2.8 The Derivative of Sine and Cosine

$y = \sin x$ $\quad \frac{dy}{dx} = \cos x$

$y = \cos x$ $\quad \frac{dy}{dx} = -\sin x$

$y = \sin u$ $\quad \frac{dy}{dx} = \cos u \cdot \frac{du}{dx}$

$y = \cos u$ $\quad \frac{dy}{dx} = -\sin u \cdot \frac{du}{dx}$

$y = \sin f(x)$
$y' = \cos f(x) \cdot f'(x)$

$\sin^2 x = (\sin x)^2$

Ex. 1 Differentiate each of the following.

a) $y = \sin(5x)$

$y' = \cos(5x) \cdot 5$

$= 5\cos(5x)$

b) $y = \sin^4(3x)$

$= [\sin(3x)]^4$

$y' = 4[\sin(3x)]^3 \cdot \cos(3x) \cdot 3$

$= 12\cos(3x)\sin^3(3x)$

c) $y = e^{\cos 3x}$

$y' = e^{\cos(3x)} \cdot (-\sin(3x) \cdot 3)$

$= -3\sin(3x)\, e^{\cos(3x)}$

d) $y = (\sin x - 2\cos x)^2$

$y' = 2(\sin x - 2\cos x)(\cos x + 2\sin x)$

e) $y = \dfrac{\sin x - \cos x}{\sin x + \cos x}$

$f' = \cos x + \sin x$

$g' = \cos x - \sin x$

$$y' = \frac{(\cos x + \sin x)(\sin x + \cos x) - (\sin x - \cos x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{2\sin x\cos x + \cos^2 x + \sin^2 x - (2\sin x\cos x - \sin^2 x - \cos^2 x)}{(\sin x + \cos x)^2}$$

$$= \frac{2\cos^2 x + 2\sin^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{2(\cos^2 x + \sin^2 x)}{(\sin x + \cos x)^2}$$

$\cos^2 x + \sin^2 x = 1$

$$= \frac{2}{(\sin x + \cos x)^2}$$

Ex. 2 Determine the equation of the tangent to $f(x) = -2\sin x$ at the point where $x = \frac{\pi}{6}$.

Need

Line in form $y = mx + b$

↳ m → Derivative → Sub $x = \frac{\pi}{6}$

↳ b → find $f(\frac{\pi}{6})$ → Sub point & m into equation and solve

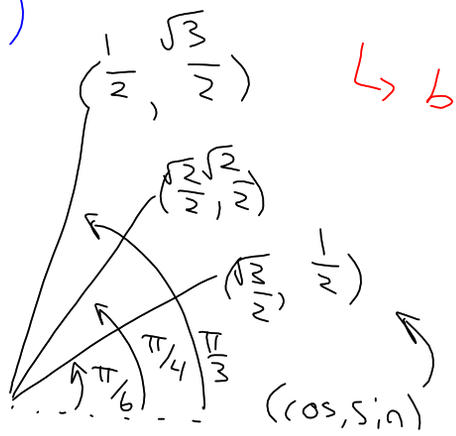
GRAPH

$f'(x) = -2\cos x$

$m_{tan} = -2\cos(\frac{\pi}{6})$

$= -2(\frac{\sqrt{3}}{2})$

$= -\sqrt{3}$



$f(\frac{\pi}{6}) = -2\sin\frac{\pi}{6}$

$= -2(\frac{1}{2})$

$= -1$

Sub $(\frac{\pi}{6}, -1)$ and $m = -\sqrt{3}$ into $y = mx + b$

$\therefore \quad -1 = -\sqrt{3}(\frac{\pi}{6}) + b$

$\frac{\sqrt{3}\pi}{6} - 1 = b$

$\therefore y = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6} - 1$

Ex. 3 Is there a difference between the derivatives of $y = \cos x^2$ and $y = \cos^2 x$?

$y = \cos x^2$

$y' = -\sin x^2 \cdot 2x$

$= -2x \cdot \sin x^2$

$y = \cos^2 x$

$= (\cos x)^2$

$y' = 2\cos x \cdot (-\sin x)$

$= -2\cos x \sin x$

$\therefore$ YES they are different.

Homework
page 256 #1 odd, 2 odd, 3de, 5, 8, 10

$$\frac{1}{n}\sin x = ?$$

$$\frac{1}{n}\sin x =$$

$$six = 6$$