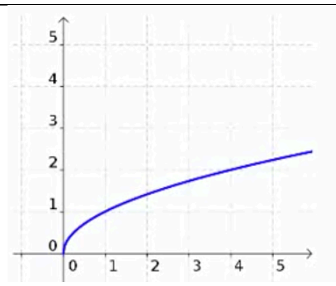
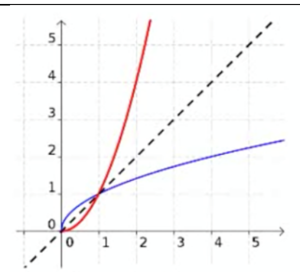
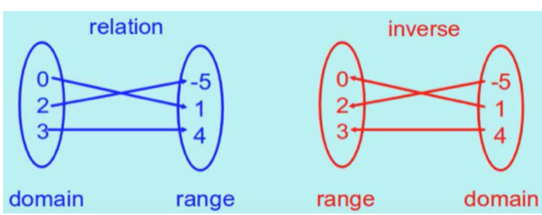
Unit 1 – Functions
Chapter 1.5: Inverse Relations & Functions

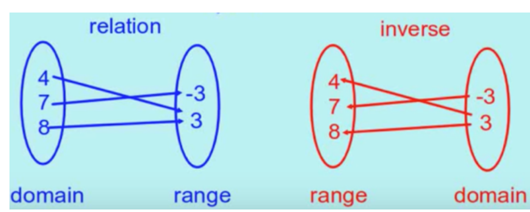
The inverse of a relation can be found by interchanging the domain and range of the relation (i.e., swap x and y).

	Original relation	Inverse relation
Points	{ (a, b), (c, d) }	{ (b, a), (d, c) }
Equation	Express in terms of independent and dependent variables	Swap x and y, rearrange to y = ……
Graph		 reflect in line y = x

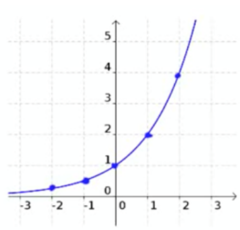
A <u>mapping diagram</u> can be used to determine if a relation is a function.

If there is only one arrow from each item in the domain, then it is a function.

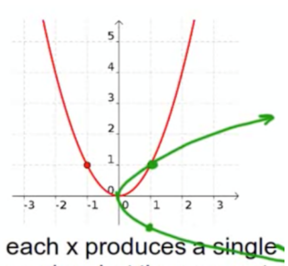




Relation is a function   Inverse is a function   Relation is a function   Inverse is NOT a function



each x produces a unique y-value, inverse is a function



each x produces a single y-value, but they are not unique: inverse is not a function

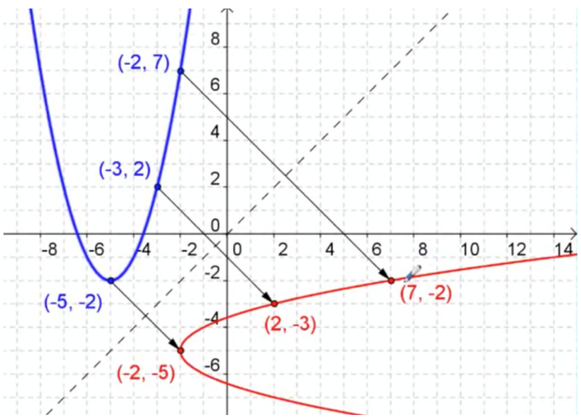
If the inverse of the function, *f(x)* is also a function, it is given the special designation of inverse function, $f^{-1}(x)$.

Note: In the inverse notation, the "-1" is not an exponent!

A function and its inverse function undo each other.

Given $f(a) = b, then\ f^{-1}(b) = a$ (assuming the inverse is a function)

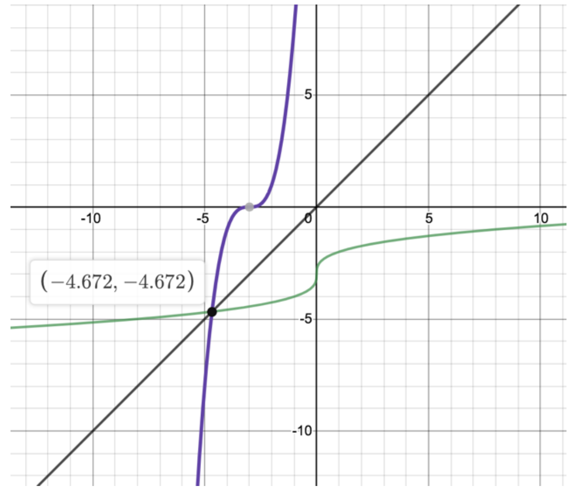
Example: Find the inverse of $y = (x + 5)^2 - 2$ graphically and algebraically.



State restrictions on the domain or range of f(x) so that its inverse is a function.

Suppose that the domain of f(x) is $2 \leq x < 5$. What is the domain of its inverse?

Practice: Find the inverse of $y = \sqrt[3]{x} - 3$.



Example:

If $f(x) = kx^3 - 1$, $f^{-1}(15) = 2$, find k.

Example:

Determine all linear functions $f(x) = ax + b$ such that if $g(x) = f^{-1}(x)$ for all values of x, then $f(x) - g(x) = 44$ for all values of x. (Note: f^{-1} is the inverse function of f.)