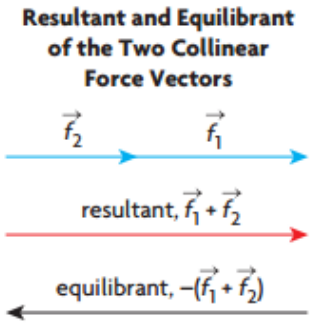
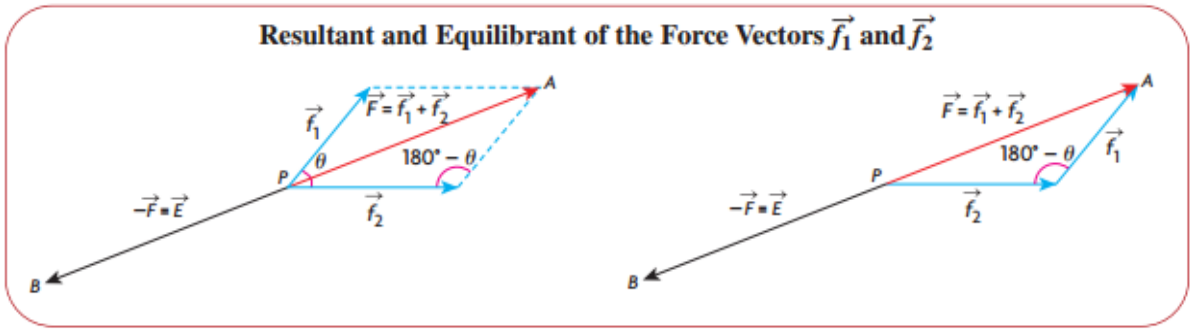
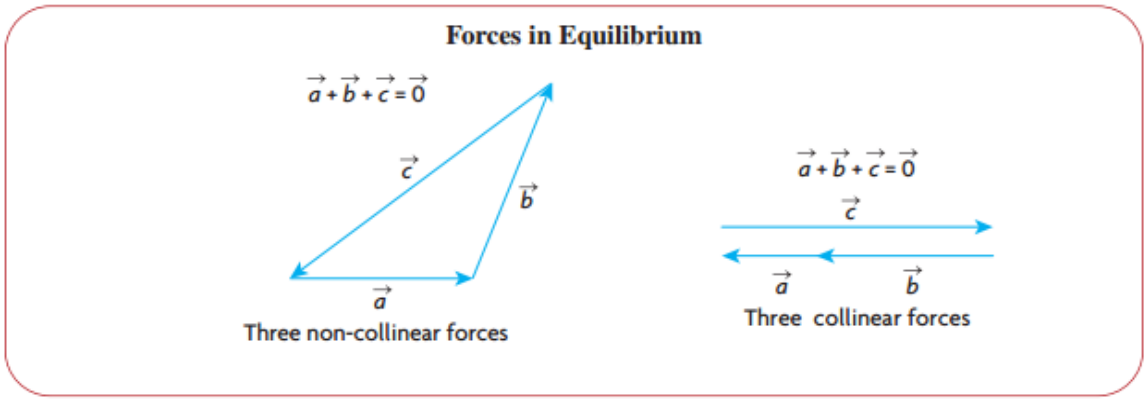
7.1 Vectors as Forces



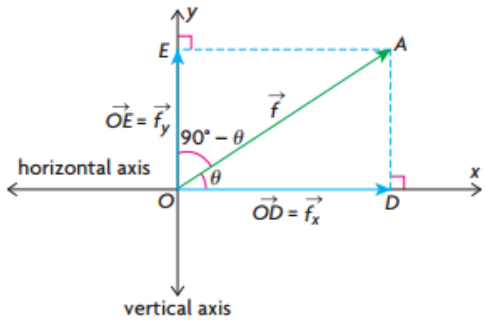




Ex1 Two forces act on a particle. f_1 acts due east and has a magnitude of 10N, f_2 acts northeast and has a magnitude of 20N. Find the force needed to keep the particle in equilibrium.

7.1 Vectors as Forces

Horizontal and Vertical Components



Resolution of a Vector into Horizontal and Vertical Components

If the vector $\vec{f}$ is resolved into its respective horizontal and vertical components, $\vec{f_x}$ and $\vec{f_y}$, then $|\vec{f_x}| = |\vec{f}|\cos\theta$ and $|\vec{f_y}| = |\vec{f}|\sin\theta$, where θ is the angle that $\vec{f}$ makes with the x-axis.

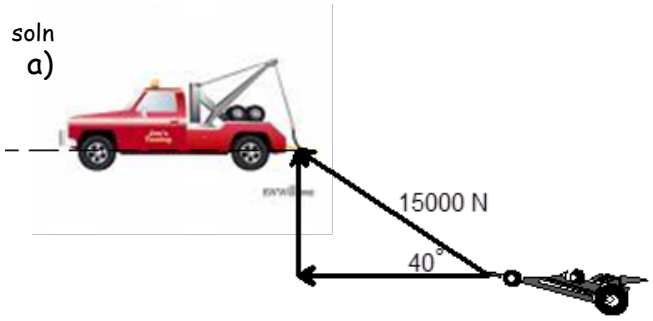
Ex2 A tow truck is pulling a car from a ditch. The tension in the cable is 15000 N at an angle of 40° to the horizontal.

a) Draw a diagram showing the resolution of the force into its rectangular components.

b) Determine the magnitude of the horizontal and vertical components of the force.

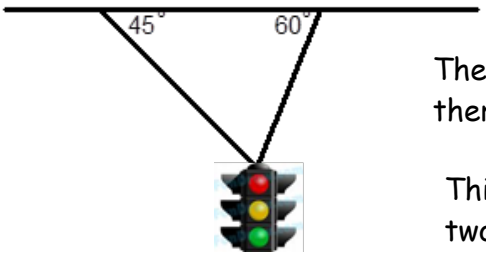
soln

a)



b)

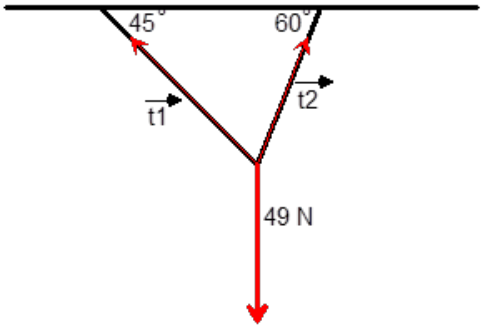
Ex3 A traffic light with a mass of 5 kg is suspended above the street as shown. Find the tensions (forces) in the wires.

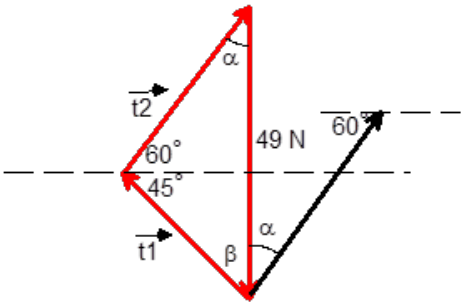


The force acting on the light is due to gravity, therefore there is a force of (5)(9.8) = 49 N down

This force is balanced by the forces (tensions) in the two wires. Let $\vec{t}_1$ and $\vec{t}_2$ represent these tensions.

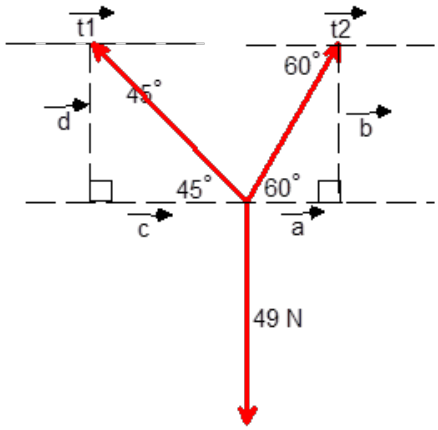
Redrawing the diagram, we get:





7.1 Vectors as Forces

We can also solve the same problem using components.



Calculate magnitudes:

$$|\vec{a}| = |\vec{t_2}| \cos 60°$$
$$|\vec{b}| = |\vec{t_2}| \sin 60°$$
$$|\vec{c}| = |\vec{t_1}| \cos 45°$$
$$|\vec{d}| = |\vec{t_1}| \sin 45°$$

For the system to be in equilibrium, the magnitudes of the horizontal components must balance each other and the magnitudes of the vertical components must balance each other.

$$|\vec{a}| = |\vec{c}|$$
$$|\vec{t_2}| \cos 60° = |\vec{t_1}| \cos 45° \quad [1]$$

$$|\vec{b}| + |\vec{d}| = 49$$
$$|\vec{t_2}| \sin 60° + |\vec{t_1}| \sin 45° = 49 \quad [2]$$

Now solve the system of equations.

$$|\vec{t_2}|\left(\frac{1}{2}\right) = |\vec{t_1}| \cos 45° \quad [1]$$

$$|\vec{t_2}| = 2|\vec{t_1}| \cos 45° \text{ or } |\vec{t_2}| = 2|\vec{t_1}|\left(\frac{1}{\sqrt{2}}\right)$$

$$|\vec{t_2}| \sin 60° + |\vec{t_1}| \sin 45° = 49 \quad [2]$$
$$\left(2|\vec{t_1}| \cos 45°\right) \sin 60° + |\vec{t_1}| \sin 45° = 49$$
$$2|\vec{t_1}|\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + |\vec{t_1}|\left(\frac{1}{\sqrt{2}}\right) = 49$$
$$|\vec{t_1}|\left(\frac{\sqrt{3}}{\sqrt{2}}\right) + |\vec{t_1}|\left(\frac{1}{\sqrt{2}}\right) = 49$$
$$|\vec{t_1}|\left(\frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 49$$
$$|\vec{t_1}| = \frac{49\sqrt{2}}{\sqrt{3}+1}$$
$$\doteq 25\,N$$

$$|\vec{t_2}| = 2|\vec{t_1}| \cos 45°$$
$$\doteq 36\,N$$

7.1 Assignment: p.363 # 5a 8 9 11 13 15 16