Worksheet: Use the properties of derivative to determine the shape of curve

1. Show that the graph of $\mathrm{f(x)} = ax^4 + bx^3$ has two points of inflection. Show that the x-coordinate of one of these points lies midway between the x-intercepts.

2. Find the value of the constant b such that the function $\mathrm{f(x)} = \sqrt{x+1} + \frac{b}{x}$ has a point of inflection at $\mathrm{x} = 3$.

3. Sketch the graph of a function with the following properties:

a)

- $f'(x) > 0$ when $x < 2$ and when $2 < x < 5$
- $f'(x) < 0$ when $x > 5$
- $f'(2) = 0$ and $f'(5) = 0$
- $f''(x) < 0$ when $x < 2$ and when $4 < x < 7$
- $f''(x) > 0$ when $2 < x < 4$ and when $x > 7$
- $f(0) = -4$

b)

- $f(-10) = f(-2) = f(8) = 0, f(0) = -6, f(12) = 4$
- $f'(-6) = 0, f''(-6) < 0, f(-6) = 6$
- $f''(x) < 0$ *when* $x \in (-\infty, 1), (1,\ 12), f''(x) > 0$ *when* $x > 12$
- $f(1) = -14, f'(1) = DNE$ *and* $\lim\limits_{x\to 1^-} f'(x) = -\infty$ *and* $\lim\limits_{x\to 1^+} f'(x) = \infty$

c)

- *Domain:* $\{\mathrm{x} | \mathrm{x} \neq -6, \mathrm{x} \in \mathrm{R}\}$
- $f(-3) = 4, f(0) = 2, f(3) = 0, f(5) = 1, f(7) = 3, f(9) = 2.5$
- $f''(x) > 0$ *when* $x \in (-\infty, -6), (0,\ 5), (9,\ \infty)$
- $\lim\limits_{x\to\pm\infty} f(x) = 2,\ \lim\limits_{x\to -6^+} f(x) = -\infty$
- $f'(-3) = 0,\ f''(-3) < 0,\ f'(3) = 0,\ f''(3) > 0,\ f'(7) = 0,\ f''(7) < 0$
- $f''(0) = f''(5) = f''(9) = 0$