#1: The function f is defined by the graph represented in the right figure. Find: [K/U 3 marks]

a) $\lim_{x \to -4^-} f(x)$ 2

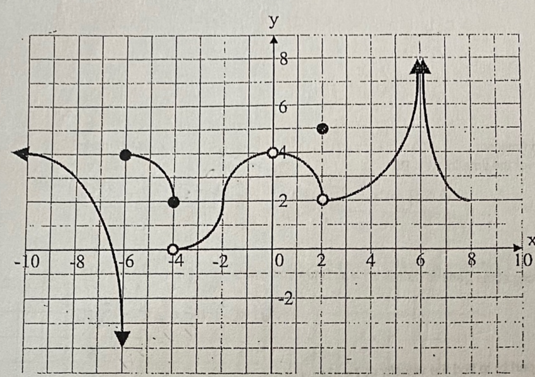
b) $\lim_{x \to -4^+} f(x)$ 0

c) $\lim_{x \to -4} f(x)$ DNE

d) $\lim_{x \to 0} f(x)$ 4

e) $\lim_{x \to 6} f(x)$ ∞

f) $\lim_{x \to -2} f(x)$ 2



#2: Given $\lim_{x \to 0} f(x) = 3$ and $\lim_{x \to 0} g(x) = -6$, use the limits properties to find $\lim_{x \to 0} \frac{3\sqrt{f(x) - g(x)}}{2g(x)}$ [K/U 2 marks]

$$\frac{3\sqrt{\lim_{x\to 0} f(x) - \lim_{x\to 0} g(x)}}{2\lim_{x\to 0} g(x)}$$

$$= \frac{3\sqrt{3-(-6)}}{2(-6)} = \frac{3 \cdot 3}{-12} = \frac{9}{-12} = \frac{3}{-4}$$

#3: Find each limit.

[1] a) $\lim_{x \to 1} \frac{x^3 - 8}{x - 2}$

$$\frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}} = 1+2+4 = 7$$

[2] b) $\lim_{x \to -4} \frac{x^2 + 6x + 8}{x^2 + 2x - 8}$

$$\frac{(x+2)\cancel{(x+4)}}{\cancel{(x+4)}(x-2)} = \frac{x+2}{x-2} = \frac{-4+2}{-4-2} = \frac{-2}{-6} = \frac{1}{3}$$

[2] c) $\lim_{x \to 2} \frac{x^4 - 16}{x^3 - 8}$

$$\frac{(x^2+4)(x+2)\cancel{(x-2)}}{\cancel{(x-2)}(x^2+2x+4)} = \frac{(4+4)(2+2)}{4+4+4} = \frac{8 \times 4}{12\,3} = \frac{8}{3}$$

[2] d) $\lim_{x \to 16} \frac{x - 16}{\sqrt{x} - 4}$

$$\frac{(\sqrt{x}+4)\cancel{(\sqrt{x}-4)}}{\cancel{\sqrt{x}-4}} = \sqrt{16} + 4 = 4 + 4 = 8$$

[2] e) $\lim_{x \to 3} \frac{x - 3}{|x - 3|}$

$$\begin{cases} 1, & x > 3 \\ -1, & x < 3 \end{cases}$$ DNE

[3] f) $\lim_{x \to 2} \frac{\sqrt{6 - x} - 2}{\sqrt{3 - x} - 1}$

$$\frac{(\sqrt{6-x}-2)(\sqrt{3-x}+1)}{(3-x)-1} = \frac{(\sqrt{6-x}-2)(\sqrt{3-x}+1)(\sqrt{6-x}+2)}{\cancel{2-x}(\sqrt{6-x}+2)} = \frac{(6-\cancel{x}-4)(2)(2)}{(\cancel{2-x})\cdot(2)}$$

$3-1-x$

$$-\frac{\cancel{\tfrac{1}{2}}(6-x)^{-\frac{1}{2}}}{\cancel{\tfrac{1}{2}}(3-x)^{-\frac{1}{2}}} = \frac{-\sqrt{3-x}}{\sqrt{6-x}} = \frac{-1}{\sqrt{4}} = \boxed{+\frac{1}{2}}$$

#4: Analyse the continuity of the following function. Graph the function. [A 4 marks]

$$f(x)=\begin{cases} x^2-2x+1, & x\le 0 \\ 1, & 0<x<4 \\ \sqrt{x}, & x\ge 4\end{cases}$$

x=1 continuous
x=4 not continuous

#5: Consider the following position function: $s(t)=t^2-t^3$. [A 6 marks]

[1] a) Find the average velocity over the time interval [1,2]

$\frac{S(2)-S(1)}{2-1} = \frac{(4-8)-(0)}{1} = \frac{-4}{1} = -4m/s$

[3] b) Find the instantaneous velocity at the generic moment $t=a$. Show your work.

$V(t) = 2t-3t^2$

[1] c) Use the formula you get at part b) to find the velocity at $t=-2s$.

$V(2) = 4-3(4) = 4-12 = -8m/s$

[1] d) Find the moments when the particle is at rest.

$\cancel{t^2-t^3=0}$
$\cancel{t^2(1-t)=0}$
$\cancel{t=0 \quad t=1}$

$2t-3t^2=0$
$t(2-3t)=0$
$t=0 \quad t=\frac{2}{3}$.

#6: Determine the instantaneous rate of change in the surface area of a spherical balloon (as it is inflated) with respect to it radius, at the point in time when the radius reaches $20cm$. [A 4 marks]

Hint: $A=4\pi r^2$

$4\pi r^2 \quad A' = 8\pi r = 8\pi(20) = 160\pi$.

#7: Use technology (a scientific calculator) to estimate the slope of the tangent line to the curve $y=x+\frac{1}{\sqrt{x-1}}$ at the point $P(2,3)$ by using $h_1=0.1$, $h_2=0.01$, and $h_3=0.001$. Show your work. [A 4 marks]

0.5

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#8: Compute the limit $\lim_{x\to 1}\frac{\frac{1}{x}-\frac{1}{\sqrt{x}}}{\sqrt[3]{x}-1}$. [TIPS 3 marks]

$\sqrt{x} \quad x^{\frac{1}{2}} = (x^{\frac{1}{6}})^3 = a^3$
$\sqrt[3]{x} \quad x^{\frac{1}{3}} = (x^{\frac{1}{6}})^2 = a^2$
$x^{\frac{1}{6}} = a$

$$\frac{\frac{1}{a^6}-\frac{1}{a^3}}{a^2-1} = \frac{\frac{1-a^3}{a^6}}{a^2-1} = \frac{-(a^3-1)}{a^6(a^2-1)} = \frac{-\cancel{(a-1)}(a^2+1+a)}{a^6(a+1)\cancel{(a-1)}} = \frac{-(a^2+1+a)}{a^6(a+1)} = \frac{-3}{2}$$