Application of Dot Product & Cross Product

Mechanical work is the dot product of force and displacement vectors.

$$W = \vec{F} \cdot \vec{d} = |\vec{F}||\vec{d}|cos\theta$$

Force is measured in Newtons (N), and displacement in metres (m). Work has units of Joules (J).

$1\,J = 1\,N \cdot m$

Notes:

(1) This determines the work done by a particular force. There could be other forces acting on an object which are also affecting its motion.

(2) Negative work means the force actually opposes the motion of the object.

Ex.1 A 20 kg box is placed at the top of a 7m long ramp inclined at 30° to the horizontal. Find the work done by gravity as the box slides down the ramp.

$W = \vec{F} \cdot \vec{d}$

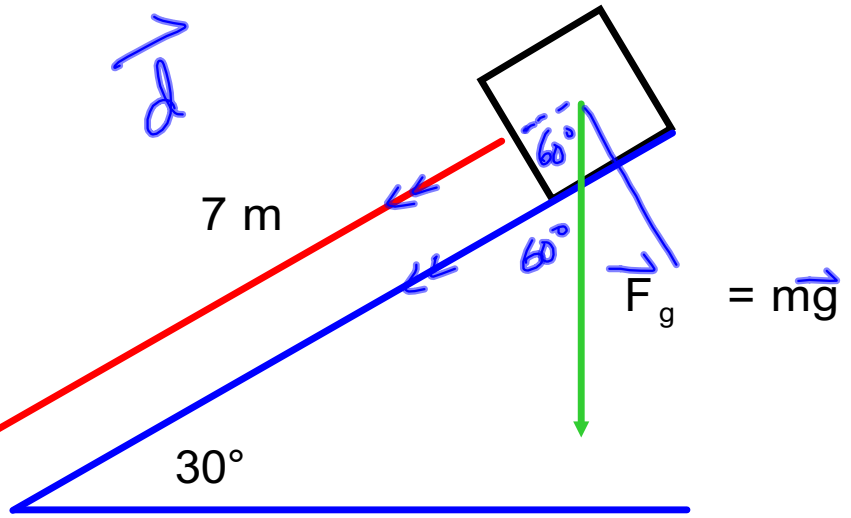
$= |\vec{F}_g||\vec{d}|\cos\theta$

$\doteq (20\,\text{kg})\left(9.8\,\frac{\text{N}}{\text{kg}}\right)(7\,\text{m})\cos 60°$

$\doteq 686\ \text{N m}$

$= 686\ \text{J}$

$\therefore$ the work done by gravity is 686 J.

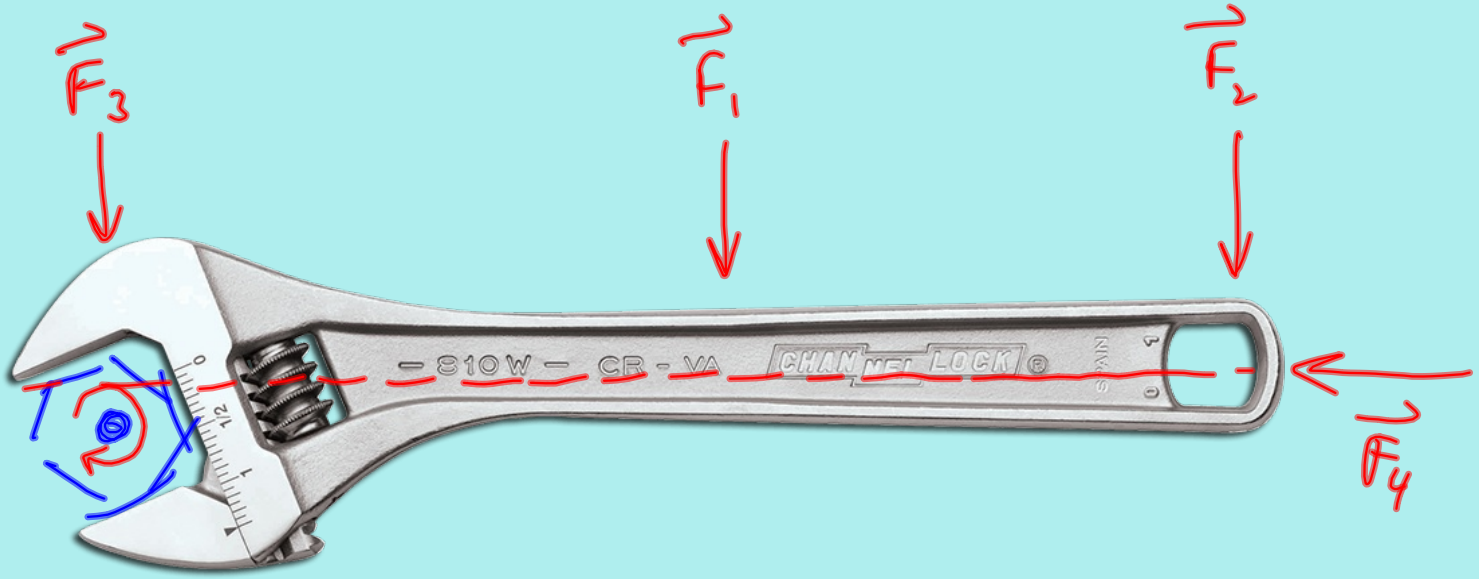


$|\vec{F}_g| \doteq (20)(9.8)$

$g \doteq 9.8\ \text{N/kg}$

Torque is a measure of how strongly an applied force will tend to rotate, or twist, an object.

It depends on the force and the effectiveness in which the force is applied.



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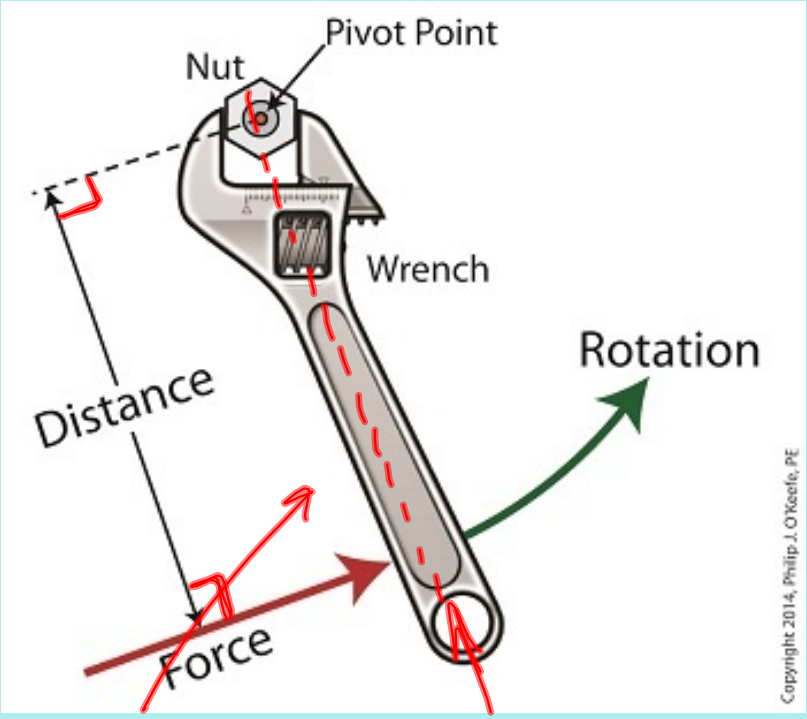

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$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$|\vec{\tau}| = |\vec{r}||\vec{F}|sin\theta$$

where $\vec{F}$ is the applied force, Newtons (N)

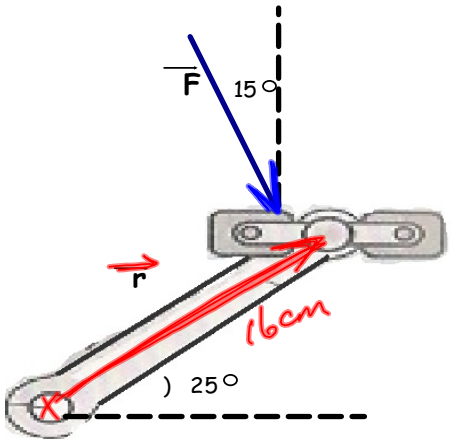
$\vec{r}$ is the moment arm, a vector from the point of rotation to the applied force, measured in metres (m)

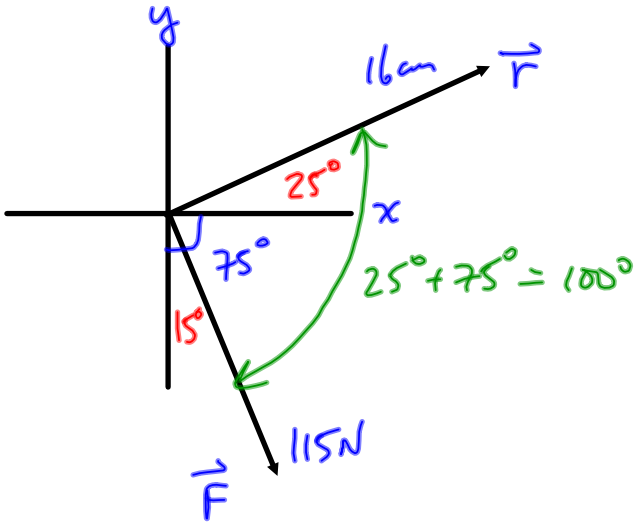
"tau"

$\vec{\tau}$ $\vec{\tau}$ is the torque, in Newton-metres (N m)

θ is the angle between the applied force and the moment arm

Ex.2 Find the magnitude of the torque produced by a cyclist exerting a force of 115N on a pedal in the position shown, if the shaft of the pedal is 16cm long.





$|\vec{\tau}| = |\vec{r}||\vec{F}| \sin\theta$

$= (16\text{cm})(115\text{N}) \sin 100^\circ$

$\doteq 1812\ \text{N cm} \times \frac{1\text{m}}{100\text{cm}}$

$\doteq 18.12\ \text{Nm}$

want $|\vec{\tau}|$ in Nm

$\therefore$ cyclist produces a torque of 18.12 Nm

Assigned Work:

p.414 # 1, 3, 5, 6, 8, 10

+

Handout (optional, posted online)