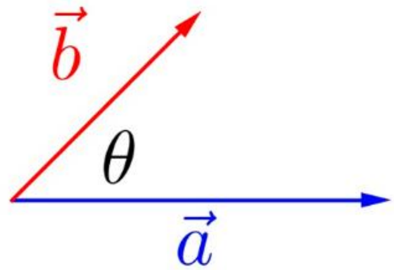
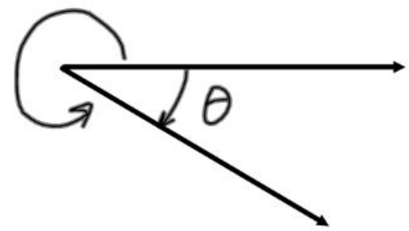
The Dot Product of Geometric Vectors

The dot product is one type of <u>vector multiplication</u>, but the product itself (i.e., the result) is a scalar.

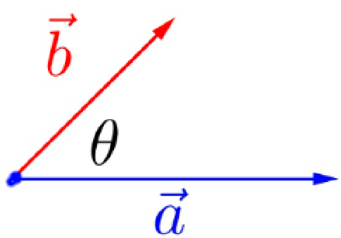




"a dot b"

$$\vec{a} \cdot \vec{b} = |\vec{a}| \left|\vec{b}\right| cos\theta \quad \text{for} \quad 0^\circ \leq \theta \leq 180^\circ$$

$\vec{a} \cdot \vec{b} = \underset{+}{|\vec{a}|}\underset{+}{|\vec{b}|} \cos\theta$

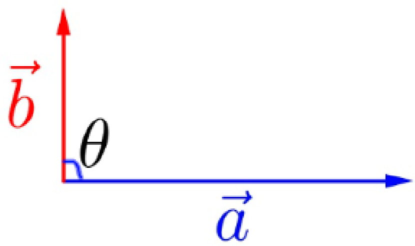


$$0^\circ \leq \theta < 90^\circ$$

$$cos\theta > 0$$

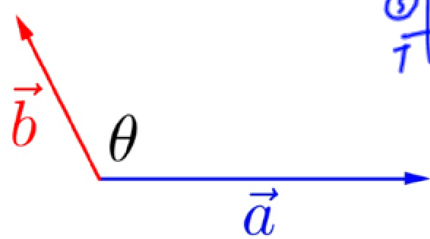
$$\therefore \vec{a} \cdot \vec{b} > 0$$

S | A
T | C



$$\theta = 90^\circ \quad cos 90^\circ = 0$$
$$\therefore \vec{a} \cdot \vec{b} = 0$$

Can be used to test for right angles.



$$90^\circ < \theta \leq 180^\circ$$
$$cos\theta < 0$$
$$\therefore \vec{a} \cdot \vec{b} < 0$$

Properties of the Dot Product:

(1) Commutative: $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

vector

(2) Distributive: $\vec{a} \cdot \left(\vec{b} + \vec{c}\right) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

vector

(3) Magnitudes: $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

$\vec{a} \cdot \vec{a}$
$= |\vec{a}||\vec{a}| \cancelto{1}{\cos 0}$
$= |\vec{a}|^2$

(4) Associative with scalar:

$$(k\vec{a}) \cdot \vec{b} = \vec{a} \cdot \left(k\vec{b}\right) = k\left(\vec{a} \cdot \vec{b}\right)$$

Ex.2 Show that

$$\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = |\vec{a}|^2 - \left|\vec{b}\right|^2$$

Ex.1 Find the angle between vectors **u** and **v** given:

(1) $|\vec{u}| = 3\,|\vec{v}|$ $\quad |\vec{u}| \neq 0 \quad |\vec{v}| \neq 0$

(2) $3\vec{u} + \vec{v}$ and $\vec{u} - 8\vec{v}$ are perpendicular.

Assigned Work

p.377 #2, 5, 6abe, 7acd, 9, 11, 12