Unit 1 – Functions
Lesson 1: Review:
Part I: Functions, Domain, and Range

A **relation** is any set of ordered pairs (x, y) relating an independent variable (typically x) to a dependent variable (typically y).

For example:

a) $y = 3x + 2$ is the equation for a set of points – linear relation (Grade 10)

b) $\{(0,1), (3,4), (2,-5)\}$ is a set of ordered pairs – scattered points

Domain: is the set of all possible values for the independent variable, x.

Range: is the set of all possible values for the dependent variable, y.

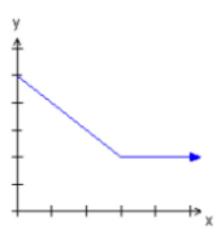
A **function** is a special type of relation where each value of the independent variable, x, yields ONLY a single value of the dependent variable, y.

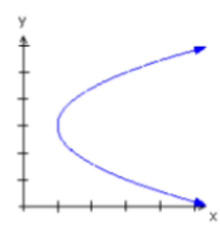
If you are presenting with:

a) Set notation: No x-value is repeated. Examples: $\{y| -5 \leq y < 3, x \in R\}$ $\{x|x = \pi k, k \in Z\}$

b) Graph: If any vertical line passes through more than one point on the graph of a relation, it is not a function – This is known as the vertical line test.

Example:





c) Equation: Rearrange for y and ensure there is only a single value produced for any x.
Example: Does the equation represent a function?

$\frac{x}{y} = 1 + x$ $x^2 + y^2 = 9$

The equation of a relation which is a function can be written using a special notation, such as $f(x) = 3x + 2$.
Under different scenario, h(t) represent a relationship between height in terms of time.

d(t) represent a relationship between distance in terms of time.

Practice from Textbook:
Pg11 / #1, 2, 3, 4, 6, 8, 11, 12, 14

Part II: Properties of Graphs of Functions

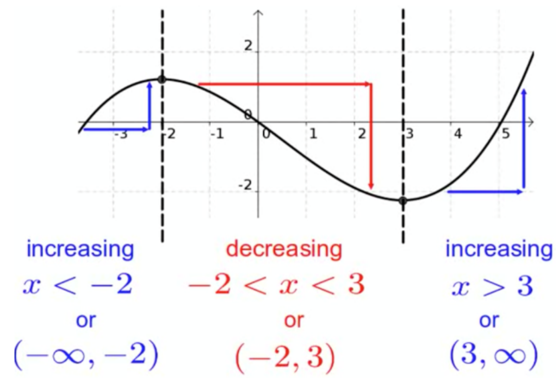
There are 7 properties/key features of functions which can be used to classify and compare them between each other:

1) Domain and range

2) x-intercepts (zeros) and y-intercept

3) Location of any discontinuities (e.g., asymptotes, holes), otherwise the function is continuous

4) Intervals of increase or decrease (always read graph left-to-right, x-values increasing)

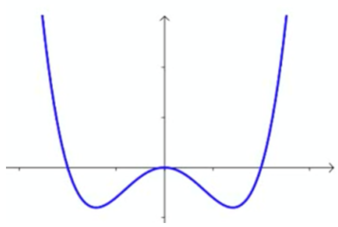


5) Turning points occur where functions change from increasing to decreasing, or vice versa

   i.e., local maximum/minimum & absolute maximum/minimum

6) Function symmetry

   a) Even symmetry can be seen graphically as a mirror image across the y-axis.



Any point (x,y) has a corresponding point (-x,y)

Algebraically:
$$f(-x) = f(x)$$

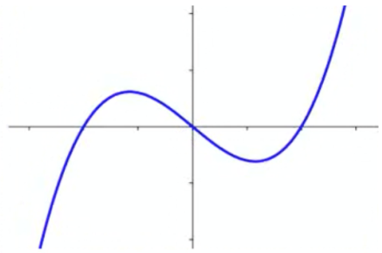
Example: Show that $f(x) = x^2(x-2)(x+2)$, shown above is an even function.

b) Odd symmetry is more difficult to see in the graph, as it represents a rotational, rather than reflective, symmetry about the origin.

Any point (x,y) has a corresponding point (-x,-y).

Algebraically,

$$f(-x) = -f(x)$$



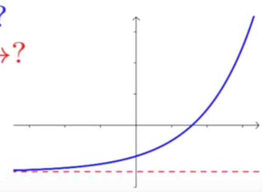
Example: Show that $g(x) = x\,(x-2)(x+2)$, shown above is an odd function.

7) End Behavior describes the tendency of the y-values as x-values approach very large positive and negative values (which we express abstractly as infinity)

Example: Describe the end behavior of $\mathrm{f(x)} = 2^x - 3$

as $x \to \infty$, $y \to ?$

as $x \to -\infty$, $y \to ?$

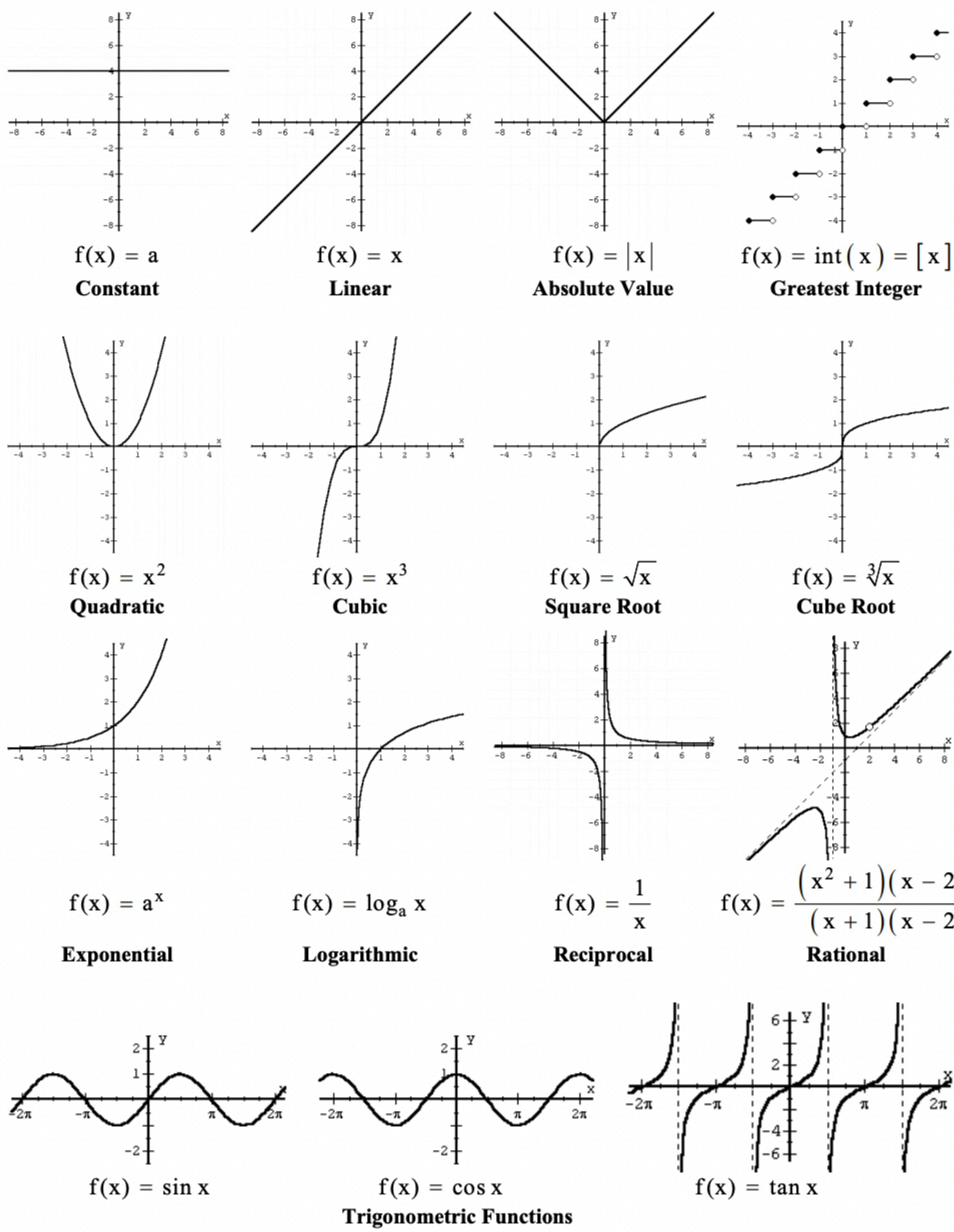


Suggested work from Textbook:
Pg23/ #3, 4, 5, 7, 8, 9, 10, 12, 14

Unit 1 – Functions

<u>Lesson 2: Transformation of Functions</u>

PARENT FUNCTIONS



Determining Transformed functions from graphs and equations:

$$y = af[k(x - d)] + c$$

For any single point, the transformations can be summarized as:

$$(x, y) \rightarrow (\frac{x}{k} + d, ay + c)$$

Example 2: Reasoning about the order of transformations

Describe the order in which you would apply the transformations defined by $y = -2f(3(x + 1)) - 4$ to $f(x) = \sqrt{x}$. Then state the impact of the transformations on the domain, range, intervals of increase/decrease, and end behaviours of the transformed function.

Suggested work from Textbook:
Pg36 / # 4, 5, 6, 7, 9, 10, 15, 16ab