THE ERINDALE ACADEMY
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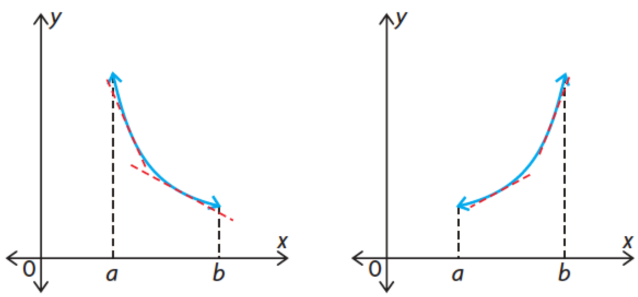
Unit 3: Curve Sketching, Optimization, and Related rates

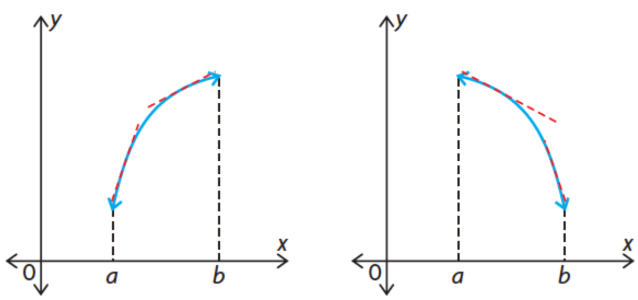
Lesson 3.2: Concavity, second derivative, and curve sketching

Concavity and the Second Derivative

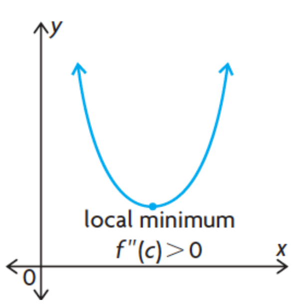
1. The graph of $y = f(x)$ is **concave up** on an interval $a \leq x \leq b$ in which the slopes of $f(x)$ are increasing. On this interval, $f''(x)$ exists and $f''(x) > 0$. The graph of the function is above the tangent at every point on the interval.



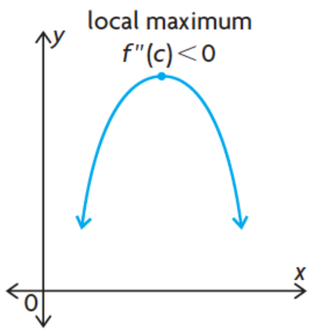
2. The graph of $y = f(x)$ is **concave down** on an interval $a \leq x \leq b$ in which the slopes of $f(x)$ are decreasing. On this interval, $f''(x)$ exists and $f''(x) < 0$. The graph of the function is below the tangent at every point on the interval.



3. If $y = f(x)$ has a critical point at $x = c$, with $f'(c) = 0$, then the behaviour of $f(x)$ at $x = c$ can be analyzed through the use of the **second derivative test** by analyzing $f''(c)$, as follows:
   a. The graph is concave up, and $x = c$ is the location of a local minimum value of the function, if $f''(c) > 0$.

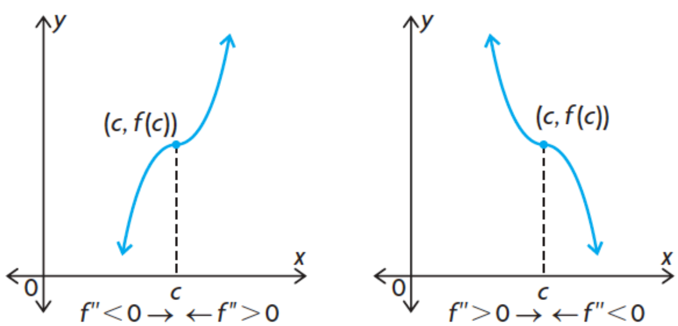


b. The graph is concave down, and $x = c$ is the location of a local maximum value of the function, if $f''(c) < 0$.



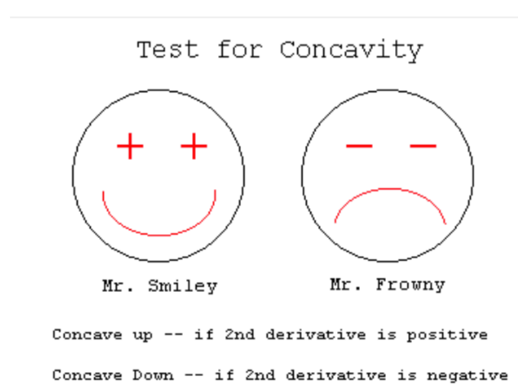
c. If $f''(c) = 0$, the nature of the critical point cannot be determined without further work.

4. A **point of inflection** occurs at $(c, f(c))$ on the graph of $y = f(x)$ if $f''(x)$ changes sign at $x = c$. That is, the curve changes from concave down to concave up, or vice versa.



5. All points of inflection on the graph of $y = f(x)$ must occur either where $\frac{d^2y}{dx^2}$ equals zero or where $\frac{d^2y}{dx^2}$ is undefined.

In the following examples, we will use these properties to sketch graphs of other functions.



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Example 1: Sketch the graph of $y = x^3 - 3x^2 - 9x + 10$

Algorithm:
Step 1: x-intercept and y-intercept
Step 2: Asymptotes and end behavior
Step 3: First derivative – Interval of increase and decrease & Point of local maximum and minimum (absolute maximum/minimum if necessary)
Step 4: Second derivative – interval of concavity & Point of inflection
Step 5: Put all together and do curve sketching

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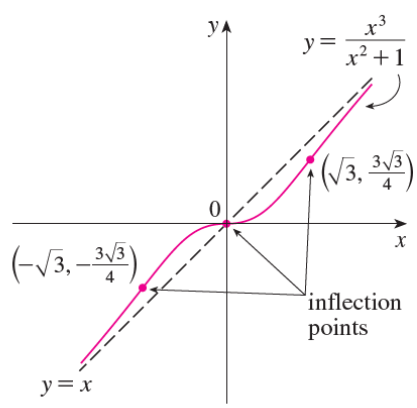
Example 2: Sketch the graph of $y = \frac{x^3}{x^2+1}$ with slant asymptotes in a form of $y = mx + b$, getting from long division.



Practice: Sketch the graph of $f(x) = \frac{4x^2-3}{x^3}$.

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Example 3: Sketch the graph of $f(x) = \frac{sinx}{1-sinx}$, $[-\pi, \pi]$

Practice: Sketch the graph of $f(x) = 4sin^2x - 1$, $[0, 2\pi]$