# 2.7 The Derivative of $y = e^x$ and $y = b^x$

Derivative of $y = e^x$ :

Proof: $f(x) = e^x$

$$f(x+h) = e^{x+h}$$

$$f'(x) = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= \lim_{h\to 0} \frac{e^x\left(e^h - 1\right)}{h}$$

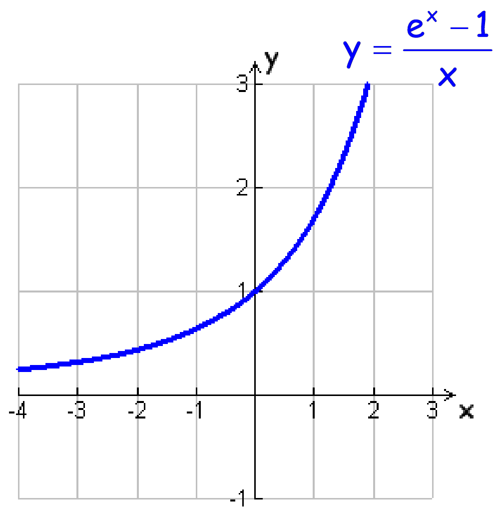
ln e

$$= e^x \cdot \lim_{h\to 0} \frac{e^h - 1}{h}$$

$$= e^x \cdot 1$$

$$= e^x$$

Look over here!




The derivative of $y = e^x$ is $\frac{dy}{dx} = e^x$.

The derivative of $y = e^{f(x)}$ is $\frac{dy}{dx} = e^{f(x)} \cdot f'(x)$.

Chain rule!

Ex. 1 Determine the derivative.

a) $f(x) = e^{5x}$

$f'(x) = e^{5x}(5)$
$= 5e^{5x}$

b) $g(x) = e^{3x^2-5x+2}$

$g'(x) = e^{3x^2-5x+2}(6x-5)$

c) $y = 3x^2 e^x$ Product rule!

$y' = 6x e^x + 3x^2 e^x$

d) $h(t) = \underbrace{3et^2}_{(3e)t^2} + e^{-2t}$

$h'(t) = 6et + e^{-2t}(-2)$
$= 6et - 2e^{-2t}$

$\frac{f'g - fg'}{g^2}$

e) $h(x) = \frac{e^x}{x}$ Quotient!

$h'(x) = \frac{e^x \cdot x - e^x(1)}{x^2}$

$= \frac{e^x(x-1)}{x^2}$

f) $k(m) = \frac{1 + e^{2m}}{2e^m}$

$k'(m) = \frac{e^{2m}(2)\cdot(2e^m) - 2e^m(1+e^{2m})}{(2e^m)^2}$

$= \frac{\cancel{2e^m}(2e^{2m} - 1 - e^{2m})}{(2e^m)^{\cancel{2}}}$

$= \frac{e^{2m} - 1}{2e^m}$

Recall: $\boxed{\frac{d}{dx}a^x = a^x \ln a}$

$$\boxed{\frac{d}{dx}a^{f(x)} = a^{f(x)}\ln a \cdot f'(x)}$$

Ex. 3 Determine the derivative.

a) $y = 7^x$

$y' = 7^x \cdot \ln 7 \cdot (1)$

b) $y = 5^{3x}$

$y' = 5^{3x} \cdot \ln 5 \cdot (3)$

$= 3\ln 5 \cdot 5^{3x}$

c) $y = 3(2)^{x+1}$

$y' = 3(2)^{x+1} \cdot \ln 2 \cdot (1)$

$= 3\ln 2 \cdot 2^{x+1}$

d) $y = 11^{5x^2 - 7x + 3}$

$y' = 11^{5x^2-7x+3} \cdot \ln 11 \cdot (10x - 7)$

e) $f(x) = 50\,(1.02)^{4x}$

$f'(x) = 50(1.02)^{4x} \cdot \ln 1.02\ (4)$

$= 200 \ln 1.02\ (1.02)^{4x}$

f) $f(x) = 7e^x$

$f'(x) = 7e^x$

g) $f(x) = \dfrac{2^{4x}}{x^3}$

$$f'(x) = \frac{2^{4x} \cdot \ln 2 \cdot 4 \cdot x^3 - 2^{4x} \cdot 3x^2}{x^6}$$

$$= \frac{2^{4x}\cancel{x^2}(4\ln 2 \cdot x - 3)}{x^{\cancel{6}\,4}}$$

$$= \frac{2^{4x}(4\ln 2 \cdot x - 3)}{x^4}$$

$f'g + fg'$

h) $f(x) = x^3 \cdot 3^x$

$f'(x) = 3x^2 \cdot 3^x + x^3 \cdot 3^x \ln 3$

$= x^2 \cdot 3^x (3 + \ln 3 \cdot x)$

Ex. 4 Determine the equation of the tangent to $f(x) = 3^x$ where $x = 2$.

$f(2) = 9$

$\therefore (2, 9)$

$f'(x) = 3^x \ln 3$

$m_{tan}\Big|_{x=2} = 3^2 \ln 3$

aka

$f'(2) = 9 \ln 3$

Sub $(2,9)$ and $m = 9\ln 3$ into $y = mx + b$

$9 = 9\ln 3(2) + b$

$b = 9 - 18\ln 3$

$\therefore y = 9\ln 3 \cdot x + 9 - 18\ln 3$

# Homework
# page 232 #2adef, 3, 7, 8
# page 240 #1, 2bd, 3, 4, 6, 8

**Top ln(e^10) reasons why e is better than pi**

10) e is easier to spell than pi.
9) pi ~= 3.14 while e ~=2.718281828459045.
8) The character for e can be found on a keyboard, but pi sure can't.
7) Everybody fights for their piece of the pie.
6) ln(pi^1) is a really nasty number, but ln(e^1) = 1.
5) e is used in calculus while pi is used in baby geometry.
4) 'e' is the most commonly picked vowel in Wheel of Fortune.
3) e stands for Euler's Number, pi doesn't stand for squat.
2) You don't need to know Greek to be able to use e.
1) You can't confuse e with a food product.

Ex. 2 Determine the equation of the tangent to $f(x) = 3xe^{-x}$ at $x = -1$.

graph