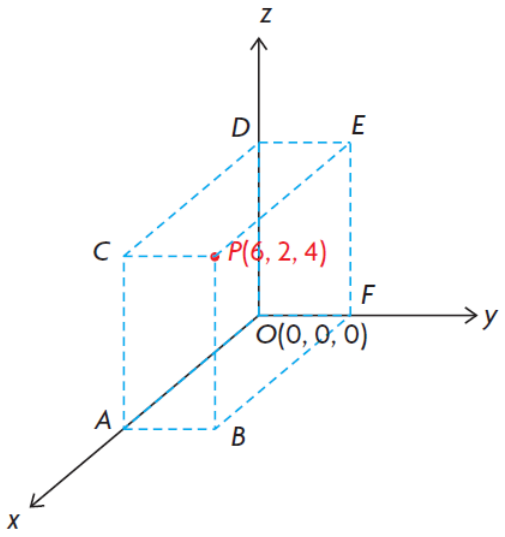


Each point $P(a, b, c)$ in R^3 has its location determined by an ordered triple. In the diagram above, the positive x-, y-, and z-axes are shown such that each pair of axes is perpendicular to the other and each axis represents a real number line. If we wish to locate $P(a, b, c)$, we move along the x-axis to $A(a, 0, 0)$, then in a direction perpendicular to the xz-plane, and parallel to the y-axis, to the point $B(a, b, 0)$. From there, we move in a direction perpendicular to the xy-plane and parallel to the z-axis to the point $P(a, b, c)$. This point is a vertex of a right rectangular prism.

Connecting the coordinates of points and vector components in R^3

a. In the following diagram, the point $P(6, 2, 4)$ is located in R^3. What are the coordinates of A, B, C, D, E, and F?

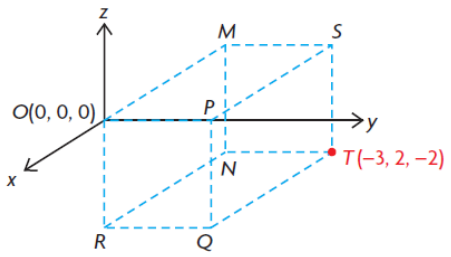
b. Draw the vector $\overrightarrow{OP}$.



3 Connecting the coordinates of points and vector components in R^3

a. In the following diagram, the point T is located in R^3. What are the coordinates of P, Q, R, M, N, and S?

b. Draw the vector $\overrightarrow{OT}$.



Representing planes in R^3 with equations

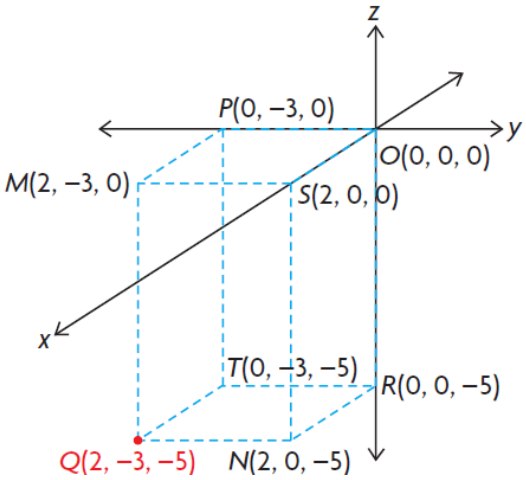
The point $Q(2, -3, -5)$ is shown in R^3.

a. Write an equation for the xy-plane.

b. Write an equation for the plane containing the points P, M, Q, and T.

c. Write a mathematical description of the set of points in rectangle $PMQT$.

d. What is the equation of the plane parallel to the xy-plane passing through $R(0, 0, -5)$?



IN SUMMARY

Key Idea

- In R^2 or R^3, the location of every point is unique. As a result, every vector drawn with its tail at the origin and its head at a point is also unique. This type of vector is called a position vector.

Need to Know

- In R^2, $P(a, b)$ is a point that is a units from $O(0, 0)$ along the x-axis and b units parallel to the y-axis.
- The position vector $\overrightarrow{OP}$ has its tail located at $O(0, 0)$ and its head at $P(a, b)$. $\overrightarrow{OP} = (a, b)$
- In R^3, $P(a, b, c)$ is a point that is a units from $O(0, 0, 0)$ along the x-axis, b units parallel to the y-axis, and c units parallel to the z-axis. The position vector $\overrightarrow{OP}$ has its tail located at $O(0, 0, 0)$ and its head at $P(a, b, c)$. $\overrightarrow{OP} = (a, b, c)$
- In R^3, the three mutually perpendicular axes form a *right-handed* system.