

8.2 Cartesian Equation of a Line

A Symmetric Equation

The parametric equations of a line in \mathbb{R}^2 :

$$\begin{cases} x = x_0 + tu_x \\ y = y_0 + tu_y \end{cases} \quad t \in \mathbb{R}$$

may be written as:

$$\frac{x - x_0}{u_x} = \frac{y - y_0}{u_y} = t, \quad t \in \mathbb{R}$$

The *symmetric equation* of the line is (if exists):

$$\frac{x - x_0}{u_x} = \frac{y - y_0}{u_y}$$

Note. The symmetric equation does exist if $u_x \neq 0$ and $u_y \neq 0$.

Ex 1. Convert the vector or parametric equations to the symmetric equation (if exists).

a) $\begin{cases} x = -3 + 2t \\ y = -5t \end{cases} \quad t \in \mathbb{R}$

b) $\vec{r} = (0,1) + t(1,-2), \quad t \in \mathbb{R}$

c) $\vec{r} = (-2,3) + t(2,0), \quad t \in \mathbb{R}$

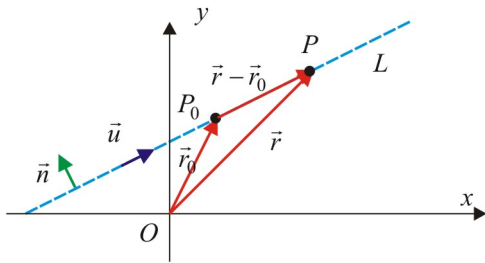
Ex 2. Convert the symmetric equation of a line L :

$$\frac{x+2}{-1} = \frac{y-3}{2}$$

to the vector equation.

B Normal Equation

Let consider a line L that passes through the specific point $P_0(x_0, y_0)$ and has the *direction vector* $\vec{u} = (u_x, u_y)$.



The vectors $\vec{n} = (-u_y, u_x) = (A, B)$ or

$\vec{n} = (u_y, -u_x) = (A, B)$ are perpendicular to the vector

\vec{u} and so they are perpendicular to the line L .

These are called *normal vectors* to the line L .

Let $P(x, y)$ be a generic point on the line L . So:

$$\overrightarrow{P_0P} \parallel \vec{u} \Rightarrow \overrightarrow{P_0P} \perp \vec{n} \Rightarrow \overrightarrow{P_0P} \cdot \vec{n} = 0$$

$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

The *normal equation* of a line is given by:

$$(\vec{r} - \vec{r}_0) \cdot \vec{n} = 0$$

C Cartesian Equation

The normal equation can be written as:

$$\vec{r} \cdot \vec{n} - \vec{r}_0 \cdot \vec{n} = 0$$

$$(x, y) \cdot (A, B) - (x_0, y_0) \cdot (A, B) = 0$$

$$Ax + By - Ax_0 - By_0 = 0$$

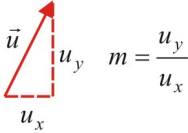
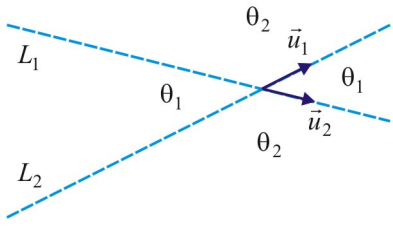
$$Ax + By + C = 0 \text{ where } C = -Ax_0 - By_0$$

The *Cartesian equation* of a line is given by:

$$Ax + By + C = 0$$

where

$\vec{n} = (A, B)$ is a *normal vector* and the constant C depends on a specific point of the line.

<p>Ex 3. Convert the vector equation of the line $L: \vec{r} = (1,2) + t(3,-2)$, $t \in R$ to the Cartesian equation.</p>	<p>Ex 4. Convert the Cartesian equation to the parametric equations and then to the vector equation. $L: -x + 2y + 3 = 0$</p>
<p>D Slope y-intercept Equation Let solve the symmetric equation of a line:</p> $\frac{x-x_0}{u_x} = \frac{y-y_0}{u_y} = t, \quad t \in R$ <p>for y:</p> $y - y_0 = u_y \frac{x - x_0}{u_x}$ $y = \frac{u_y}{u_x}x + y_0 - \frac{u_y}{u_x}x_0$ <p>The <i>slope y-intercept equation</i> of a line in R^2 is given by:</p> $y = mx + n$ $m = \frac{u_y}{u_x}$  <p>where m is the <i>slope</i> and n is the <i>y-intercept</i> which depends on a specific point of the line.</p>	<p>Ex 5. Convert the vector equation to the slope y-intercept equation: $L: \vec{r} = (-2,3) + t(1,-2)$</p> <p>Ex 6. Convert the slope y-intercept equation to the vector equation. $L: y = -\frac{2}{3}x + 4$.</p>
<p>E Angle between two Lines The <i>angle</i> between two lines is determined by the angle between the <i>direction vectors</i>:</p> $\cos \theta = \frac{\vec{u}_1 \cdot \vec{u}_2}{\ \vec{u}_1\ \ \vec{u}_2\ }$ <p>Note. There are two pairs of equal angles between two lines (see the figure below).</p>  <p>Note:</p> $\theta_1 + \theta_2 = 180^\circ$	<p>Ex 7. Find the acute angle between each pair of lines.</p> <p>a) $L_1: \vec{r} = (0,1) + t(1,-3)$, $t \in R$; $L_2: \frac{x-2}{3} = \frac{y+1}{-2}$</p> <p>b) $L_1: 2x - 3y + 6 = 0$; $L_2: y = -\frac{1}{3}x + 2$</p>

Reading: Nelson Textbook, Pages 435-442

Homework: Nelson Textbook: Page 443 #1, 3, 4, 5, 6, 7, 8, 10ab, 11, 14