EXAM REVIEW FOR MHF 4U
ADVANCED FUNCTIONS

ADDITIONAL REVIEW EXERCISES

Expressions

1. Factor fully.
   a) $x^3 + 3x^2 - 25x - 75$
   b) $x^3 + x^2 - 14x - 24$
   c) $64x^3 + 27y^3$
   d) $30x^3 + 17x^2 - 8x - 4$
   e) $x^6 - 124x^3 - 125$
   f) $x^7 - 8x^4 - 16x^3 + 128$

2. Simplify: $\sqrt{\dfrac{(x^a)^4}{x^{3a+b}}} \bullet \dfrac{(x^b)^4}{x^{a+3b}}$

3. Find the exact value of the following.
   a) $7^{\log_7 \sqrt{5}}$
   b) $\log_{64} \sqrt[6]{8}$
   c) $\log_8 6 - \log_8 3 + \log_8 4$
   d) $\log_9 \left(3^7 \bullet \sqrt[5]{81}\right)$

4. Write as a single logarithm: $a\log_5(x-7) - \dfrac{2}{3}\log_5 w + 2$

5. Let $f(x) = \{(3,2),(5,1),(7,4),(9,3),(11,5)\}$ and $g(x) = \{(1,3),(2,5),(3,7),(4,9),(5,11)\}$. Determine:
   a) $f(g(3))$
   b) $(g \circ f)(9)$
   c) $(f-g)(x)$
   d) $(f+g)(x)$

6. Convert to degrees.
   a) $\dfrac{11\pi}{15}$ radians
   b) 56 radians

7. Convert to radians.
   a) 420˚
   b) -24˚

8. Find the exact value of the following.
   a) $\cos\dfrac{3\pi}{4}$
   b) $\csc\left(\dfrac{-3\pi}{2}\right)$
   c) $\tan\dfrac{11\pi}{6}$
   d) $\sin\dfrac{7\pi}{12}$
   e) $\sec\left(\dfrac{5\pi}{6}\right)\cos\left(\dfrac{7\pi}{4}\right) - \cot\left(\dfrac{-\pi}{3}\right)$

9. Given: $\sin A = \dfrac{-6}{7}$, $\dfrac{3\pi}{2} \le A \le 2\pi$, and $\tan B = \dfrac{2}{3}$, $\pi \le B \le \dfrac{3\pi}{2}$

   Find the exact value of the following.
   a) $\sec A$
   b) $\cos 2B$
   c) $\sin(A+B)$
   d) $\tan(A-B)$

10. Given: $f(x) = \dfrac{1}{x-5}$ and $g(x) = x^2 + 8$

    Find:
    a) $(f-g)(x)$
    b) $\left(\dfrac{g}{f}\right)(x)$
    c) $(f \circ g)(x)$
    d) $(g \circ f)(x)$
    e) $(g \circ (g))(x)$
    f) $f^{-1}(x)$
    g) $g^{-1}(x)$
    h) $(f \circ f^{-1})(x)$
    i) $(g-f)(3)$
    j) $(fg)(-1)$
    k) $(f \circ g)(5)$
    l) $(g \circ f)(5)$

Equations, Inequalities & Identities

11. Solve. Exact answers are required, where possible. Otherwise, express answers correct to one decimal place. Where necessary, state restrictions.

a) $x^3 - 3x^2 = 4x - 12$

b) $x^3 - 5x = 5x^2 - 1$

c) $x^3 + 4x^2 + 9x + 10 = 0$

d) $x + \frac{1}{x-4} = 0$

e) $\frac{2x}{x-1} + \frac{1}{x-3} = \frac{2}{x^2-4x+3}$

f) $4(7^{x-2}) = 8$

g) $\log_4(x+3) = 2$

h) $\log_7(x+2) = 1 - \log_7(x-4)$

i) $\log_9(x-5) + \log_9(x+3) = 1$

j) $\log_5(x+1) + \log_5 2 - \log_5(x+3) = \log_5(x-1)$

k) $5 \cdot 8^{x+2} = 5^{7x}$

l) $\left(4^2\right)\left(2^{2x-3}\right) = \left(16^{x-2}\right)\left(\frac{1}{\sqrt{2}}\right)$

m) $3^{2x} - 2(3^x) - 15 = 0$

n) $\sin^2 x - 2\sin x - 3 = 0 \ (0 \le x \le 2\pi)$

o) $\cos 2x = \cos x \ (0 \le x \le 2\pi)$

p) $\sqrt{2}\tan x \cos x = \tan x \ (0 \le x \le 2\pi)$

q) $2\cos 2x = 1 \ (0 \le x \le 2\pi)$

r)

12. Solve.

a) $x(x+1)(x-2)(x-4) > 0$

b) $(x+7)^2(x-3)^3 < 0$

c) $2x^3 + 3x^2 - 11x \ge 6$

d) $\frac{x+2}{x^2-9} \ge 0$

e) $\frac{5}{x+3} + \frac{3}{x-1} < 0$

13. Prove.

a) $\cos\theta + \sin\theta = \frac{1+\tan\theta}{\sec\theta}$

b) $\frac{1}{1-\sec\theta} + \frac{1}{1+\sec\theta} = -2\cot^2\theta$

c) $\cos^2 2\theta - \cos^2\theta = \sin^2\theta - \sin^2 2\theta$

d) $\sin\left(\pi + x\right) + \cos\left(\frac{\pi}{2} - x\right) + \tan\left(\frac{\pi}{2} + x\right) = -cotx$

e) $\frac{\sin\left(\pi - x\right)\cos\left(\pi + x\right)\tan\left(2\pi - x\right)}{\sec\left(\frac{\pi}{2} + x\right)\csc\left(\frac{3\pi}{2} - x\right)\cot\left(\frac{3\pi}{2} + x\right)} = \sin^4 x - \sin^2 x$

f) $\cos\left(x+y\right)\cos\left(x-y\right) = \cos^2 x + \cos^2 y - 1$

14. If $\log_b a = \frac{1}{x}$ and $\log_a \sqrt{b} = 3x^2$, show that $x = \frac{1}{6}$.

15. If $h^2 + k^2 = 23hk$, where h>0, k>0, show that $\log\left(\frac{h+k}{5}\right) = \frac{1}{2}\left(\log h + \log k\right)$

Graphs

16. Determine whether each of the following functions are even, odd or neither.

a) $f\left(x\right) = \frac{1}{x^3+1}$

b) $g\left(h\right) = 2x^4 + 3x^2$

c) $h\left(x\right) = \left(\frac{1}{x^3+x}\right)^5$

17. Graph the following functions. Determine and label all key features.

a) $y = -x(x-3)(x+4)$

b) $y = (x-2)^2(x+3)^3$

c) $y = \dfrac{-2}{x-1}$

d) $y = \dfrac{5x-3}{2x+1}$

e) $y = 3^{x+2} - 1$

f) $y = \log_2(8x^2)$

g) $y = 2\sin\left(x - \dfrac{\pi}{3}\right)$, $(-2\pi \le x \le 2\pi)$

h) $y = \cos\left(\dfrac{1}{2}x + \dfrac{\pi}{4}\right) - 1$, $(-2\pi \le x \le 2\pi)$

18. When is the function, $f(x) = \dfrac{4}{x-1} - 3 + \dfrac{-3x^2}{5-4x-x^2}$, below the horizontal asymptote?

19. State the range, period, amplitude, phase shift and equations of the asymptotes for each of the following functions for $0 \le x \le 2\pi$. (State only the properties that each function has.)

a) $y = -3\cos\left(3x - \dfrac{\pi}{4}\right) - 2$

b) $y = \cot\left(x - \dfrac{\pi}{6}\right)$

Applications

20. When $x^4 - 4x^3 + ax^2 + bx + 1$ is divided by (x-1), the remainder is 7. When it is divided by (x+1), the remainder is 3. Determine the values of a and b.

21. An open box, no more than 5 cm in height, is to be formed by cutting four identical squares from the corners of a sheet of metal 25 cm by 32 cm, and folding up the metal to form sides. The capacity of the box must be 1575 cm^3. What is the side length of the squares removed?

22. Consider all rectangles with an area of 200 m^2. Let x be the length of one side of the rectangle.
    a) Express the perimeter as a function of x.
    b) Find the dimensions of a rectangle whose perimeter is 70 m.

23. Determine the intercepts, holes and the equations of all asymptotes with behaviour of $y = \dfrac{x^3 - 2x^2 - x + 2}{x^2 - x - 6}$ then sketch.

24. Estimate instantaneous rate of change of each function at the given x value using a centered interval of ± 0.001.

a) $f(x) = x^3 + x^2$ at $x = 2$

b) $f(x) = -x^4 + 1$ at $x = 3$

25. The population of a town is modelled by $P(t) = 6t^2 + 110t + 3000$, where P is the population and t is the number of years since 1990. Find the average rate of change in population between 1995 and 2005.

26. Energy is needed to transport a substance from outside a living cell to inside the cell. This energy is measured in kilocalories per gram molecule, and is given by: $E = 1.4(\log C_1 - \log C_2)$, where C_1 represents the concentration of the substance outside the cell and C_2 represents the concentration of the substance inside the cell.
    a) Rewrite the formula as a single logarithm.
    b) Find the energy needed to transport the exterior substance into the cell if the concentration of the substance outside the cell is double the concentration inside the cell.
    c) What is the sign of E if $C_1 < C_2$? Explain what this means in terms of the cell.

27. A ferris wheel with a radius 10 m makes 2 rotations in 4 minutes. What is the speed of the ferris wheel in meters per second.

28. A circular arc has length 3 cm, and the radius of the circle is 2 cm. What is the measure of the angle subtended by the arc, in both radians and in degrees?