THE ERINDALE ACADEMY
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Unit 2: Derivatives

Lesson 1: Rules of derivatives

We have seen that derivatives of functions are of practical use because they represent instantaneous rates of change.

Computing derivatives from the limit definition, first principle, is tedious and time-consuming. So in this unit, we will study on previously developed rules that simplify the process of differentiation.

Name	Rules	Leibniz Notation
Constant Function Rule:	*If* $f(x) = k$, where k is a constant, then $\boldsymbol{f'(x) = 0}$.	$\frac{d}{dx}(k) = 0$
Power Rule:	*If* $f(x) = x^n$, where n is a real number, then $\boldsymbol{f'(x) = nx^{n-1}}$	$\frac{d}{dx}(x^n) = nx^{n-1}$
Product Rule:	If $y = f(x)g(x)$, where $f(x)$ *and* $g(x)$ are two generic functions, then $\boldsymbol{y' = f'(x)g(x) + f(x)g'(x)}$. Similarly, If $y = f(x)g(x)h(x)$, where $f(x)$, $g(x)$ and $h(x)$are three generic functions, then $\boldsymbol{y' = f'(x)g(x)h(x) + f(x)g'(x)h(x) +}$ $\boldsymbol{f(x)g(x)h'(x).}$	$\frac{d}{dx}(f \times g) =$ $\frac{df}{dx} \times g + f \times \frac{dg}{dx}$
Quotient Rule:	If $y = \frac{f(x)}{g(x)}$, where $f(x)$ *and* $g(x)$ are two generic functions, then $\boldsymbol{y' = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}}$	$\frac{d}{dx}(f \div g)$ $= \frac{\frac{df}{dx} \times g - f \times \frac{dg}{dx}}{g^2}$
Chain Rule:	If *f and g* are functions that have derivatives, then the composite function $h(x) = (f \circ g)(x) = f(g(x))$ has a derivative given by: $\boldsymbol{h'(x) = f'(g(x)) \times g'(x)}$	$\frac{d}{dx}(h) = \frac{df}{dg} \times \frac{dg}{dx}$

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Part I: Constant rule, power rule, and product rule.

Example 1: Find the derivative of each function.

a) $f(x) = -x^2$

b) $f(x) = \sqrt[3]{x} + 5x + 8$

c) $f(x) = (2x + 4)(3x - 5)$

d) $f(x) = (3x^2 + 4x - 6)(2x^2 - 3x - 9)$

Pause a sec and practice a bit before we move on: Textbook pg90. #1abcde, 6, 7b, 12

1. Use the product rule to differentiate each function. Simplify your answers.

a. $h(x) = x(x - 4)$

b. $h(x) = x^2(2x - 1)$

c. $h(x) = (3x + 2)(2x - 7)$

d. $h(x) = (5x^7 + 1)(x^2 - 2x)$

e. $s(t) = (t^2 + 1)(3 - 2t^2)$

f. $f(x) = \dfrac{x - 3}{x + 3}$

2. Use the product rule and the power of a function rule to differentiate the following functions. Do not simplify.

a. $y = (5x + 1)^3(x - 4)$

b. $y = (3x^2 + 4)(3 + x^3)^5$

c. $y = (1 - x^2)^4(2x + 6)^3$

d. $y = (x^2 - 9)^4(2x - 1)^3$

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6. Determine the equation of the tangent to the curve $y = (x^3 - 5x + 2)(3x^2 - 2x)$ at the point $(1, -2)$.

7. Determine the point(s) where the tangent to the curve is horizontal.

   a. $y = 2(x - 29)(x + 1)$
   b. $y = (x^2 + 2x + 1)(x^2 + 2x + 1)$

8. Use the extended product rule to differentiate the following functions. Do not simplify.

   a. $y = (x + 1)^3(x + 4)(x - 3)^2$
   b. $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

9. A 75 L gas tank has a leak. After t hours, the remaining volume, V, in litres is $V(t) = 75\left(1 - \frac{t}{24}\right)^2$, $0 \le t \le 24$. Use the product rule to determine how quickly the gas is leaking from the tank when the tank is 60% full of gas.

10. Determine the slope of the tangent to $h(x) = 2x(x + 1)^3(x^2 + 2x + 1)^2$ at $x = -2$. Explain how to find the equation of the normal at $x = -2$.

PART C

11. a. Determine an expression for $f'(x)$ if $f(x) = g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x)$.
    b. If $f(x) = (1 + x)(1 + 2x)(1 + 3x) \dots (1 + nx)$, find $f'(0)$.

12. Determine a quadratic function $f(x) = ax^2 + bx + c$ if its graph passes through the point $(2, 19)$ and it has a horizontal tangent at $(-1, -8)$.

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Part II: Quotient rule

Example 2: Find the derivative of each function.

1) $f(x) = \frac{x^2-5x}{x^3+1}$

2) $g(x) = \frac{x^2-4x-12}{\sqrt{x}-2}$

3) Find the coordinate(s) on the curve of $f(x) = \frac{2x+8}{\sqrt{x}}$, where the tangent line is horizontal.

Practice from textbook: pg97. # 4, 5cd, 6, 7, 8, 9

4. Use the quotient rule to differentiate each function. Simplify your answers.

a. $h(x) = \frac{x}{x+1}$

b. $h(t) = \frac{2t-3}{t+5}$

c. $h(x) = \frac{x^3}{2x^2-1}$

d. $h(x) = \frac{1}{x^2+3}$

e. $y = \frac{x(3x+5)}{1-x^2}$

f. $y = \frac{x^2-x+1}{x^2+3}$

6. Determine the slope of the tangent to the curve $y = \frac{x^3}{x^2 - 6}$ at point $(3, 9)$.
7. Determine the points on the graph of $y = \frac{3x}{x - 4}$ where the slope of the tangent is $-\frac{12}{25}$.
8. Show that there are no tangents to the graph of $f(x) = \frac{5x + 2}{x + 2}$ that have a negative slope.

9. Find the point(s) at which the tangent to the curve is horizontal.

   a. $y = \frac{2x^2}{x - 4}$

   b. $y = \frac{x^2 - 1}{x^2 + x - 2}$

Part III: Chain rule

Take the derivative of the outer function, leave the inner function unchanged, then multiply by the derivative of the inner function.

Example 3: Find the derivatives of the following.

a) $y = (x^3 + 2x^2 - 3x + 5)^4$

b) $f(x) = \sqrt{x^2 - 5}$

c) $g(x) = \frac{2}{(x^3 - 27)^4}$

d) $f(x) = x(2x + 7)^4(x - 1)^2$

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Practice:

Textbook: Pg106

8. Differentiate each function. Express your answer in a simplified factored form.
   a. $f(x) = (x + 4)^3(x - 3)^6$
   b. $y = (x^2 + 3)^3(x^3 + 3)^2$
   c. $y = \dfrac{3x^2 + 2x}{x^2 + 1}$
   d. $h(x) = x^3(3x - 5)^2$
   e. $y = x^4(1 - 4x^2)^3$
   f. $y = \left(\dfrac{x^2 - 3}{x^2 + 3}\right)^4$

9. Find the rate of change of each function at the given value of t. Leave your answers as rational numbers, or in terms of roots and the number π.
   a. $s(t) = t^{\frac{1}{3}}(4t - 5)^{\frac{2}{3}}, t = 8$
   b. $s(t) = \left(\dfrac{t - \pi}{t - 6\pi}\right)^{\frac{1}{3}}, t = 2\pi$

10. For what values of x do the curves $y = (1 + x^3)^2$ and $y = 2x^6$ have the same slope?

11. Find the slope of the tangent to the curve $y = (3x - x^2)^{-2}$ at $\left(2, \frac{1}{4}\right)$.

12. Find the equation of the tangent to the curve $y = (x^3 - 7)^5$ at $x = 2$.

13. Use the chain rule, in Leibniz notation, to find $\dfrac{dy}{dx}$ at the given value of x.
    a. $y = 3u^2 - 5u + 2, u = x^2 - 1, x = 2$
    b. $y = 2u^3 + 3u^2, u = x + x^{\frac{1}{2}}, x = 1$
    c. $y = u(u^2 + 3)^3, u = (x + 3)^2, x = -2$
    d. $y = u^3 - 5(u^3 - 7u)^2, u = \sqrt{x}, x = 4$

14. Find $h'(2)$, given $h(x) = f(g(x)), f(u) = u^2 - 1, g(2) = 3$, and $g'(2) = -1$.

Put it ALL together:

Textbook pg110 - 113. #1, 4, 5, 6, 7, 8, 9, 11, 12, 13, 17, 18, 19, 22, 23, 26, 27, 28, 29
Pg114. #1, 3, 4, 5, 6, 7, 8, 9, 10, 11