2.3 The Product Rule!

Ex.1 Find the derivative of $h(x)= (x^2+1)(x^3-2)$

$$= x^5 - 2x^2 + x^3 - 2$$

$$h'(x) = 5x^4 - 4x + 3x^2$$

The Product Rule:

If $p(x) = f(x)g(x)$, then $p'(x) = f'(x)g(x) + f(x)g'(x)$.

Leibniz:

If u and v are functions of x, $\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$

Ex.1 (redo) using the Product Rule.

$$h(x) = \underbrace{(x^2+1)}_{f}\underbrace{(x^3-2)}_{g}$$

$$h'(x) = f'g + fg'$$

$$= (2x)(x^3-2) + (x^2+1)(3x^2)$$

$$= 2x^4 - 4x + 3x^4 + 3x^2$$

$$= 5x^4 + 3x^2 - 4x$$

Proof of Product Rule

$$F(x)=f(x)g(x)$$

$$F'(x)=\lim_{h\to 0}\frac{F(x+h)-F(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\frac{f(x+h)g(x+h)-f(x)g(x+h)+f(x)g(x+h)-f(x)g(x)}{h}$$

$$=\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}g(x+h)+f(x)\frac{g(x+h)-g(x)}{h}\right]$$

$$=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\lim_{h\to 0}g(x+h)+\lim_{h\to 0}f(x)\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$$

$$=f'(x)g(x)+f(x)g'(x)$$

Ex. 2 Find $\frac{dy}{dx}$ if $y = (2x^3+5)(3x^2-x)$

$f \quad g$

$y' = f'g + fg'$

$= 6x^2(3x^2-x) + (2x^3+5)(6x-1)$

$= 18x^4 - 6x^3 + 12x^4 - 2x^3 + 30x - 5$

$= 30x^4 - 8x^3 + 30x - 5$

Ex.3 Differentiate $f(x) = \sqrt{x}\,(2-3x)$ and simplify.

$f(x) = x^{\frac{1}{2}}(2-3x)$

$f \quad g$

$f'(x) = f'g + fg'$

$= \frac{1}{2}x^{-\frac{1}{2}}(2-3x) + x^{\frac{1}{2}}(-3)$

$= x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}} - 3x^{\frac{1}{2}}$

$= x^{-\frac{1}{2}} - \frac{9}{2}x^{\frac{1}{2}}$

$= \frac{1}{x^{\frac{1}{2}}} - \frac{9}{2}x^{\frac{1}{2}}$

Ex.4 Find the slope of the tangent to the graph of the function $f(x) = (3x^2+2)(2x^3-1)$ at the point (1,5).

$s \quad t$

$f'(x) = s't + st'$

$= 6x(2x^3-1) + (3x^2+2)(6x^2)$

$= 12x^4 - 6x + 18x^4 + 12x^2$

$= 30x^4 + 12x^2 - 6x$

Sub in $x = 1$

$f'(1) = 30 + 12 - 6$

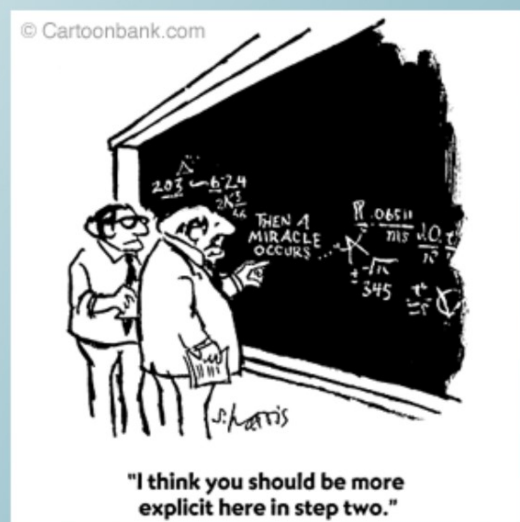
$= 36$

$\therefore m_{tan} = 36$

Extended Product Rule for a product of three functions

NOTE: If $p(x)=f(x)g(x)h(x)$ then

$p'(x)= f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$



Hmk.
Pg. 90 # 1abcde,4,5abcf,6,7,12