



## Unit 2: Derivatives

### Implicit differentiation

You might ask that what is implicit differentiation? And how implicit and explicit differ from each other?

**Explicit function** is a function where the dependent variable can be separated with independent variables. For example,  $x + 2y = 0$  is explicit because eventually  $y$  can be isolated in terms of  $x$  as  $y = -\frac{x}{2}$ .

However, **implicit function** is when the dependent variable,  $x$ , cannot be detached with  $y$ . For instance,  $\sin(x + e^y) = 3y$ .

How do I perform implicit differentiation?

In implicit differentiation, we differentiate each side of an equation with two variables (usually  $x$  and  $y$ ) by treating one of the variables as a function of the other (usually  $y$  is a function of  $x$ ). This calls for using the chain rule.

Example 1: Find  $\frac{dy}{dx}$  if  $2x^5 + x^4y + y^5 = 36$ . And find the slope of the tangent to the curve at the point (1, 2).

Example 2: Find  $y'$  if  $x^2 + \sqrt{y} = x^2y^3 + 5$

Example 3: If  $\sin x + \sin y = 1$ , find the derivative of  $y$  with respect with  $x$ .



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### Method of Logarithmic differentiation

The calculation of derivatives of complicated functions involving products, quotients, and powers can often be simplified by taking logarithms. The method used in the following example is called logarithmic differentiation.

Steps:

1. Take logarithms of both sides of an equation  $y = f(x)$ .
2. Differentiate implicitly with respect to  $x$ .
3. Solve the resulting equation for  $y$ .

Example: Find  $y$  prime.

a)  $y = \frac{e^x \sqrt{x^2+1}}{(x^2+2)^3}$

b)  $y = \sqrt[3]{\frac{x \cos x}{x^2-1}}$

c)  $y = x^{\sin x}, x > 0$