The Dot Product of Algebraic Vectors

Given $\vec{a} = (a_x, a_y) = a_x\vec{i} + a_y\vec{j}$ $\quad \vec{b} = (b_x, b_y) = b_x\vec{i} + b_y\vec{j}$

$$\vec{a} \cdot \vec{b} = (a_x\vec{i} + a_y\vec{j}) \cdot (b_x\vec{i} + b_y\vec{j}) \quad \text{FOIL}$$

$$= a_x b_x \underbrace{\vec{i} \cdot \vec{i}}_{1} + a_x b_y \underbrace{\vec{i} \cdot \vec{j}}_{0} + a_y b_x \underbrace{\vec{j} \cdot \vec{i}}_{0} + a_y b_y \underbrace{\vec{j} \cdot \vec{j}}_{1}$$

$$= a_x b_x + a_y b_y$$

$\vec{i} \cdot \vec{i} = |\vec{i}|^2$

$= 1$

$\vec{i} \cdot \vec{j} = 0$

$\vec{i} \perp \vec{j}$

In R^2: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$

In R^3: $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$

Ex.1 Given $\vec{a} = (\underset{x}{-1}, \underset{y}{4}, \underset{z}{6})$

$\vec{b} = (\underset{x}{2}, \underset{y}{1}, \underset{z}{-3})$

Determine $\vec{a} \cdot \vec{b}$

$\vec{a} \cdot \vec{b} = (-1)(2) + (4)(1) + (6)(-3)$

$= -2 + 4 + (-18)$

$= -16$

Ex.2 Find the angle between: $\vec{a} = (0, 5, 7)$
$\vec{b} = (2, 4, 6)$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
① ② ③ ④

① $\vec{a} \cdot \vec{b} = (0)(2) + (5)(4) + (7)(6)$
$= 0 + 20 + 42$
$= 62$

② $|\vec{a}| = \sqrt{0^2 + 5^2 + 7^2}$
$= \sqrt{25 + 49}$
$= \sqrt{74}$

③ $|\vec{b}| = \sqrt{2^2 + 4^2 + 6^2}$
$= \sqrt{4 + 16 + 36}$
$= \sqrt{56}$
$= 2\sqrt{14}$

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$
$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$\theta = \cos^{-1}\left[\dfrac{62}{\sqrt{74}\sqrt{56}}\right]$

$RAA \doteq 15.6°$

S (A)
T (C)
Reject QIV

$\boxed{\theta \doteq 15.6°}$

Ex.3 For what value(s) of k are the vectors perpendicular? $\vec{u} = (-1, 3, -4)$
$\vec{v} = (3, k, -2)$

1 : $\vec{u} \cdot \vec{v} = 0$

$(-1)(3) + (3)(k) + (-4)(-2) = 0$

$-3 + 3k + 8 = 0$

$3k = -5$

$k = \frac{-5}{3}$

Assigned Work:

p.385 # 2, 4, 6bd, 7b, 9b, 10a,
11, 12, 14

Read Example 5 on page 384